MANUAL OF SURVEYING

FOR

FIELD AND OFFICE

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BY

RAYMOND E. DAVIS, C. E.
INSTRUCTOR IN CIVIL ENGINEERING
UNIVERSITY OF ILLINOIS
MEMBER SOCIETY FOR PROMOTION
OF ENGINEERING EDUCATION

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PREFACE

This book is a manual of the practice of surveying designed for the use of civil engineering students in their preparation for field and office exercises. Primarily it is intended for use in conjunction with a treatise on surveying if the field and office practice is developed with the theory, or for use alone if practice is given after the theory has been studied. It is felt that the book is also adapted for use in short elementary courses in surveying in which the time devoted to field and office work is comparatively long, such as the courses usually given to mechanical and electrical engineering students.

The aim in the preparation of this volume has been to provide an efficient means of teaching students the proper use of surveying instruments, the proper procedure in making surveys, and the proper methods of computing and mapping. Attempt has been made to treat the practical phases of the subject in great detail, for the author's experience in teaching leads him to believe that superior results are attained only when the student is enabled before beginning a work to see clearly its purpose, a method of accomplishing it, and the reason for using such method. Much of the material of the book has been suggested to the author through questions asked and mistakes made by students and inexperienced surveyors over a period of several years.

The first chapter is designed to prepare the student for the practice of surveying as developed in the succeeding chapters. It deals with general instructions for field and office work, and emphasizes the more difficult and important parts of surveying, such as note keeping, precision of measurements, consistent accuracy, and efficient methods of computing.

The remainder of the book, through the explanation of problems of field and office, progressively builds up the practice of surveying from elementary work with tape to complete topographic surveys.

Attention is called to the following features:

1. The book is of a size and weight to be conveniently carried in the coat pocket.

2. The explanation of each problem is complete in itself, unless some portion has been developed in a prerequisite problem; and therefore there is no necessity for seeking elsewhere in the book for additional information concerning methods, the use of instruments, etc.

3. The suggestions following each problem contain much information not usually included in text-books, but which is usually brought to the attention of the inexperienced surveyor either by reason of his own blunders or through oral instructions. It is believed that these suggestions will be particularly helpful in field classes where the parties are so scattered that it is impossible for each student to receive the constant attention of an instructor.

4. The precisions of measurements necessary for various surveys have been emphasized by establishing limits of error or by stating the errors allowed in practice.

5. Where there is more than one method of performing a surveying operation and the methods are not radically different, the variations follow the detailed explanation of the most common case and are compared one with another.

6. The practical applications of field and office exercises and the dependence of methods on conditions are brought to the attention of the student.

7. Emphasis is placed on methods of checking, especially those methods that require little additional labor and may be applied as the work progresses.

8. The sample pages of notes and computations—of which there are a liberal number—are examples of clear, systematic forms used in actual practice, and are not unduly elaborate.

9. The problems in the determination of latitude, longitude, and azimuth may be performed with little or no previous knowledge of astronomy. The methods explained give results sufficiently accurate for any but the most precise surveys.

10. The book contains all tables which will ordinarily be of use in plane and topographic surveying. The Polaris tables make the use of the Nautical Almanac unnecessary, and render the determination of the meridian a very simple process.

11. The method of explaining the relation between size of angle, angular error, and ratio of precision of the trigonometric functions has not, to the author's knowledge, before appeared in any text-book. This important phase of the

precision of measurements is usually not well understood, even by the experienced surveyor.

12. In the description of the adjustments of the transit and level, distinction has been made between the type of instrument having adjustable objective slide and the type in which the objective slide is permanently fixed as far as lateral motion is concerned. Also, in the description of the adjustments of the plane-table alidade, distinction is made between the type of alidade having telescope tube rigidly attached to the horizontal axis and the type having a collar in which the telescope may be turned about its axis.

13. The plane-table, as the most efficient instrument for general topographic surveying, has received due consideration, and plane-table methods for large-scale work have been given more attention than is usually accorded them.

14. The portions of the book dealing with fieldwork and office work are closely co-ordinated, problems in computing and drafting in general being the continuation of field problems.

15. The cross references to related subject matter are numerous; the running titles are suggestive of page contents; and the index is sufficiently complete to make ready reference possible.

The author is indebted to Prof. George L. Hosmer, Mr. H. H. Jordan, and Mr. W. H. Rayner for valuable criticisms of portions of the manuscript; and to Mr. A. R. Alger for a rough draft of a considerable number of the problems and notes of Chapters III and IV.

Much credit is due Prof. Ira O. Baker for his keen interest in the work, for the painstaking care with which he reviewed the manuscript, and for valuable suggestions without which the book would have been very imperfect.

Acknowledgments are also due Mr. O. H. Tittman, Superintendent of the U. S. Coast and Geodetic Survey, for furnishing data for the Polaris tables; John Wiley and Sons for permission to use Table X; and Mr. C. W. Reinhardt for the careful preparation of the sample pages of notes and computations and of the plates of letters.

R. E. D.

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CONTENTS

CHAPTER I

FIELD AND OFFICE WORK

SECTION 1. THE FIELD NOTE-BOOK

SECTION 2. FIELD PROBLEMS

SECTION 3. SURVEYING INSTRUMENTS

SECTION 4. OFFICE COMPUTATIONS

CHAPTER II

ELEMENTARY FIELD PROBLEMS

CHAPTER III

COMPASS AND TRANSIT PROBLEMS

CHAPTER IV

LEVEL PROBLEMS

CHAPTER V

USE OF THE PLANE-TABLE AND SEXTANT

CHAPTER VI

FIELD ASTRONOMY

CHAPTER VII

TOPOGRAPHIC SURVEYING

CHAPTER VIII

OFFICE PROBLEMS

LIST OF TABLES

MANUAL OF SURVEYING
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FIELD AND OFFICE

CHAPTER I

FIELD AND OFFICE WORK

SEC. 1. THE FIELD NOTE-BOOK

1. **Kind of Book.**—The note-book should be of good quality paper, with stiff board or leather cover, made to withstand hard usage, and of a size convenient to slip in the coat pocket. There are several special field note-books sold by engineering supply companies which are intended for particular kinds of notes. For general surveying or for students in fieldwork where the problems to be done are general in character, an excellent form of note-book has the right-hand page divided into small rectangles with a red line running up the middle, and the left-hand page divided into several columns. In general, tabulated numerical values are to be written on the left-hand page; sketches and explanatory notes on the right.

2. **Field Notes.**—No matter what the character of the survey, all field notes should be taken in the field at the time the work is being done. Notes made later, from memory or copied from other field notes, may be useful but they are not field notes. Notes should be neat and intelligible to anyone having a knowledge of surveying. They are generally recorded in pencil, but are permanent work, and should not be regarded as memoranda to be used only for the time being or in the immediate future.

The observer should also realize that the notes will very likely be used by other persons not familiar with the locality; they must rely entirely upon what he has recorded. For this reason not only should the note-book contain all necessary information, but data should be recorded in a form which will admit of only one interpretation, and that the correct one.

A good sketch will help more than anything else to convey a correct impression to others, and for this reason sketches should be used freely. In general, sketches are not drawn to scale, but in most work it is better to construct them approximately to scale. It is to aid in accurate sketching that the right-hand page of the note-book is divided into rectangles. If the note keeper will bear in mind what use is to be made of the notes, it will aid him greatly in deciding what data are necessary and what are not. The student should understand at the beginning of his work that it is not an easy thing to take good notes. It is possibly the most difficult thing he will be called upon to do. Too many beginners seem to think that a page of neat letters, figures, and lines constitute good notes; but although neatness is one of the primary requisites, it must be borne in mind that neatness alone can not make the record clear and complete. To make the notes clear, the student should strive to put himself in the place of one who is not on the ground at the time the survey is made; and to make the notes complete, he should thoroughly understand for what purpose they are to be used. He should strive at all times to make the notes on the present problem clearer, neater, and more complete than the notes of the previous problem. Before making any survey, the necessary data to be collected should be carefully considered; and when doing the fieldwork, all such data should be obtained, but no more.

Various problems throughout the following pages are accompanied by forms of notes similar to those used by many engineers. The student should not look upon these as standard, but rather as examples of clear notes, and should feel free to introduce a form of his own if he believes his system is the better. With those problems unaccompanied by sample pages of notes, the student will be expected to exercise his own ingenuity.

3. **Suggestions for Keeping Field Notes.**—Use a 4H pencil and keep it well pointed. Make a neat title either on the flyleaf or on the cover, showing the owner of the note-book, the number and name of the course, and the year in which the notes are taken. Leave several pages in the front of the book for an index. Show in the index, the number, name, date, and pages of each problem done in the field. Also show whether or not the work has been accepted by the instructor. Always keep the index up to date.

Take all notes in the note-book while in the field. Leave nothing to memory or guesswork.

For each problem make a title at the top of the page, showing name and number of problem, date, weather, names of the members of the party and the duty of each, and time occupied in doing the fieldwork. Use the left-hand page for a tabulated list of measurements; the right-hand page for sketches and explanatory notes.

If a page of notes is abandoned for any reason, do not obliterate the notes or cut out the page, but write diagonally across the page in large letters "*abandoned*" and designate the page number of the continuation of the notes. Also show same in the index. Notes may be abandoned because they are illegible or because they contain such erroneous or useless data that another survey is necessary.

Do not change notes without first being *certain* that a change should be made, and above all, go at the work with a truly scientific spirit, being absolutely honest with yourself.

Do not make the notes appear *more* accurate or *less* accurate than they really are.

In general do not erase numerical data. If a number is in error, draw a line through it and write the corrected value above. Portions of sketches and explanatory notes may be erased when there is a good reason for doing so. Make the sketches freehand and of liberal size.

REINHARDT'S style of slant lettering is generally conceded to be the best form of lettering for taking notes rapidly and neatly, because of its simplicity. (See Fig. 48, page 223, for examples of form.)

4. Explanatory Notes.—The purpose of explanatory notes should be to make clear that which the numerical data and sketches fail to do. In some surveys explanatory notes entirely take the place of sketches, in which case they are placed on the right-hand page in the same line with numerical data they explain. In other surveys they are used in conjunction with sketches and numerical values, and are placed in such position as not to interfere with other data. Sketches themselves frequently contain features which are only intelligible through some note. When used in this capacity, notes should be placed in such position on the page with the sketch as not to interfere with other figures or letters. At the same time they should be placed in close proximity to that which they explain. Certain abbreviations of the more common artificial features met with by the surveyor should be utilized by the student. A few are enumerated below:

stk.	Stake.	*cf.*	Crow-foot.
tk.	Tack.	*c.cf.*	Cut crow-foot.
mon.	Monument.	*C*	Center.
S.B.	Stone bound.	℄	Center line.
d.h.	Drill-hole.	*sec.cor.*	Section corner.
tel.	Telegraph pole.	*¼ sec.cor.*	Quarter section corner.
spk.	Spike.	*twp.cor.*	Township corner.
na.	Nail.	△	Triangulation station.
cb.	Curb.	⊡	Stadia station.
C.B.	Catch basin.	⊙	Transit traverse station.
M.H.	Manhole.	*B.M.*	Bench mark.
b.b.	Base-board.		

5. **Numerical Data.**—The figures used should be plain; one figure should never be written over another. If the numerical data are to be tabulated in columns on the left-hand page, place the numbers so that all figures in the tens column will be on a straight line, thus at a glance one can tell what numbers are in tens or in hundreds or in thousands. Where decimals are used, the decimal point should never be omitted; and the number should always show with what degree of precision the measurement was taken. Thus an angle of 64° when measured to the nearest minute should be recorded 64° 00′ and a distance measured to the nearest 0.1 ft. should be recorded not as 142.20 ft. but as 142.2 ft. Numbers placed on sketches should always be in such position as to clearly indicate to what they refer. Frequently this is impossible except by the aid of dimension lines or arrows.

6. **Sketches.**—Sketches are very rarely made to scale, but in many cases they are made approximately to scale. In other cases, however, certain distances are exaggerated for the sake of clearness. Sketches should be made large enough to prevent crowding of details. Paper is cheap, and clearness of details will often save hours of time in the office.

A sketch should be made when data can not be clearly described by *short* explanatory notes, or when it will aid in any way in the correct interpretation of the notes. What the sketch should show must be determined by the note keeper, for the most part before a line is drawn. To make good sketches one must not only be able to judge distances and angles with a fair degree of accuracy, but one must also know what features to include. A sketch crowded with unnecessary data is often very puzzling even though all necessary features are included. Culture and many of the public and private works may be most readily shown by conventional signs (Figs. 44, 45, 46, and 47, pages 218–222).

Sec. 2. Field Problems

7. Fieldwork.—It is not possible, in the ordinary course in surveying, to develop the student into an expert instrumentman; however, it is expected that the course will give the student a working knowledge of surveying instruments and their uses. The work is not always carried out as it would be in actual practice, because of the brief time allowed for field problems and because during this time it is necessary for all students to obtain as much practice as possible in the manipulation of field instruments and the taking of notes. In elementary fieldwork no long surveys of any sort are attempted, but a number of short problems are taken up which might in practice become parts of extended surveys.

Members of the student field parties should alternately assume the various duties involved in the field problems, but during any single period should not change positions. The ability to hold the rod properly should be looked upon as being just as essential as the knowledge of how to manipulate the level, for intelligent direction of work makes necessary a thorough understanding of all its details.

8. Study the Problems.—Before going into the field, the student should understand exactly what he is to do and why he is to do it. This can be accomplished only by a thorough study of the problem for the day, noting first the object of the problem, and then conducting a critical examination of the course of procedure. In his mind he should go through the various steps involved until he has completely mastered them, so that while in the field he may spend all his time and attention in putting into practice that of which he has already learned the theory. A student who fails to do this is seriously handicapped in his fieldwork, and, though he may work conscientiously while in the field, will have but a superficial knowledge of what he is doing. The problems should be regarded as instructions which are to be rigidly followed, and a thorough understanding of the work to be accomplished is as essential as the skill to carry it out. After making a thorough study of the problem the student should prepare a list of the equipment (instruments, stakes, etc.) necessary for its performance.

9. Habit of Correctness.—The student should aim to do his work thoroughly, introducing checks wherever possible, and no measurement should be regarded as correct until verified. In as far as it is practicable, methods of verification should differ from the methods used in original measurements. All persons are liable to mistakes, but mistakes in fieldwork become discreditable to the maker if he allows any other than himself to discover the discrepancy. When the student is tempted to neglect his work in this manner in order to save time, he should realize that habits are very easy to form but very difficult to break; and that nothing, unless it be wilful dishonesty, is so injurious to the reputation as habitual carelessness.

10. Consistent Accuracy.—On the other hand extreme accuracy should not be thought necessary at all times, but rather an accuracy consistent with the purposes of the survey. Beginners often fail to comprehend the different degrees of precision necessary for the different kinds of work, or worse yet, fail to maintain a consistent degree of accuracy throughout any one survey. There are no set rules for the relative accuracy of different classes of surveys, nor could there be, for the objects and conditions are too many and too complicated, but common sense can always be resorted to. Thus one can see the absurdity of taking the measurements of farming land, worth only a few dollars an acre, with the same precision as the measurements of small city lots, worth thousands of dollars. The student should then, before beginning a problem, establish the limit of error for himself, using his own common sense and the experience of others to guide him. The best surveyor is not the one who is extremely accurate, but the one who can make a survey just accurate enough to serve its purpose without waste of time or money.

11. Relation Between Angles and Distances.—If measurements are to be consistent, it is evident that the precision of angles should correspond to the precision of related distances —or in other words, the percentage of error in angle should equal the percentage of error in distance. Oftentimes it is impracticable to maintain an exact equality between these two percentages; but with one or two exceptions, which will be considered presently, surveys should be so conducted that the variation between precision of angle and precision of distance will not be large. Since the accuracy of a survey is often judged by the number of significant places in the recorded distances, it is always best to measure angles with a precision at least equal to the precision of the distances. It is common practice before beginning a survey or any distinct portion of a survey to fix the permissible error of linear measurement, this being expressed as a ratio, as 1/1000; sometimes it is desired to locate a given point within a specified distance of its true position. The following table shows for various angular errors the corresponding ratios of precision and the errors for a length of 1000 ft. For a length other than 1000 ft. the linear error is in direct proportion.

Angular error	Linear error in 1000 ft.	Ratio of precision
10′	2.9089	1/344
5′	1.4544	1/688
1′	0.2909	1/3 440
30″	0.1454	1/6 880
20″	0.0970	1/10 300
10″	0.0485	1/20 600
5″	0.0242	1/41 200

To illustrate the use of the preceding table suppose distances are to be chained with a precision of 1/10,000; the corresponding permissible angular error is 20″. Again, the distance from the instrument to a desired point is determined as 660 ft. with a probable error of 2 ft. For an angular error of 10′ the corresponding linear error is $0.66 \times 2.9 = 1.91$ ft. Therefore the angle need be determined only to the nearest 10′.

The *exceptions* referred to in the first part of this article are surveys in which the distances are roughly determined and the angles are measured with more than the required accuracy without increased effort or loss of time. For example, in rough chaining the ratio of precision might be 1/1000, corresponding to an angular error of 03′. But with the ordinary transit, the angles could be determined to the nearest 01′ as quickly as to the nearest 03′.

12. Angles Used in Trigonometric Computations.—Very often field measurements are made the basis of computations involving the trigonometric functions, and it is necessary that the computed results be of a required precision. If the values of these functions were exactly proportional to the size of the angles—or in other words, if any increase in the size of an angle was accompanied by a proportional increase or decrease in the value of a function—the problem of determining the precision of angular measurements would resolve itself into that explained in the preceding article; but since the ratios of the rates of change to the values of the sines of small angles, the cosines of angles near 90°, and the tangents and cotangents of small and large angles are relatively large, it is evident that the precision with which an angle is determined should be made to depend upon the size of the angle and the function to be used in the computations. Usually too little attention is paid to this important phase of the precision of measurements, even by experienced surveyors, and as a consequence computed results are very often assumed to be more nearly correct than they really are. It is not practicable to measure each angle with exactly the precision necessary to insure sufficiently accurate computed values, but at least the surveyor should have a sufficiently comprehensive knowledge of the purpose of the survey and of the properties of the trigonometric functions to keep the angles within the required precision.

The curves of Plates I and II, pages 9 and 10, show the ratios of precision corresponding to various angular errors from 05″ to 01′ for sines, cosines, tangents, and cotangents.* For the function under consideration these curves may be utilized to determine the ratio of precision corresponding to a given angular error and angle, or to determine the maximum or minimum angle that for a given angular error will furnish the required ratio of precision, or to determine the precision with

* This method of representing the effect of angular error on the ratio of precision was suggested by an article in "Engineering Record," Oct. 18, 1913, by Mr. W. H. Rayner.

which angles of a given size must be measured to maintain a required ratio of precision in computations.

The following examples illustrate the use of the curves:

1. In computations involving the use of cosines a ratio of precision of 1/10,000 is to be maintained. It is desired to know with what precision angles must be measured. On the

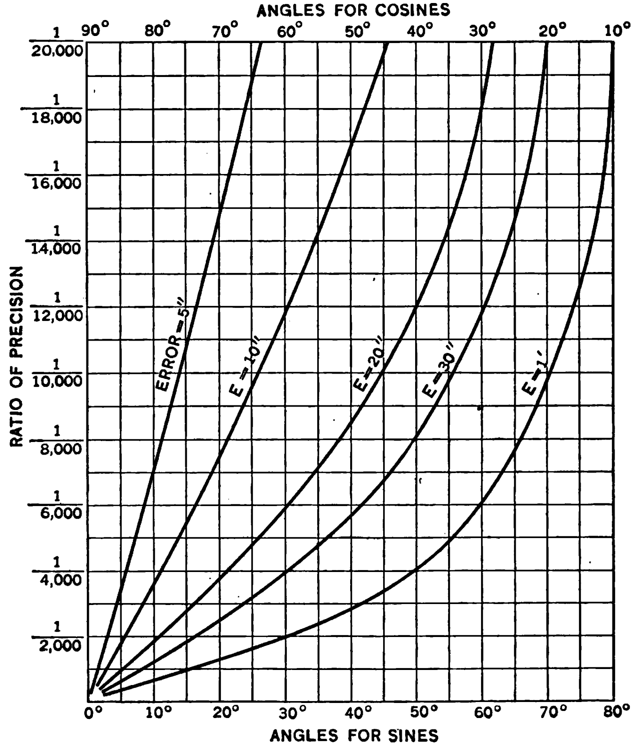


PLATE I.—RATIO CURVES FOR SINES AND COSINES.

Ratio Curves for Cosines opposite 1/10,000 it will be seen that for angles of about 76° the angular error can not exceed 05″, for angles of about 64° the angular error can not exceed 10″, and so on.

2. An angle measured with a one-minute transit is recorded as 32° 00′. The probable error of the angle is 30″. It is

desired to know the ratio of precision of a computation involving the tangent of the angle. On the Ratio Curves for Tangents it will be seen that the ratio of precision opposite the intersection of the curve $E = 30''$ and a line corresponding to 32°, is 1/3000.

3. In a triangulation system the angles can be measured with an error not exceeding 05″. Computations involving the use of sines must maintain a ratio of precision of not less

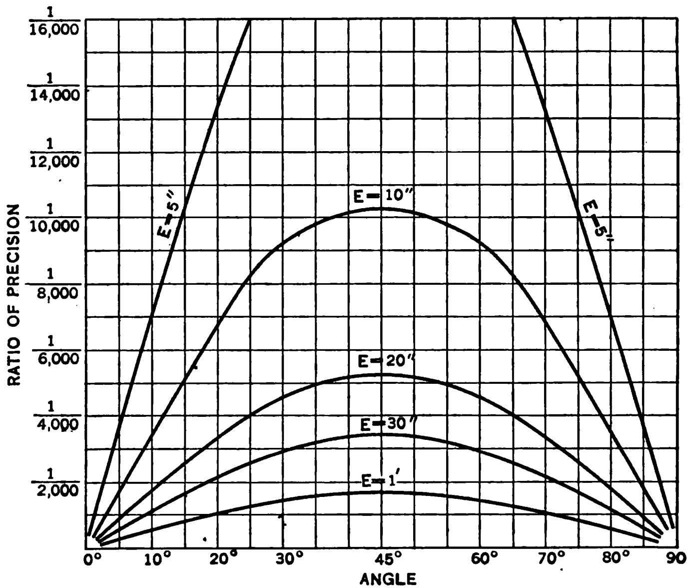


Plate II.—Ratio Curves for Tangents and Cotangents.

than 1/20,000. It is desired to determine the minimum allowable angle. On the Ratio Curves for Sines the angle corresponding to a ratio of precision of 1/20,000 and an angular error of 05″, is 26°.

It should here be noted that linear as well as angular measurements must always be taken with the required precision of the computed results, for no result can be more nearly correct than the data from which it was obtained.

13. **Speed.**—Speed in fieldwork depends to a large extent upon practice in handling instruments; but no amount of

practice will secure rapid work unless the work be carefully planned, systemized, and carried out with consistent accuracy. To attain rapidity, the student should be constantly on the lookout for little ways of saving time. He must have system in every thought and move. If he is handling an instrument, he must know exactly what he is to do some time before he does it. He must acquaint himself with every part of the instrument. The turning of the right screw the right way at the right time should not require a cessation of mental activity from other things, but should be a mere automatic movement. Although rapidity is essential, when obtained at the expense of accuracy it is a thing to be avoided. The surveyor who is really quick in his work is he who never hurries beyond the speed at which he can maintain the required accuracy, but who utilizes all his time to the best advantage.

14. **Signals.**—Except for short distances a good system of hand signals between different members of the party makes a more efficient means of communication than is possible by word of mouth. The number of signals necessary will depend upon the character of the work and the nature of the country. A few of the more common are given below:

"*Right*" or "*Left.*"—The arm is extended in the direction of the desired movement, the right arm being extended for a movement to the right and the left arm for a movement to the left. A long, slow, sweeping motion of the hand indicates a long movement; a short, quick motion indicates a short movement. This signal may be given by the transitman in directing the chainman on line, by the leveler in directing the rodman for a turning-point, by the chief of the party to any member, or by one chainman to another chainman.

"*All Right.*"—Both arms are extended horizontally and the forearms waved vertically. The signal may be given by any member of any party.

"*Plumb the Flag*" or "*Plumb the Rod.*"—The arm is held vertically and moved in the direction that the flag or rod is to be plumbed. It is given by the transitman or leveler.

"*Give a Foresight.*"—The instrumentman holds one arm vertically above his head.

"*Establish a Turning-point*" or "*Set a Hub.*"—The instrumentman holds one arm above his head and waves it in a circle.

"*Give Line.*"—The flagman holds the flag horizontally in both hands above his head and brings it down and turns it to a vertical position. If he desires to set a hub, he waves the flag with one end in the ground from side to side.

"*Turning-point*" or "*Bench Mark.*"—In profile leveling the rodman holds the rod horizontally above his head and then brings it down on the point.

"*Wave the Rod.*"—The leveler holds one arm above his head and moves it from side to side.

"*Pick up the Instrument.*"—Both arms are extended downward and outward, then inward and up, as one would do in grasping the legs of the tripod and shouldering the instrument. It is given by the chief of the party or by the head chainman when the transit is to be moved to another point.

Care should be taken to make the signals so clear that they may be readily understood. Where long sights are taken or where the peculiar color of the background renders hand signals indistinct, colored flags similar to those used by railroad trainmen, may be used to good advantage. Of course the color should be in contrast with that of the background. Red can be seen very well against snow, while white can be distinguished clearly against the dark green of the forest.

SEC. 3. SURVEYING INSTRUMENTS

15. General Remarks.—As the use of the various surveying instruments is made necessary in the field problems explained in the following chapters, suggestions and precautions for the care and manipulation of these instruments are given. These should be heeded, not only because the student is responsible for the equipment he is using, but because he will establish the foundation of a very desirable qualification —that of carefulness.

Poor instruments should not be made the object of excuses for inferior work, instead a special endeavor should be made to maintain the required accuracy. The instruments used by elementary classes in surveying are rarely in good condition, and very seldom in good adjustment owing to the rough handling they receive; but if the problem is properly performed, instrumental errors will be eliminated to a great extent. The student will find later in practice that he will be called upon many times to do good work with poor equipment, and should be glad of an occasional chance of becoming acquainted with poor and defective instruments.

On taking the equipment from the locker, the student should *thoroughly examine each piece*, and in case of loss or injury, should *immediately report the loss to his instructor* so that the liability may be fixed.

16. Care of Transit, Level, and Compass.—(1) Handle the instrument carefully when removing it from its box. (2) See that it is securely fastened to the tripod head. (3) Avoid carrying the instrument on the shoulder while passing through doorways or beneath low-hanging branches. Before climbing over a fence or fallen tree place the instrument on the other side of the obstacle with the tripod legs well spread. (4) Never leave the instrument while it is set up in the street, on the sidewalk, in the vicinity of buildings in the process of construction, or in any other place where there is the slightest possibility of accident. (5) Do not set the tripod legs too close together; and see that they are firmly planted. (6) Tighten the leveling screws until they come to a firm bearing, but not enough to warp the leveling head. *The general tendency is to tighten these screws far more than is necessary.* (7) Always put on the sun shade. Do this by a screw motion in a clockwise direction, thus doing away with any possibility of loosening the objective lens. (8) *When the magnetic needle is not in use, see that it is raised off the pivot.* When the needle rests on the pivot, jars are apt to blunt the point of the pivot or to chip the jewel, thus causing the needle to be sluggish. (10) Dust caps for both the objective and the eyepiece should be put in place and the instrument wiped clean before returning it to the box. (11) If the instrument is not to be returned to its box, see that it is protected from dust with a cloth hood.

17. Care of Small Equipment.—(1) Keep the chain or tape straight when in use. (2) Do not use the flag pole as a bar to loosen stakes or stones. Such usage bends the steel point and soon renders the flag pole unfit for lining purposes. (3) The brass or steel shoe on the foot of the leveling rod should not be struck against hard objects, as this, if continued, will round off the foot of the rod, thus introducing a possible error in leveling work. (4) Long rods when not in use should be placed in an upright position or supported their entire length, for otherwise they are apt to warp. Jointed leveling rods should have all clamps loosened when not in use to allow for possible expansion of the wood. (5) To avoid losing pins, tie a piece of colored cloth (preferably bright red) through the ring of each. (6) Remember that hard knots in the plumb-bob string indicate a slovenly or inexperienced instrument-man; learn to make a sliding bowknot that can be easily undone.

Sec. 4. Office Computations

18. General Remarks.—All computations should be preserved in a note-book for that purpose. This book may be the same as that used for field notes, but preferably it should be larger, say 8″ × 10 ½″. This size will give considerable space for a problem without turning to a new page, and the columns of tabulated values will not have to be crowded as will often be the case with the smaller book. The pages should be divided by light lines into 3/16-in. squares. With such an arrangement columns can be kept straight without additional ruling, sketches can easily be made, and it will be an easy matter to give the computations a neat, systematic appearance.

In general, computations in the office will be a continuation of some fieldwork. They will be required for purposes of obtaining areas, plotting maps, profiles, and cross-sections, calculating dimensions to be laid off in the field, or ascertaining other desired information concerning the survey. It is desirable that these computations be easily accessible for future reference, and for this reason the pages of the book should be numbered and the contents of the book should be shown by a complete index. Parts of problems separated by other computations should be cross referenced. Each problem should have a clear heading which should include the name of the survey, the kind of computations, the field book and number of page of the original notes, the name of the computer, the name of the checker, and the dates of computing and checking. Enough of the field notes should be transcribed to make computations possible without further reference to the field book.

A systematic form should be used, and all calculations should be shown. Similar quantities should be tabulated in columns, and each column should have a heading showing what it contains so that the work may be inspected at a glance. Multiplications, divisions, and logarithmic computations should be neatly and systematically recorded, so that others as well as the computer can trace the solutions without waste of time. Similar problems should be calculated in similar form. The student will soon become accustomed to a particular form, and the likelihood of mental confusion will decrease. Such of the field notes as are needed in the computations should be transscribed to the office book as neatly as possible.

19. Checking.—In practice the necessity of checking every computation is recognized. *No confidence is placed in results that have not been checked*, and important results are checked by more than one method. The student should see the necessity for this, for from experience he knows that a computation of any considerable length is rarely made without some error. He should early form the desirable habit of checking his own work, and should feel certain that his results are reasonably correct before handing them to his instructor. *Students should depend upon themselves.* It is true that students may check results by comparing work, but to do the work together comparing each step as it is completed is not checking in the true sense of the word, and is a weakness to be avoided. Such methods would not be countenanced in practice.

Most problems can be solved by more than one exact method. Since by using the same method in checking, the same error is likely to occur, the student should check his results by a different method when this is feasible. Approximate checks may be obtained in many cases through such mechanical devices as the slide-rule, the planimeter, and the protractor. The use of the slide-rule as a means of approximately checking each step as it is computed can not be too strongly advocated. Large arithmetical mistakes are almost sure to be discovered in this way, though of course mistakes due to confusion of numbers or to wrong methods will not be shown. Graphical methods may often be used, as an approximate check, to good advantage, both for final results and intermediate steps. They generally take less time than arithmetical or logarithmic solutions, and possible wrong assumptions in the precise solution will be avoided.

20. Precision of Computations.—For the most part, data forming the basis of computations in surveying consist of distances and angles determined from measurements in the field. No measurement can be exact; therefore all measurements contain errors of unknown magnitude, though methods of observation will confine these errors within certain limits. If a measurement recorded as 100 ft. contained a probable error of 1 ft., this measurement multiplied by two would contain a probable error of 2 ft., and the ratio of precision would in each case be 1/100; if the 100 ft. with a probable error of 1 ft. were multiplied by 10 ft. containing a probable error of 1 ft. the result could not have a greater ratio of precision than 1/10. From these examples it is obvious that in computations involving multiplication or division the result can not have a higher ratio of precision than that of the factor having the lowest ratio of precision. The computer should therefore bear in mind that results can not be more accurate than the data from which they are obtained. The general tendency among students is to disregard this most important phase of computations by making their work unnecessarily refined and by leaving in their final results an unwarranted number of significant figures. In lengthy computations it is sometimes necessary to carry out the intermediate results to one extra place, but final results should show only the consistent number of significant figures. Not only is much time wasted in these useless refinements, but the final results are deceptive as well. The student should bear in mind that computations, like field measurements, should maintain an accuracy consistent with their purpose, and that though the precision of computations may be less than the precision of field measurements, it can never be greater.

Before beginning a computation, the purpose of its results should be known and the data upon which it is based should be critically examined to determine the precision of field measurements. Without this preliminary knowledge and study, the computation is likely either to indicate that the results are of a higher precision than the original data or it is likely to be unnecessarily refined. It has been before pointed out that field measurements should be taken with a precision dependent upon the purpose of the survey and that angles should have at least as great precision as related distances; nevertheless to presume such consistent accuracy would be unwarranted, for often data taken in the field serves more than one purpose, and very often the surveyor has paid little attention to securing consistent measurements. The following examples illustrate the value of examining the given data and of knowing the purpose of the computation:

1. Notes of a land survey indicate that the distances were measured with a precision of 1/10,000, while angles were measured with a probable error of 01′. It is desired that computations for area (involving sines and cosines) be as precise as the data warrants. Referring to the *Ratio Curves for Sines and Cosines* (Plate I, page 9), it will be seen that such an angular error for angles of average size corresponds to a precision of about 1/3000. It is this precision which will govern the accuracy of the computed area, and the computed area should contain four significant figures. Had it been presumed that the precision of angles was consistent with that of distances, doubtless five significant figures would have been shown in the result.

2. The notes of the preceding example are to be used in plotting a map of the area. The scale of the map is to be 100 ft. = 1 in. and the computations will involve sines and cosines. Investigation shows that at the scale mentioned distances can not be plotted closer than to the nearest foot, therefore the computations should contain but three significant figures. Had four significant figures been used, the length of computations would have been nearly doubled.

21. Trigonometric Tables.—The number of places to be used in trigonometric tables will depend upon the ratio of precision of the particular value of the function involved rather than upon the angular error. The number of significant figures for a particular angle, angular error, and function may be readily determined by inspecting either the Ratio Curves (Plates I and II, pages 9 and 10), or trigonometric tables. *Assuming that the precision of angle is the governing precision*, for tangents and cotangents three places will be sufficient for angles in error more than 02′; four places, for angles in error less than 02′ but more than 10″; and five places, for angles in error less than 10″ but more than 01″.

For sines and cosines of angles of *average size* it will be seen that four places are sufficient for angles in error not less than 20″; five places for angles in error less than 20″ but more than say 05″; and six places for angles in error less than 05″ but more than ½″. Since angles are likely to be other than average size, the preceding limits should in general be doubled, though for sines of very large angles and for cosines of very small angles the necessary number of places should be ascertained by first determining the ratio of precision corresponding to the given angular error. For example the ratio of the tabular difference for 10″ to sin 80° (or cos 10°) is about 1/120,000, therefore six places will be necessary for angles as small as 10° when the error is 10″; or from the Ratio Curve for Sines it will be seen that for angles greater than 70° when the angular error is 01′, five places will be required.

22. Computations for Angles and Distances.—When, as is often the case, computations for angles or distances may be made in one of several ways, each of which depends upon data of like precision, it is best to compute angles by using functions which change rapidly, *i.e.*, by using tangents or cotangents; and to compute distances by using functions which change slowly, *i.e.*, by using sines or cosines.

For example the Ratio Curves for Tangents and Cotangents (Plate II, page 10) show that if the precision of data is 1/10,000 the maximum error of angles computed by tangents or cotangents is about 10″; the Ratio Curves for Sines and Cosines (Plate I, page 9) show that with data of a similar precision, angles between 27° and 63° could not have been computed by either sines or cosines with a precision as great as 10″.

23. Logarithms *vs.* Natural Functions.—As to whether or not logarithms shall be used will depend upon the computations under consideration. It takes less time to multiply two numbers of three digits each by arithmetic than by logarithms; possibly these numbers might be extended to four digits each, if it were only for a single computation. But for multiplying, dividing, squaring, cubing, or taking the roots of numbers, it is doubtful if arithmetic can ever be used to advantage beyond four significant figures. Where there are a number of similar computations, as is quite often the case in surveying, there is a *decided advantage* in using logarithms for *four* significant figures, for not only will less time be consumed in computing by logarithms but the liability of mistakes will be lessened and the mental strain on the computer will be decidedly decreased. Short arithmetical methods of multiplication and division are useful at times, but more often numbers that have enough significant figures to make these methods economical are large enough to make the use of logarithms more so.

Students in general either fail to appreciate the advantages of logarithms or feel that they are not sufficiently versed in the use of logarithms to use them with facility. Facility in the use of logarithms is not hard to acquire provided the work is carefully planned. Before attempting any computations a form should be prepared which will show an orderly, systematic arrangement of all logarithms and numbers that are to enter the computations, and the given data should be recorded in this form. Like or similar operations of computing should be grouped. The order of the work should be as follows: (1) determination of logarithms of all known numbers; (2) determination of the logarithms of the functions of all known angles; (3) addition or subtraction of known logarithms; and (4) determination of numbers corresponding to computed logarithms. In using the logarithmic tables the work should be so planned that the least possible number of pages will be turned at one time. For example, if logarithms of 5623, 1213, 6157, and 2142 were to be found, it would be a waste of time to look them up in the order written. Obviously a better way would be to look them up in the order 1213, 2142, 5623, 6157. Logarithms are frequently arranged so that sines, cosines, tangents, and cotangents for the same angles appear on the same page. If, as is often the case, more than one function of the same angle is to be used, the corresponding logarithms should be determined at the same time even though these functions may be some distance apart in the computations.

The method here outlined may not seem to be the natural course of procedure to the student, because he has been used to working with arithmetic where each step is completed before another is begun; but if he will systematize his work so as to change the train of his thought as seldom as possible, he will find that his efficiency has increased, and he will be loath to use arithmetic in lengthy computations.

24. Graphical and Mechanical Methods.—Though graphical and mechanical methods are chiefly used as checks on precise methods of computing, the surveyor will find that through their use many results may be determined with sufficient precision and in much less time than they could be obtained by computation.

By plotting, areas, omitted measurements, etc., may be found graphically. Often the data given in tables may be shown by curves or diagrams.

The more usual mechanical devices are the slide-rule (of which there are several kinds for different purposes) which will in general give values within less than 1 per cent., the planimeter for determining areas of plats, the pantograph for reducing drawings, and the protractor for measuring or laying off angles.

Sec. 5. Maps

25. General Remarks.—Maps may be divided into two classes: those that become a part of public records of land division, and those that form the basis of a study. The best examples of the former are the plats filed as parts of deeds in the county registry of deeds (in most states); and good examples of the latter are the preliminary maps along the proposed route of a railroad of which the location is to be projected. It is evident that the dividing line between these two classes is very indistinct, since many maps might serve both purposes. The requirements of a map will depend upon its purpose.

In general the things that should appear on a map that is to become a part of a public record are:

1. The lengths of all lines shown.
2. The bearings of all lines shown, or the angles between intersecting lines.
3. The location of the tract either with reference to principal meridian and base-line, if the tract is within the boundaries of the United States public land surveys; or with reference to co-ordinate axes, if within a city having a co-ordinate system.
4. The number of each block and lot, if the plat is of a city, town, or addition.
5. The position and kind of each monument set, with distances to reference marks.
6. The positions and names of all roads, streams, landmarks, etc.
7. The names of adjacent property owners, and the bearings of all property lines intersecting lines of the tract mapped.
8. The direction of the meridian (true or magnetic or both).
9. A legend or key to symbols shown on the map.
10. A diagram of the scale used with a corresponding note stating the scale. (The latter is usually shown in the title.)
11. A full and continuous description of the boundaries of the tract by bearing and length of sides; and the area of the tract.
12. The witnessed signatures of those possessing title to the tract mapped; and if the tract is an addition to town

or city, a dedication of all streets and alleys to the use of the public.

13. A certification by the surveyor that the plat is correct to the best of his knowledge.

14. A neat and explicit title showing the name of the tract, or its owner's name, its location, the scale of the drawing, the surveyor's name, and the date.

Of maps made the basis of studies, there are so many varieties and the requirements are so varied that a definite statement of all that each should include would be impossible. In general, maps of this kind show very few dimensions (often not any), the value of the map depending upon the correct representation of the position of features of the land rather than directly upon field measurements or computed values. Maps of this class may be divided into two kinds: (1) those that graphically represent such natural and artificial features as condition and culture of land, and public and private works (often called flat topographic maps); and (2) those that include some or all of the preceding features, but also represent the relief or contour of the ground (called topographic maps). Other things which should *always* appear on these two kinds of maps are:

1. The direction of the meridian.

2. A legend or key to symbols used, if they are other than the common conventional signs (Figs. 44–47, pages 218–222).

3. A diagram of the scale of the map with a corresponding note stating the scale.

4. A neat and appropriate title generally stating the kind or purpose of the map, the name of the tract mapped or the name of the project for which the map is to be used, the location of the tract, the scale of the drawing, the name of the engineer or draftsman or both, and the date.

26. Meridians.—The direction of the meridian is indicated by a needle or feathered arrow pointing north, of sufficient length to be transferred with reasonable accuracy to any part of the map. A meridian needle 4 in. or 5 in. long will be sufficient for large maps; 2 in. or 3 in. long for small maps. Preferably it should be near the middle of the drawing, but it should not interfere with features of the map. The true meridian is usually represented by an arrow with *full* head; the magnetic meridian by an arrow with *half* head. When both are shown the angle between them should be indicated.

The requisites of a meridian needle or arrow are that it shall be simple, graceful, and of such a size and in such location on the map that it will readily catch the eye; but it should not be of such proportions that it will be the *first thing* that strikes the eye. The general tendency is to make needles and arrows too large, blunt, and heavy. It is a waste of time to construct elaborate, ornate needles; moreover such needles do not add to the value of the maps which they adorn, but on the contrary often make such a marked contrast with the rest of the drawing as to detract from its general appearance.

27. **Lettering.**—All lettering should be free-hand. The style of lettering to be used will largely depend on the purpose of the map.

For office drawings or drawings which are not to become more or less public property, the REINHARDT style of letters, of which there are two kinds—inclined and vertical (Fig. 48, page 223), are used almost exclusively (except for titles). Although not as beautiful as some other styles of letters, this style is extremely practicable, since the letters may be rapidly constructed and easily read. Frequently the two kinds with their variations (extended and compressed forms) may be advantageously combined on a single drawing. Thus the names of streets and streams may be shown in extended form, inclined capitals; the names of property owners in normal form, lower-case vertical letters; and notes in compressed form, lower-case inclined letters.

Italic, *Roman*, and *Gothic* letters (Fig. 49, page 224) are more beautiful in appearance than the REINHARDT letters, and although more difficult of construction their use is often justifiable *on drawings to be used by the general public.* Of these three styles, Roman letters are the most difficult to construct on account of their fine lines and shadings; and slight deviations in their form are most quickly apparent, possibly on account of familiarity with perfect forms through printed pages. Gothic letters, both inclined and vertical, are most readily constructed; and unless the draftsman through considerable practice first familiarizes himself with the forms of letters of the other two styles he had best stick to Gothics. Italics are very little different from Roman letters except that they are inclined, but for this reason deviations in their form are not so apparent. Frequently all these styles may be used on a single drawing. For example the maps of the U. S.

Geological Survey show the names of civil divisions in Roman letters, hydrography in Italics, public works in inclined Gothics, and hypsography in vertical Gothics.

28. **Titles** should be so constructed that they will readily catch the eye, and the sizes of different parts should be in proportion to their importance. The best position for the title is the lower right-hand corner of the sheet. In general each line should be centered and the distance between lines should be such that the title as a whole will appear well balanced. The space occupied by the title should be in proportion to the size of the map. The general tendency is to make the title too large. One author has expressed the opinion that the length of the rectangle enclosing the title should not exceed 1/5 that of the longest border-line of the sheet, and that the width of the enclosing rectangle should not exceed 1/4 the length of the shortest border-line. These limits for most maps seem to be high. The different parts of the title should be in straight lines, and only the common styles of capital letters should be used. The scrolls and flourishes to be seen in the titles of many old plans are now obsolete, and few well-designed titles contain lower-case letters. It is best to construct the title of all vertical or all inclined letters, but a change of style of lettering between different parts is permissible when such change will serve to accentuate the important parts of the title. Thus the principal part may be constructed in Roman letters, while the remainder is in Gothics. It is bad practice to change the style for each line, for it conveys the impression that the draftsman was trying to show what he could do. In general, titles should be constructed free-hand, as well as the letters on the body of the map. The use of REINHARDT letters for the important parts of titles, even those of office plans, is not prevalent, though they may be appropriately used for the less important parts where small letters are used. The use of Gothic capitals is most common, for not only are they effective but they are easily made.

CHAPTER II

ELEMENTARY FIELD PROBLEMS

Problem 1. Pacing

Object.—To determine the length of normal pace, to test the reliability of an assumed 3-ft. pace, and to determine an unknown distance by pacing.

Procedure.—(1) Walk over an assigned course of known length ten times at an ordinary gait, counting the paces each time. Record the observed number in the field note-book as shown in accompanying form. Compute the average length of the natural pace and average number of paces for 100 ft. (2) With a tape mark a course 30 ft. long, every 3 ft. Walk over this course several times to obtain the proper stride; then pace the assigned course with this stride, recording the data and making computations as previously explained. (3) Walk over a course of unknown length several times at a natural pace, then several times at an assumed 3-ft. pace. Estimate the distance by each method; and then find the true distance with a steel tape. Note the error.

Hints and Precautions.—(1) In attempting to walk at a natural rate, avoid the general tendency to exceed that rate. (2) Count the paces carefully, estimating to the nearest one-tenth pace at the end of the course. (3) Reject observations that vary from the mean by more than 3 per cent. (4) Remember that field notes are a permanent record and should show clearly all the work done in the field. If an observation is rejected, draw a line through it, but do not erase. (5) Never fail to include in the notes of any survey, a complete description or sketch of the work accomplished.

Practical Applications.—Pacing is employed to a considerable extent in reconnaissance surveys, since for that sort of work approximate distances are in accord with the other rough methods used. It is also used by topographers and map sketchers on small-scale work.

A small pocket instrument, called a pedometer, automatically registers the number of paces. It is used to avoid the

Prob. 1. Pacing.

Standardization of Pace.

Natural Pace.			3 ft. Pace.		
Trial.	Paces. per 300 ft.		Trial.	Paces. per 300 ft.	
1	116.5		1	98.5	
2	117.3		2	101.4	
3	117.8		3	102.4	
4	115.9		4	101.6	
~~5~~	~~121.4~~		5	97.1	
6	118.6		6	100.2	
7	117.1		7	101.6	
8	117.2		8	103.0	
9	119.7		9	100.1	
10	118.5		10	102.4	
11	115.0		Mean	100.8	
Mean	117.4		Paces per 100 ft = 33.6		
Paces per 100 ft = 39.1			Length of Pace = 2.98′		
Length of Pace = 2.56′					

Estimating Length. Line 251-252

Trial	1	2	3	Mean	Length in ft.
Nat. Paces	371	377	375	374	957
3 ft. Paces	318	326	324	323	969
Tape					948
	Error in Natural Pace 1 in 100				
	" " 3 ft. " 1 in 45				

1.

F. F. Smith.
Oct. 1, 1913.
Fair & Cold.

Measurements taken with Steel Tape from Locker No. 46 by F. F. Smith, H.C. and J. C. Brown, R.C.

300 ft. Course over Smooth Ground. Strong Wind at Right Angles with Course.

Line 251-252 over Slightly Rolling Ground. Crosses Highway and two Rail Fences.

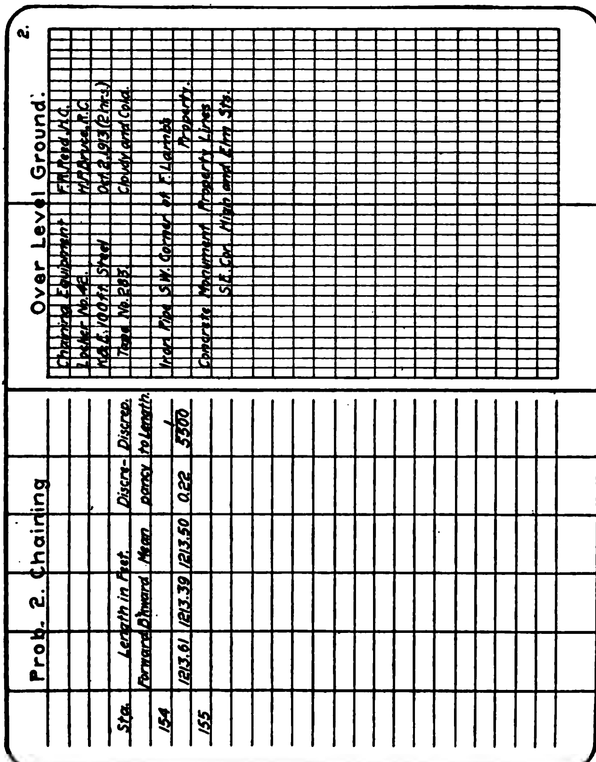
SAMPLE NOTES I.

necessity of counting paces, which for a long distance is not only very monotonous, but absorbs the mind of the observer as well. This little device can often be used to good advantage.

Problem 2. Chaining over Level Ground with 100-ft. Steel Tape

Object.—To chain over an approximately level course about 1200 ft. long, and to check the distances by chaining in the opposite direction.

Procedure.—(1) Having set a flag pole at one end of the line *B*, the rear chainman goes to the other end *A*, taking one pin. (2) The head chainman with a flag pole, ten pins, and the zero end of the tape moves toward *B* until the one hundred-foot end of the tape approaches *A*. The rear chainman then calls out "*chain*," whereupon the head chainman with the flag pole in a perpendicular position at the zero end of the tape (or with a pin at the zero end if the ground is smooth and a clear sight can be obtained) allows himself to be put on line by the rear chainman. (3) When this condition has been fulfilled the rear chainman holds the one hundred-foot mark at *A* until the head chainman has stuck a pin at the zero point. The rear chainman, when ready for the measurement, calls "*stick*," and when the pin is firmly placed and the distance checked the head chainman replies "*stuck*." (4) The rear chainman then drops his end of the tape, calls "*forward*," pulls the pin at *A*, and both men move rapidly forward, the head chainman keeping approximately on line by sighting over the flag pole at *B* to some advanced point. (5) As the rear end of the tape approaches the pin in the ground the rear chainman calls "*chain*," a signal for the head chainman to stop, and the operation of setting the pin is repeated as before. (6) As the eleventh pin is stuck the head chainman calls "*out*" and waits until the rear chainman comes up. The chainmen then count the pins to make sure that none are lost and record the tally. (7) In observing fractions at the end of the course, first note the number of feet, and with the zero end of the tape held at the end point estimate the number of tenths; then slide the tape forward until, with a footmark at the last pin set, the station falls within the finely graduated foot at the end of the tape.



SAMPLE NOTES 2.

The distance should then be read to tenths of feet and estimated to hundredths. Record the data as shown on page 27. (8) Compute the ratio of discrepancy to length. Measurements should check within one in five thousand.

Hints and Precautions.—(1) The number of pins held by the rear chainman at any time indicates the number of hundreds of feet chained since the last tally. *The pin in the ground should not be counted.* (2) The rear chainman should refrain from holding onto the tape as he moves from station to station, for, if he moves too slowly, the head chainman is retarded, and if he moves too fast the chain is liable to become kinked. (3) Be careful not to disturb the "*stuck*" pin by allowing the tape to press against it. (4) Avoid injury to the tape by always keeping it straight while in use. (5) Avoid inconsistent errors by checking every measurement. (6) In order not to obstruct the view of the rear chainman in lining, the head chainman should stand beside the tape facing the line. This position also enables the chainmen to better withstand the pull on the tape. (7) If a reel is not provided for winding the tape, do it up in 5-ft. lengths in the form of a figure eight. To do this, stand beside one end of the tape, take the end of the tape in the left hand, and, allowing the tape to slide loosely through the right, extend the arms. As the 5-ft mark comes along, grasp it with the right hand, and, bringing the hands together, lay it in the fingers of the left hand without permitting the tape to turn over. Then grasp the loop with the left hand and again extend the arm for another 5-ft. length. When the last mark is reached, tie the loop lightly where the ends of the tape come together. By properly twisting the tape it may then be put in circular form.

Practical Applications.—Linear measurement by tape is the basis of all surveying in which a considerable degree of accuracy is desired. This method of chaining is the one used in general engineering practice, such as land, city, railroad, and highway work.

Problem 3. Standardization of Tape and Chaining over Uneven Ground

Object.—To standardize the tape before going into the field and on returning from work; to find the horizontal length of an assigned course about 800 ft. long over uneven ground; and to correct for the error in length of tape as determined by the standardization tests.

Procedure.—(1) Standardize the tape by comparing it with the official standard. Maintain the required pull by means of a spring balance attached to one end of the tape. Determine the error with a finely divided scale. (2) Chain the course as in the previous problem, except that, since the ground is uneven, the tape must be held horizontally and the point transferred to or from the ground at the low end of the line by means of a plumb-bob. Maintain a pull estimated to be equal to that used during standardization. (3) For measurements in which the line slopes faster than 5 or 6 ft. in a hundred, it will be necessary to "*break chain;*" that is, to choose some intermediate point on the tape at which it will be possible to hold the tape horizontal. (4) Correct for the error in length of tape by adding to or subtracting from the observed distances, the observed error multiplied by the number of tape lengths in the course. *The correction should be added when the tape is longer than the standard, and subtracted when it is shorter.* Data should be recorded as shown in sample notes 3, page 30.

Hints and Precautions.—(1) The general tendency is to hold the downhill end of the tape too low. Avoid this by comparing the tape with some level line, or if possible, have the two ends in line with the horizon. (2) In "breaking chain" the head chainman should first pull the tape forward its *full length*, then go back to the footmark necessary to hold the tape level. After this point has been projected to the ground and marked with a pin obtained from the rear chainman, he again moves forward (without moving the tape) to the footmark necessary to make the tape level from the intermediate point just set. The rear chainman arriving at the point, holds on the footmark previously held by the head chainman. In this way the chain is always advanced its full length each time it is moved, except at the end of the course, and thus there is no chance of making a mistake of a fraction of a tape length. The intermediate points, being marked by pins obtained from the rear chainman introduce no opportunity for a mistake in tally. (3) It is usually more accurate to chain downhill than to chain uphill, since in chaining downhill the rear end may be held firmly on the ground.

Prob. 3. Chaining Over Uneven Ground.

5

Sta.	Length in feet.			Correct-ion.	Cor. Length
	Forward	B'kward	Mean		
351					
	311.73	311.65	311.69	+0.07	311.76
352					
	~~198.40~~ 198.60	198.62	198.61	+0.04	198.65
353					
	213.84	213.78	213.81	+0.05	213.86
354					

Standardization.

Before	100.021
After	100.023
Mean	100.022

Chaining Equipment	J. Pratt, H.C.
Locker No. 35.	S. Fulton, R.C.
Chicago 100 Ft. Steel-	Oct. 3, 1913 (2 hrs.)
Tape	Cold and Windy.

Tk. in Stk. Ctr. Line M.C.R.R. at Haskell St.
East on Ctr. Line Haskell St. up steep Hill.
Nail in pavement.
North over Creek and R.R. Embankment.
S.B. S.E. Corner Jones' Lot.
North up hill (Rough).
1" Iron pipe N.E. Corner Jones' Lot.

Tape compared with Standard at
City Hall.

SAMPLE NOTES 3.

PROBLEM 4. SURVEY OF FIELD WITH STEEL TAPE

Object.—To collect sufficient data for calculating the area of the field by two sides and included angle, by three sides, and by base and perpendicular methods. See Sample Notes 4, page 32. (The data may be used in Problem 69, page 180.)

Procedure.—(1) Divide the field into triangles, avoiding as far as possible any construction which will result in forming a very acute angle; that is, make the triangles as nearly equilateral as the shape of the field will readily permit.

(2) Measure the sides and altitude of each of the triangles. Erect perpendiculars by one of the following methods:

3 : 4 : 5 *Method.*—To erect a perpendicular to *AB* that shall include point *C* in the triangle *ABC* (Fig. 1) by the 3 :4 :5 method. Assume point *a* to be on the perpendicular.

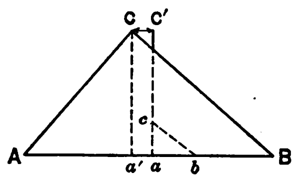
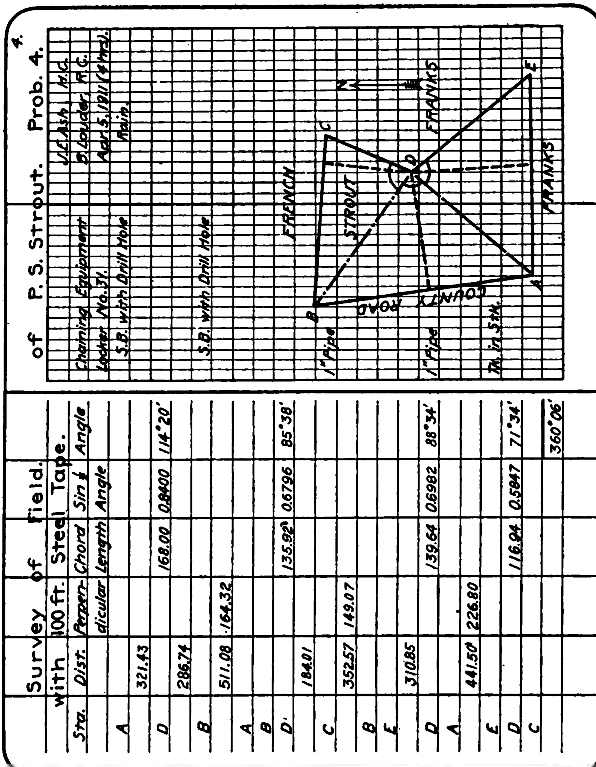


FIG. 1.—PERPENDICULAR BY 3 : 4 : 5 METHOD.

Set a pin at *a* on line *AB*. With sides 3, 4, 5 ft., or a multiple of these numbers, such as 24, 32, and 40 ft., make a right triangle *abc* with the tape as follows: Set a pin at *b* on line *AB* 24 ft. from *a*. Loop the tape over the pins at *a* and *b* so that the 32-ft. mark on the tape coincides with *a*, and the 56-ft. mark coincides with *b*. The head chainman with the zero and 96-ft. marks of the tape grasped in one hand, moves to *c* setting a pin at the point where the zero and 96-ft. marks coincide. The rear chainman at *a* or *b* prevents the chain from slipping over the set pins as the head chainman sets the pin at *c*, and sights along *ac* to set *C'* by *C*. If *C* does not lie on this prolongation, measure the perpendicular distance from the point *C* to the line *aC'* and move the foot of the perpendicular *a* along the line *AB* by an equal amount, to the point *a'*. If the trial perpendicular fails to include the point *C* by several feet, repeat the process for *a'*, the new point. If, however, the distance *C'C* is small, the new foot of the perpendicular may be set by making *aa'* equal to *C'C* and it may be assumed as correct without further test.



Survey of Field of P. S. Strout. Prob. 4.
with 100 ft. Steel Tape.

Sta.	Dist.	Perpen-dicular	Chord Length	Sin ½ Angle	Angle
A					
	321.43				
D			168.00	0.8400	114°20'
	286.74				
B					
	511.08	164.32			
A					
B					
D'			135.92	0.6796	85°38'
	184.01				
C					
	352.57	149.07			
B					
E					
	310.85				
D			139.64	0.6982	88°34'
A					
	441.50	226.80			
E					
D			116.94	0.5847	71°34'
C					
					360°06'

Sample Notes 4.

Chord-bisection Method.—Estimate the position of the perpendicular from C upon AB (Fig. 2), and set a pin at d on this estimated perpendicular, less than one tape length from the line AB. With d as center and length of tape as radius,

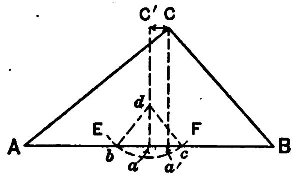


FIG. 2.—PERPENDICULAR BY CHORD-BISECTION METHOD.

the head chainman describes the arc of a circle. The rear chainman stationed at A or B determines the position of pins at b and c, the points of intersection between the line AB and the arc of the circle EF. The point a bisects the line bc. Prolong ad, place C' by C, and move point a as described in the 3 : 4 : 5 method.

(3) Determine one angle of each triangle by the chord method as follows: With the vertex of the angle as the center,

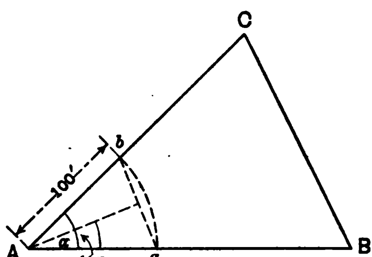


FIG. 3.—ANGLE WITH TAPE.

swing the tape, setting pins a and b (Fig. 3) at points where the arc intersects the lines AB and AC. Measure the chord distance ab.

$$\sin \tfrac{1}{2}\alpha = \frac{ab}{200}$$

Hints and Precautions.—(1) The chord-bisection method of erecting perpendiculars is the more accurate, while the

3 : 4 : 5 method requires less time. (2) If the perpendicular and the segments of the base on each side of its foot are measured, sufficient data will have been obtained for determining the angles by the tangent method. However, the chord method of measuring angles is more accurate, and the tangent method should be used only as a check. (3) Considerable care should be taken in lining in points, and intersections should be determined as closely as the eye of the observer will allow. (4) A very good way of finding the approximate position of the perpendicular is to stand on the line *AB* with arms extended horizontally in the directions of *A* and *B*. Now, with eyes closed, bring the arms to the front, palms together, and opening the eyes, see if the vertex of the triangle lies in line with the hands. If it does not, the observer should change his position to some other point along *AB* nearer the desired perpendicular.

Practical Applications.—It is obvious that surveying by this method would be too slow and inaccurate to be utilized to any great extent except in surveys covering small areas; hence this method is, in general, restricted to rough estimates or checks. In this problem the elementary steps are the more important, since the construction of perpendiculars and the measuring of angles is often necessary when the transit is not at hand. Angles measured by the tape method are usually thought of as being less accurate than those measured with the transit; but for small angles, the reverse is likely to be true.

Problem 5. Survey of Field with Irregular Boundary

Object.—To collect sufficient data for calculating the area and for plotting the assigned field. (The data may be used in Problems 70, 71, and 72, pages 180 and 182.)

Procedure.—(1) Divide as much of the field as its shape will permit into triangles as in Problem 4, having in view simplicity of field operations. Lay out the triangles so that no long offsets will be necessary. (2) Collect data for finding the area of the triangles by one of the methods explained in Problem 4. (3) From the triangle sides nearest the boundary, take offsets at such intervals as will insure a sufficiently accurate plat. Record the data as shown in sample page of notes.

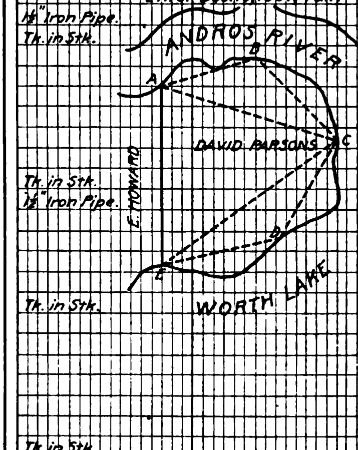
Hints and Precautions.—(1) To take the offsets accurately and quickly, set several pins at the desired intervals along the

Prob. 5. Survey of Land of David Parsons, Smithville. 3.

With 100 ft. Steel Tape.

Sta.	Dist.	Offset L	Sta.	Dist.	Offset L
A	0	0			
	25	21.4	D	0	0
	50	37.8		25	10.0
	75	44.6		50	26.2
	100	27.2		75	34.6
	111	0		100	34.5
	125	15.5		125	25.2
	140	16.7		150	14.0
B	159 06 / 0	0		175	4.1
	30	21.6	E	208.64 / 0	0
	50	42.0	A	275.98	
	90	51.4	AC	281.14	
	140	27.8	EC	273.61	
	165	13.0			
C	187 88 / 0	0			
	14	0			
	45	16.3			
	55	31.6			
	75	61.0			
	90	14.2			
	100	7.0			
D	109 89	0			

F. Pierce, H.C.
Steel Tape No. 2. F.S.N. Mooney, R.C.
2 hrs. Dec. 10, 1907. Fair.



SAMPLE NOTES 5.

side of the triangle before measuring the offsets. If the distance to the boundary is not more than 50 ft. and the boundary is fairly regular, erect perpendiculars by estimation with the eye. (2) If the boundary changes abruptly at any point, be sure to take an offset at that point. (3) Do not measure the offsets with unnecessary accuracy. For example, if the sides of the triangles are measured to the nearest 0.01 ft., measure the offsets no closer than the nearest 0.1 ft. (4) Do not record offsets as being on the right of the line when they are on the left.

Practical Applications.—This problem is frequently made use of in land surveying where the boundary line is a stream or lake. Lines are run by compass or transit as near to the stream or lake as is practicable. Where the boundary changes direction frequently, offsets are taken at short intervals; but as long as the boundary continues as one straight line, offsets need not be taken. However, because of the greater ease with which the areas can be computed, offsets are frequently taken at regular intervals.

Problem 6. Obstructed Distance with Chaining Equipment

Object.—To determine the distance between two points with an obstacle on line.

Procedure.—(1) On an assigned course about 800 ft. long, assume that there is some obstacle which makes direct measurement and intervision impossible, and find the distance by the swing-offset method. (2) Find the distance by constructing a line parallel to the given line. (3) Assume that the points are intervisible but direct chaining is impossible, and determine the distance by methods other than those used above. (4) As a check, measure the distance directly. Compare the results.

Swing Offsets.—To find the distance by the swing offset method, the head chainman attaches the end of the tape to one end of the line as *B* (Fig. 4), and describes an arc of a circle the radius of which is 100 ft. and the center of which is the point *B*. The rear chainman stationed at *A* lines in the end of the tape with some distant object as *O*, and sets points *a* and *b* where the head chainman crosses line *AO*. A point *C* midway between *a* and *b* lies on the perpendicular *CB*. Bisect *ab*, set a pin at *C*, and measure the distance *AC*, to obtain the necessary data for calculating *AB*.

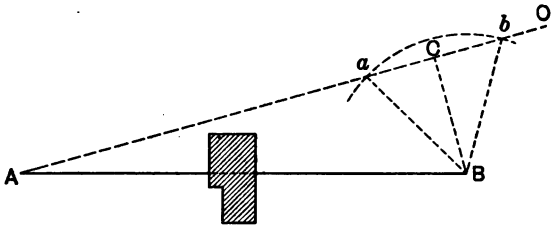


FIG. 4.—SWING OFFSET WITH TAPE.

Parallels.—Parallels may be laid off with a tape by one of the following methods: 1. By equal distances; 2. by alternate angles; 3. by similar triangles.

Method 1.—If points are intervisible, as will be the case when the obstruction is a stream or lake, erect perpendiculars at *A* and *B* (Fig. 5) by one of the methods of Problem 4,

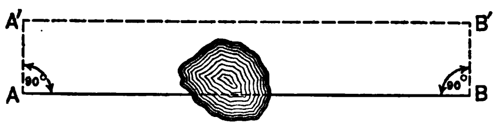


FIG. 5.—INACCESSIBLE DISTANCE.

chain distance $AA' = BB'$ to clear the obstacle, and measure the distance $A'B'$. If the perpendiculars have been carefully erected and the distance $AA' = BB'$ is not great, the measured distance should agree closely with the length of the line *AB*. If the points are not intervisible, this method will be impossible.

Method 2.—If the offset necessary to avoid the obstacle is considerable, the preceding method will become inaccurate because of the uncertainty of right angles measured with tape. If the points are intervisible, establish *AC* (Fig. 6, page 38), so that the estimated value of α will be less than 45°. Determine the chord length of α for radius of 100 ft. (or greater if precision makes it necessary). Measure *CD*, it being roughly perpendicular to *AB*. At *C* lay off an angle equal to the angle at *A* so that *CE* will be parallel to *AB*. Measure *CE*, *E* being any convenient point such that β will be less than

45°. Measure *BE* and *EF*. $AB = AD + CE + FB$. Solve the right-angle triangles *ADC* and *BFE* for *AD* and *BF*, and compute the length of the line *AB*.

The precision of this method over that of the preceding method is due to the fact that the angles laid off are small and that the distances *AD* and *BF* are *computed* rather than measured. The reason for computing these distances will

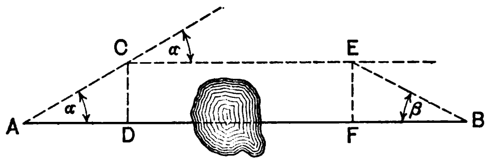


FIG. 6.—INACCESSIBLE DISTANCE (LONG OFFSET).

readily be seen when it is considered that any variation of the line *DC* from the true perpendicular will make little difference in its length as compared with the corresponding change of length of *AD*. To illustrate, suppose α (Fig. 6) equal 45°, that the true length of *AD* and *CD* is 200 ft. and that *D*, supposedly on the perpendicular through *C*, is 10 ft. in error. As *computed* from *AC* and *CD*, *AD* is about 0.3 ft. in error, but its *measured* length is 10 ft. in error.

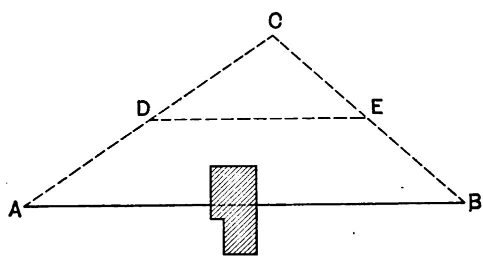


FIG. 7.—OBSTRUCTED DISTANCE.

Method 3.—If the points are not intervisible, both the preceding methods are impossible. Let *C* (Fig. 7) be a point from which both ends of the line *AB* are visible. Measure *AC* and *BC*. Lay off *CD* and *CE* so that *CD* will bear the same relation to *CA* that *CE* bears to *CB*. That is $CD/CA = CE/CB$. (It will generally be convenient to make this ratio a simple one such as ½ or ⅛.) The triangles *ACB* and *DCE* are similar. Measure *DE*, and compute *AB*.

CHAPTER III

COMPASS AND TRANSIT PROBLEMS

PROBLEM 7. ADJUSTMENTS OF THE SURVEYORS' COMPASS

1. **Bubble Adjustment.**—*To make the axes of the bubble-tubes perpendicular to the vertical axis of the instrument.* Level up the instrument, and turn 180° about the vertical axis. If the bubbles move from the center, bring each *halfway* back by means of the adjusting screws. Level the instrument, and repeat the operation to verify the adjustment.

Detection of Errors of Adjustment in Needle or Pivot.—Note the readings of both ends of the needle for various positions of the compass. If the end readings always differ by the same amount, then the needle only is out of adjustment, but if the difference in the end readings varies for different positions of the needle, there may be an error in adjustment of both needle and pivot. In making these adjustments it is necessary to *first adjust the needle,* since its error can be detected independently of the error in the pivot.

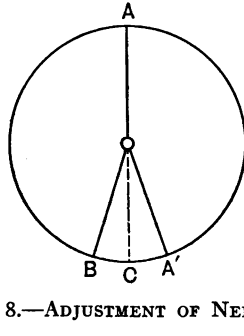


FIG. 8.—ADJUSTMENT OF NEEDLE.

2. **Adjustment of the Needle.**—*To make the middle of the needle lie in the same vertical plane with the ends.* With the instrument set up and the needle free, remove the glass cover.

Steady the needle with a piece of wood or copper or brass wire, and read both ends, *A* and *B* (Fig. 8, p. 39). Turn the compass needle until the end *B* takes the position previously occupied by *A*. If the needle is out of adjustment, the end *A* will fall at *A′*. *BA′* in the figure is *twice* the error. Remove the needle from the compass box, and with a pair of pliers bend it by an amount estimated to be equal to its error. When it is replaced on the pivot with one end at *A*, the other end should fall on *C*, a point midway between *A′* and *B*. Repeat the operations until the desired condition is realized.

3. **Adjustment of the Pivot Point.**—*To make the pivot point lie exactly in the center of the graduated circle.* Having already straightened the needle, turn the compass box until the ends of the needle are 180° apart (*AB* in Fig. 9). Now

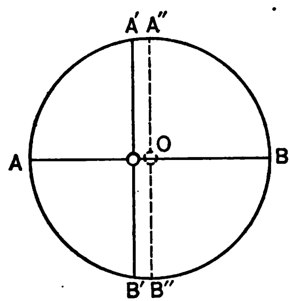


FIG. 9.—ADJUSTMENT OF PIVOT.

turn the needle through 90° to the position *A′B′* in Fig. 9, and read both ends. The difference between the end readings will be the maximum and equal to *twice* the distance of the pivot from the true center. Remove the needle, and with a pair of pliers bend the pivot in a direction perpendicular to the last position of the needle, to the point *O*. The ends of the replaced needle will take the positions *A″* and *B″*, but will read the same. Repeat the operations until adjustment is obtained.

4. **Adjustment of the Sights.**—*To make the plane of the sights perpendicular to the plane of the bubbles.* Level up the instrument *very carefully*, and sight first through one vane and then through the other at a suspended plumb-bob string. If the slit in either vane makes an angle with the plumb line,

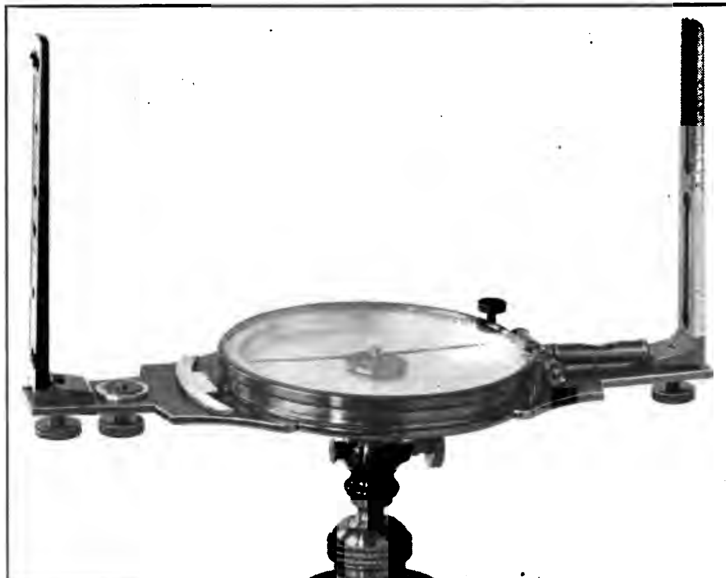


PLATE III.—SURVEYORS' VERNIER COMPASS.
(From photograph furnished by W. & L. E. Gurley.)

(*Facing page* 40.)

the sights are out of adjustment and should be corrected by filing off the base of the vane or by putting strips of paper under one side of the base sufficient to make the slit truly vertical.

PROBLEM 8. DETERMINATION OF MAGNETIC DECLINATION

Object.—To determine the magnetic declination with the surveyors' compass. See Sample Notes 8, page 42.

Procedure.—(1) See that the compass is in good adjustment; if it is not, adjust as explained in Problem 7. (2) Set the compass over one end of a meridian that has been determined by astronomical observations, sight along the line, and clamp the compass in that position. (3) By means of the tangent-screw, move the compass circle until the needle reads zero. (4) From the declination arc read the declination to the nearest minute; record the declination and the time of observation. (5) Take observations as above every five minutes over a period of half an hour or more, resetting the line of sight and compass circle for each observation. (6) If possible take a series of observations at about the same time on each of several days. (7) Determine the most probable value of the declination for the hour of observation and the probable error of a single observation and of the mean. (See Problem 34, p. 98.) (8) Determine the mean declination by adding to or subtracting from the observed declination the average solar-diurnal variation (Table VI, p. 252) for the place nearest the place of observation.

Hints and Precautions.—(1) Care should be taken to keep sources of magnetic disturbance away from the instrument. *Great care should be taken not to produce static charges of electricity by rubbing the glass.* A moistened finger pressed against the glass will remove such charges. (2) At each observation the compass box should be tapped lightly as the needle comes to rest that there may be no adhesion between the jewel of the needle and the pivot. (3) Between 6 and 7 p.m. is the best time for taking magnetic observations, because at this time the magnetic declination reaches approximately its mean value for the day, as will be seen by looking at Table VI. Between 10 and 11 a.m. the declination also reaches its mean value, but the rate of change of declination is more rapid at

Prob. 8 Determination of Declination at Philips, Mass. 6.

With Vernier Compass.

Obs. No.	Time A.M.	Vernier Read.	V	V^2	
1	$8^h 15^m$	12°13′	4	16	
2	20	22	5	25	
3		24	7	49	
4		23	6	36	
5		12	5	25	
6		19	2	4	
7		10	7	49	
8		18	1	1	
9		14	3	9	
10	9 00	12°15′	2	4	
Mean	$8^h 37^m$	12°17′		218	$= \Sigma V^2$

$E_s = \pm 0.67\sqrt{\frac{\Sigma V^2}{n-1}} = \pm 0.67\sqrt{\frac{218}{9}} = \pm 03.'2$

$E_m = \pm \frac{E_s}{\sqrt{n}} = \pm \frac{03.2}{\sqrt{10}} = \pm 01.'0$

s.d. Variation (Table VI) = +03′

Mean Mag. Declin. = 12°20′W

Gurley Compass from Locker E.

M. R. Beck.
G. Bolton.
May 4, 1912.
Fair and Warm.

Meridian Determined by Observations on Polaris, Probable Error ±0°00′02″.

Vernier Arm and Graduated Circle turned Counter-Clockwise.

SAMPLE NOTES 8.

this time than between 6 and 7 p.m. (4) The values given in Table VI to correct observations taken at other times of day are approximate, since this diurnal change varies from year to year, and is greater in summer than in winter. However such approximation is in accord with other rough methods of the problem. (5) If the compass circle has been turned in a clockwise direction the declination is east. If the diurnal variation is positive for the time of the observation, the north end of the needle is deflected more to the eastward than when the declination is at a mean. Hence, the mean is determined by algebraically subtracting the diurnal variation from the observed declination, east being considered as positive and west as negative. (6) The probable error shows the precision with which the measurements were taken, and may be taken as indicative of the precision of future observations under similar conditions.

Practical Applications.—In re-running old surveys or in making new surveys the lines of which are referred to the true meridian, it is necessary that the declination be known. An isogonic chart will furnish the surveyor with an approximate value of the declination say within ten or fifteen minutes, but this will be hardly close enough for the average survey, nor is such a chart always available. The above method is the one in common use among surveyors and should furnish a determination within 03′ of the true declination.

PROBLEM 9. SURVEY OF FIELD WITH SURVEYORS' COMPASS AND 66-FT. CHAIN

Object.—To find the lengths and bearings of the sides of an assigned field. (These data may be used in Problem 73, page 183.)

Procedure.—(1) Set off the magnetic declination for the place where the survey is to be made, on the declination arc of the compass. (2) Set up at one corner of the field, *A*, and sight along the line *AB*, being careful to have the south end of the compass box nearest the eye. Lower the needle, and as it comes nearly to rest, tap the compass lightly with the end of a lead pencil. When the needle becomes stationary read the north end, estimating the bearing to the nearest 05′. The direction is indicated by the meridianal and longitudinal letters on the compass circle between which the north end of the needle stands. (3) Take a back bearing from *A*. (4)

Prob. 9 Survey of Wood-Lot of R.D. Fly, Bemis, Me. 11.

With Surveyors' Compass and 66 Ft. Chain.

Sta.	Obj.	Dist.	Bear.	Int.Ang.	Cor.Bear.
A	E		N28°00'W		
	B	24.93	S30°40'W	238°40'	S30°40'W
B	A		N30°40'E		
	C	37.56	S83°30'E	65°30'	S83°45'E
C	B		N84°30'W		
	D	48.42	N 2°00'W	82°30'	N 1°10'W
D	C		S 2°15'E		
	E	35.26	S89°30'W	91°45'	N89°20'W
E	D		East		
	A	25.77	S28°30'E	61°10'	S28°05'E
				539°35'	

W.F. Dunn 木

Gurley Vernier Compass. F. Arseneault, Chain.

No. 89. Decln. 15°20' W. Nov. 13, 1906.

Set off with Vernier. Snow.

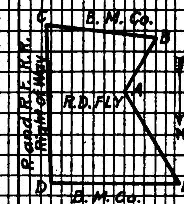
Spruce Tree 18"⌀ Blazed B.M.Co/R.D.F.

Cedar Stump 12"⌀ " " / "

Rough Stone, d.h.

Ledge, d.h. and cross.

Cedar Stake 4"⌀ 4' high.



Note: Bearings are Referred to the True Meridian.

SAMPLE NOTES 9.

Chain the line AB, and record the distance in chains, to the nearest link. (5) Set up the compass at B. Observe the back bearing of the line AB and the forward bearing of the line BC. Chain the distance BC. (6) Continue in the same manner around the field, taking both back and forward bearings from each point. (7) Calculate the interior angles of the field from the back and forward bearings measured at the vertex of each of the angles, and correct the observed bearings for local attraction, if such exists. See Sample Notes 9.

Hints and Precautions.—(1) The upright sight vanes are usually unlike, the one to be attached nearest the north point of the compass box being marked for reading vertical angles, and the one nearest the south point being fitted with "peep" sights. It is well to bear this in mind, both when assembling the compass and when using it in the field. (2) Since the accuracy with which angles can be read depends upon the delicacy of the needle, special care should be taken to avoid any concussion between the jewel bearing of the needle and the pivot point. *Never, under any condition, move the instrument without first making certain that the needle is clamped.* (3) Keep sources of attraction such as chaining pins, axe, and pocket knives away from the instrument when bearings are being observed. (4) Be sure to set off the declination in the right direction. If the declination is east, the magnetic meridian lies to the east of true north; and since the point of zero bearing must lie on the magnetic meridian, the graduated circle must be turned in a clockwise direction. (5) In order to balance the needle or to counteract the magnetic dip, a fine brass or copper wire is wound about the southern end of the needle (in the northern hemisphere). With the instrument level if the needle does not remain horizontal when it is free to swing on the pivot, the glass should be removed and the wire moved along the needle until it balances perfectly. (6) In order not to confuse the north and south ends of the needle when taking bearings, always note the position of the counterbalancing wire. (7) The test for local attraction is to observe the forward and back bearing of each line. If these bearings fail to agree by an appreciable amount; local attraction exists at one and possibly both points from which observations were taken. To correct for local attraction, start from some line in the traverse, the forward and back bearings of which have the same angular value (that is, the ends of which are free from local attraction) and calculate the bearings of the other sides by means of the interior angles at each point. (8) The interior angles will be correct regardless of local attraction, since at any one point the needle will be deflected as much when the compass box is in one position as when it is in any other. If n is the number of sides of the field and $\Sigma\theta$ the sum of the interior angles, then $\Sigma\theta = (n - 2)\ 180°$. (9) In chaining, observe the precautions stated in Problems 2 and 3, pages 26 and 28.

Practical Applications.—Formerly the compass was the most important of surveying instruments, being utilized in nearly all classes of plane surveying. At the present time, however, the transit has in a great measure taken its place, and the compass is little used except in the rougher class of surveys in which speed and not accuracy is the first requirement. Some land surveyors of the "old school" still retain the compass, but since land in this country, which a few years ago was worth but little, is now in many cases quite valuable, more accurate methods of measurement are required. For the same reason the old link chain is rapidly giving way to the light but more accurate steel tape. However, the compass is not without some advantages. It is light, and is easily and quickly set up, and an error in the bearing of one line does not affect the bearings of other lines in the same survey. It is especially suitable for preliminary surveys on lines through wooded country. Where clear sights are obtainable, only alternate stations need be occupied; and if a tree is on line, it is only necessary to set up on the other side of it, finding by trial the position which, with the backsight on the tree, will give the back bearing of the line prolonged. In retracing old lines where no greater degree of accuracy is required than was obtained in the original survey, and where there is a possibility of undetected local attraction in the old bearings, the compass may be used with economy.

Problem 10. Retracing Survey with Compass and 66-ft. Chain. Two Adjacent Corners Known

Object.—To retrace property lines from the notes of an old compass survey. The bearings given in the original notes are magnetic, and the declination at the time of the original survey is unknown. Two adjacent corners of the plot can still be identified.

Procedure.—(1) Measure the length of the known side and compare with the original. (2) By proportionate distances compute the lengths of the other sides in terms of the re-survey chain. (3) Set up the compass at one end of the known line, sight along the line, and clamp the compass. (4) Release the needle, and as it comes to rest move the compass circle by means of the tangent-screw until the original bearing is read. (5) Proceed to lay out the field, chaining distances in terms of the re-survey chain as computed above and laying off the original bearings. Examine the ground for rotten stakes or other evidence that would have precedence over bearings and lengths of sides. (6) Reference the new corners by bearings and distances to nearby permanent objects.

Hints and Precautions.—(1) Examine the ground well; careful digging with a spade will often reveal unmistakable evidence of the location of the original corner. Discolored earth may mark the former position of a stake long since rotted away. (2) If the original corners were witnessed, the witness points should be sought for as carefully as for the original corners. If such points are found, the corner should be re-established from them rather than from bearings and distances of the sides. (3) Observe the hints and precautions of the previous problem.

Practical Applications.—The above method of re-running a survey is in common use by land surveyors. It is the duty of the surveyor to re-establish corners as near to their original position as possible, regardless of whether the old survey was correctly made or not. If he discovers errors in the original survey he may point them out, but the re-adjustment of property lines is not within his scope.

PROBLEM 11. ADJUSTMENTS OF THE ENGINEERS' TRANSIT

Note.—In the description of the following adjustments it is assumed that the objective slide does not admit of adjustment, but is permanently fixed in the telescope tube so far as lateral motion is concerned; and that the maker has done his part by so constructing the instrument that the optical axis and the axis of the objective slide coincide and are perpendicular to the horizontal axis. This ideal construction is never exactly attained; but in most modern instruments the departure is so slight that it need not be considered in ordinary transit work, and in precise surveys the resulting errors are eliminated by methods of procedure.

For transits with adjustable objective slides, the adjustments are the same as for those with permanently fixed objective slides except as noted on page 51.

(1) To Make the Vertical Cross-hair Lie in a Plane Perpendicular to the Horizontal Axis.

Test.—Sight at a well defined point about 200 ft. from the instrument. Swing the telescope through a small vertical arc, so that the point will appear to traverse the length of the vertical cross-hair. If the point appears to move continually on the hair, the cross-hair lies in a plane perpendicular to the horizontal axis.

Correction.—If the point does not appear to move always on the cross-hair, loosen two adjacent reticule screws and turn the reticule in the telescope until the point appears to traverse the entire length of the hair. Tighten the *same* two screws.

(2) To Make the Axes of the Plate Bubbles Perpendicular to the Vertical Axis.

Test.—Turn the alidade until each bubble-tube is parallel to a pair of foot-screws. Bring the bubbles to the centers of the tubes by means of the foot-screws. Reverse the alidade 180° in azimuth. If the bubbles remain in the centers of their tubes, the axes of the bubble-tubes are in a plane perpendicular to the vertical axis.

Correction.—If the bubbles do not remain in the centers of the tubes, bring them *half way* back by means of the adjusting screws. Level the instrument again, and repeat the process to verify the results.

(3) To Make the Line of Sight Perpendicular to the Horizontal Axis.

Test.—Set up and level the instrument. Sight on a point *A* (Fig. 10) about 500 ft. away, plunge the telescope and set another point *B*, on the line of sight and about the same distance away on the opposite side of the transit. Turn the instrument in azimuth and again sight at *A* (with telescope inverted). Plunge the telescope; if *B* is on the line of sight, the desired relation exists.

Correction.—If *B* is not on the line of sight, set a point *C*, beside it on the line of sight. Mark a point *D*, *one-quarter*

PLATE IV.—ENGINEERS' COMPLETE TRANSIT.
(From photograph furnished by C. L. Berger & Sons.)

(Facing page 48.)

of the distance from C to B, and adjust the reticule by means of the two opposite horizontal screws, until the new point is on the line of sight.

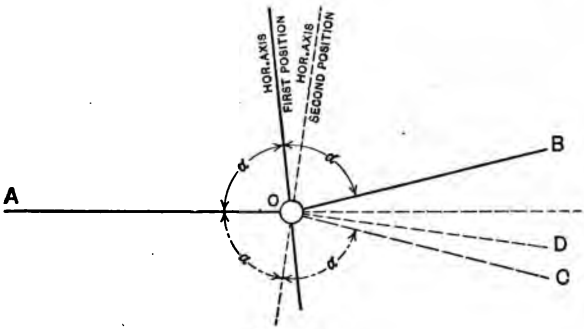


FIG. 10.

(4) To Make the Horizontal Axis Perpendicular to the Vertical Axis.

Test.—Set the transit near a building, flag staff, or other structure upon which is some well defined point at a considerable angle of elevation. Level the instrument very carefully, thus making the vertical axis truly vertical. Sight at the high point A (Fig. 11, page 50); and with the horizontal motions clamped, depress the telescope and set a point B on the ground. If the horizontal axis is truly horizontal, A and B will be in the same vertical plane, since the line of sight has been made perpendicular to the horizontal axis. Plunge the telescope, reverse in azimuth, and again sight on A. Depress the telescope as before; if the line of sight falls on B, the horizontal axis is perpendicular to the vertical axis.

Correction.—If the desired relation does not exist, set C on the line of sight beside B. A point D *one-half* the distance from B to C will lie in the same vertical plane with the high point. Loosen the screws of the bearing cap, and raise or lower the adjustable end of the horizontal axis until the line of sight will pass through both A and D when the telescope is swung in a vertical arc. *The high end of the axis is always on the same side of the vertical plane through the high point as the point last set.* In re-adjusting the bearing cap, care should be taken not to bind the horizontal axis, but it should not be left so loose as to allow the objective end of the telescope to drop of its own weight when not clamped.

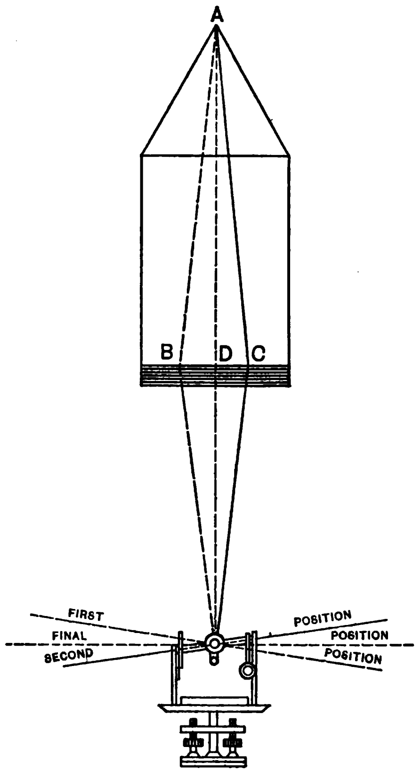


FIG. 11.—ADJUSTMENT OF HORIZONTAL AXIS.

(5) To Make the Line of Sight Coincide with the Optical Axis.

Test.—Set two stakes, one about 25 ft. and the other 300 or 400 ft. from the transit. With the horizontal axis clamped, take a rod reading on the distant point, and without disturbing the horizontal axis read the rod on the near point. Plunge the telescope, turn through 180°, and set the horizontal cross-hair at the last rod reading with the rod held on the near point.

Sight to the distant point. If the desired relation exists, the first and last readings will be the same.

Correction.—If there is a difference between the rod readings and it is considerable, move the horizontal cross-hair by means of the upper and lower reticule screws until it has apparently traversed over several times the apparent error. Repeat the process, gradually reducing the movement of the cross-hair as the rod readings to the distant point approach each other, until by successive approximations the error is reduced to zero. The rod when held on the near point should be read with great care, for a small difference in the position of the cross-hair on the near rod will be sufficient to indicate a considerable error on the distant rod.

(6) To Make the Axis of the Telescope Bubble Parallel to the Line of Sight.

Test and Correction.—Same as the peg adjustment of the dumpy level (Problem 25, p. 79) except that the correction is made by moving the bubble-tube until the axis is horizontal after having set the line of sight on the rod reading for a level line.

(7) To Make the Vertical Circle or Arc Read Zero when the Telescope Bubble is in the Center of the Tube.*

Test.—With the plate level, bring the telescope bubble to the center, and read the vernier of the vertical circle.

Correction.—Loosen the vernier, and move it until the desired relation is established.

(8) To Adjust the Auxiliary Bubble on the Vernier of the Vertical Circle so that its Axis is Parallel to the Axis of the Telescope Bubble when the Vernier Reads Zero.

Test.—Bring the telescope bubble to the center of its tube, and move the vernier by means of the tangent-screw until it reads zero.

Correction.—By means of the capstan-headed screws, move the bubble attached to the vertical vernier until it is in the center of its tube.

Transit with Adjustable Objective Slide.†—Some transits have objective slides which move in adjustable rings, and it is claimed by the manufacturers of such instruments that the optical axis will require no further attention after the factory adjustment has been made unless through accident some of

* This adjustment is not applicable to a transit having a bubble attached to the vertical vernier.

† See Note on page 47.

the parts of the telescope become bent. However, it is well to test the adjustment occasionally, and the instrumentman should be able to make corrections if necessary. Having performed the adjustment of the vertical cross-hair (Adjustment (3)) for an average length of sight, focus the vertical cross-hair on a distant point. Move the objective out, and bring it to a focus on some very well-defined point near the instrument. Plunge the telescope, turn 180° about the vertical axis, and again set the vertical cross-hair on the near point. Sight toward the distant point. If the objective slide is in adjustment, the line of sight will strike the first point sighted. If the line of sight does not strike the distant point, move the ring controlling the objective slide by means of the screws on the sides of the telescope until by estimation the line of sight has moved over one-half the apparent error at the distant point. The relation between the adjustments of the vertical cross-hair and optical axis is such that the adjustments must be repeated alternately until both are found to be correct. The vertical adjustment of the objective slide may be performed in a similar manner with reference to the horizontal cross-hair, but it is usually best to make corrections with the horizontal cross-hair as described in Adjustment (5) unless one is reasonably sure that the factory adjustment, through accident or otherwise, has been altered.

To Center Eyepiece Slide.—A transit telescope which shows objects erect may have an eyepiece slide one end of which moves through an adjustable ring. When the transit has otherwise been adjusted, the cross-hairs may not appear in the center of the field of view owing to the axis of the eyepiece slide not being coincident with the optical axis. This is a convenient relation but is unnecessary as far as the proper working of the transit is concerned. The adjustment is made by loosening and tightening the screws which project from the telescope tube between the eye end of the telescope and the screws controlling the cross-hair ring.

Hints and Precautions.—(1) The adjustments of the transit are more or less dependent one upon another. For this reason, if a transit is badly out of adjustment, time will be saved by first making the corrections roughly for those adjustments which are dependent one upon another until all the tests have been tried, and then repeating the tests and corrections (beginning at the first). The plate bubbles will not be disturbed by other adjustments and may be exactly corrected before other adjustments are tested. The adjustments of the telescope bubble and vertical circle depend upon the correct position of the line of sight and for this reason should not be tested until the adjustments of the cross-hairs and standards have been tested and found correct. (2) In making the corrections be sure that no screw is left loose or unduly strained. (3) Always have the tripod shoes planted firmly in the ground. (4) *Repeat the test of each adjustment after the correction has been made.* (5) If the transit has an erecting eyepiece which is permanently centered, Adjustment (5) may usually be made with sufficient precision for ordinary spirit or trigonometric leveling by simply moving the horizontal cross-hair until it appears to be in the middle of the field of view. If the transit has an inverting eyepiece, the cross-hair ring will limit the field of view and the cross-hair will appear in the middle whether or not it intersects the axis of the telescope. (6) In place of performing Adjustment (5) as previously described, some instrumentmen prefer to remove the telescope from the standards and to revolve it about *its* axis (not the horizontal axis) in improvised wyes. With the telescope in these wyes and the bubble down, the position of the horizontal cross-hair on some object is noted. The telescope is then carefully revolved about its axis until the buble is over the telescope. If the horizontal cross-hair no longer remains on the point, it is moved, by means of the reticule screws, over a distance estimated to be equal to one-half the displacement, and the test is then repeated. The "wyes" may be V-shaped notches in two thin boards, and they may be kept in position by nailing the boards to the edge of a third board. With some transits it will be necessary to remove the telescope bubble before the telescope can be placed in the wyes. This method presupposes that the geometrical axis through the centers of the two sections of the telescope tube over the improvised wyes coincides with the optical axis, and that this geometrical axis does not change position as the telescope is turned in the wyes. Instruments with coatings of nonconducting material over the metal would be least likely to fulfil these conditions. (7) Remember that the threads of the reticule screws *are in the reticule* and not in the barrel of the telescope; hence the reticule will be drawn toward the screw that is tightened. (8) In making Adjustment (4) it is not sufficient to adjust until the mid-point on the ground is sighted, as in so doing the plane of sight is moved away from the high point at the same time.

Problem 12. Angles of a Triangle with Transit. Linear Ties

Object.—To determine the value to the nearest half minute of each angle of a triangle, and the angular error of closure; and to witness the corners.

Procedure.—(1) Drive stakes at the vertices of a triangle with sides paced about 300 ft. Tack each stake. (2) Set the transit over one stake, level the plate bubbles by means of the foot-screws; set the horizontal vernier at 0°, and clamp the upper motion. (3) Sight at the flag pole held vertically on one of the other points, by turning the transit on the lower motion. When the vertical cross-hair is nearly on the flag pole, clamp the lower motion and bisect the flag pole by use of the lower motion tangent-screw. (4) Unclamp the upper motion and turn the telescope to the other point. Clamp and bisect the flag pole making the final setting with the tangent-screw controlling the upper motion. (5) Read and record the angle turned off on the horizontal limb. (6) By the same process determine the value of the other angles. (7) Determine the angular error of closure. (8) Reference the corners by two or more linear ties to nearby permanent objects of very definite character.

Hints and Precautions.—(1) To set up the transit, first set it approximately over the point, then adjust the legs to make the tripod head nearly level. Now pick up the instrument bodily, set it carefully over the point, and press each shoe firmly into the ground, at the same time adjusting each leg separately until the plumb-bob falls within ½ in. of the tack and the instrument is approximately level. Finally loosen two adjacent foot-screws, and slide the instrument on the adjusting head until the plumb-bob is over the tack. See that the adjusting head is well centered before beginning to set up. (2) Attach the plumb-bob to the instrument by a sliding bowknot if an adjusting device is not provided. Hard knots should never be tied in the plumb-bob string. (3) The plumb bob should be suspended at such a height as to just clear the tack. (4) Protect the plumb-bob from the wind while completing the "set up." (5) The flagman should stand directly behind the flag pole, holding it lightly with the fingers of both hands at the height of the chin, and allowing it to balance on the tack head. (6) The flag pole should never be stuck behind the point, in place of being held upon it, if accurate results are desired. (7) Before taking a sight, be sure that the eyepiece is accurately focused on the cross-hairs. The test for parallax is to move the head slightly from side to side while looking through the telescope. The eyepiece slide should be adjusted until the cross-hairs are very distinct and the objective should be moved until there is no apparent motion of the cross-hairs on the image. (8) When sighting, focus the objective carefully upon the flag pole so that it appears very distinct in outline. Do not allow the hand to rest upon the instrument or the clothing to brush against the tripod. (9) The vertical cross-hair should be sighted at the middle of the flag pole (not at the side) and as near to the ground as conditions will permit. (10) If it is necessary to sight at the top of the flag pole, the flagman should be extremely careful to hold it vertical. (11) Do not forget to clamp the motion before using the tangent-screw. (12) Be sure the right tangent-screw is being used (a common source of error with beginners). (13) In reading the angle, be sure to read all of the scale reading before adding the vernier reading (that is do not omit a 30′ space or a 20′ space). When reading the vernier, *have the eye directly over the coinciding graduations.* The reason for this is that, if the eye is not held directly over the coinciding graduation marks and the upper surfaces of the two plates are not in the same plane (as may be the case through wear), the graduations will appear to coincide when in reality they do not. Observe that the graduations on either side of those coinciding, fail to concur by the same amount. (14) The least reading, or fineness of reading of a vernier, is determined by dividing the value of the smallest division on the scale by the number of divisions of the vernier on one side of the index. (15) The plate bubbles should be leveled before measuring an angle, *but the foot-screws should not be disturbed between initial and final settings of the line of sight.* (16) The instrumentman should carefully avoid coming in contact with the tripod legs while he is sighting. (17) Tie lines should intersect at angles between 30° and 150°. If the lines were to intersect at lesser or greater angles, the re-location of the obliterated corner would become uncertain. (18) Make clear and complete notes. Tabulate measurements on the left-hand page; and in the sketch on the right-hand page show the location of the triangle with reference to important surroundings, such as buildings and roads, and show the exact length of reference ties.

Practical Applications.—The transit is almost universally used by engineers and surveyors for measuring angles on work requiring precision, such as city, railroad, mine, and highway surveys. On such surveys it is important that angles be read quickly and accurately. The vernier makes accurate readings possible, and the principle on which it is based should be understood.

Problem 13. Prolongation of a Line by Double Sighting with Transit

Object.—To produce a straight line with precision, setting stakes at intervals of approximately 300 ft.

Procedure.—(1) Set two points about 300 ft. apart in such position as to afford an open field for a thousand feet or more in advance. (2) Set the instrument on the forward point with the telescope inverted, using the lower horizontal motion, the upper motion being clamped. (3) Plunge the telescope and set a stake on the line a hundred paces in advance. Mark a point on the stake exactly on line. (4) Take a second backsight upon the rear stake in the same manner as before, with the telescope normal. Plunge the telescope again, and mark a point on the advance stake. (5) If this point is not coincident with the first point set, a point midway between them is on the line. (6) Set the transit over this point, and advance by the same process, backsighting upon the next point in the rear. Continue in this way for the desired distance. (7) Check the work by setting the instrument upon the rear point, sighting carefully upon the next point, and then noting the linear error of points set by double sighting, without moving either horizontal motion.

Hints and Precautions.—(1) Be sure that one backsight from each station is taken with the *telescope inverted* and one with the *telescope direct.* (2) Tacks should be set in all stakes, and after being set should be checked. A finely divided scale should be used for bisecting the distances. (3)

Observe all precautions suggested under Problem 12 which apply to this problem. In the field notes tabulate the approximate distances (pacing) between points set by double sighting and also the deviation of these points from a straight line.

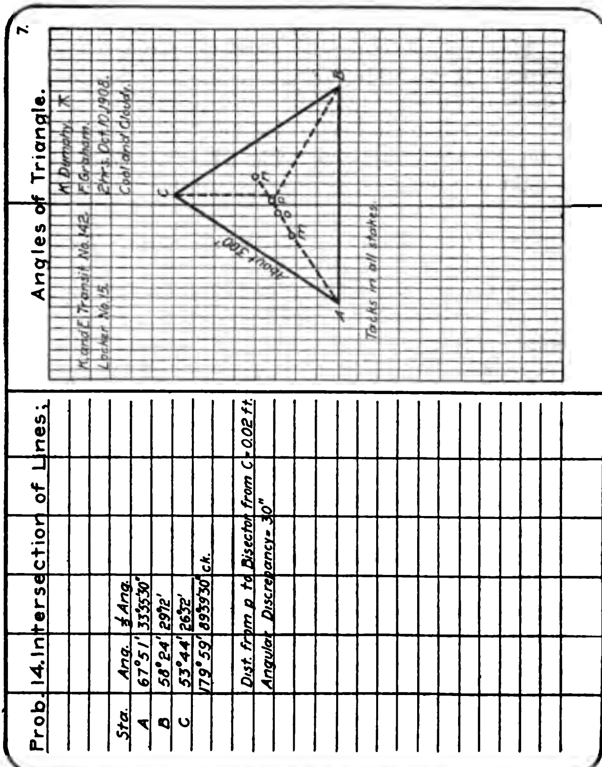
Practical Applications.—The method of double sighting is used when it is desired to set a point in advance accurately on line. The process of double sighting eliminates instrumental errors. It is used in prolonging lines of a considerable length, such as railroad tangents. Frequently a line prolonged by simply plunging the telescope with a transit supposed to be in perfect adjustment has later been found to be not a straight line but a curve of large radius.

PROBLEM 14. INTERSECTION OF LINES WITH TRANSIT

Object.—To bisect the three angles of a triangle, and to mark the point of concurrence of the bisectors.

Procedure.—(1) Drive three stakes, *A*, *B*, and *C*, at the vertices of a roughly equilateral triangle, with sides 300 or 400 ft. in length. Tack each stake. (2) Set up the transit and measure the angle at *A*. Lay off one-half of the measured amount, thus establishing the bisector of angle *A*. On the bisecting line of sight and on an estimated bisector of angle *B* drive a stake *o*, and partially drive a tack. Set two more tacked stakes, *m* and *n*, on the bisecting line of sight about 10 ft. from and on opposite sides of *o*. (3) Set the transit over *B* and locate the position of the bisector as at *A*. Drive a stake on this line and under a cord stretched from *m* to *o* or *n* to *o* as the conditions require. Tack the exact point of intersection, *p*. (4) Set the transit on *C*, measure the angle, and bisect as at *A* and *B*. (5) Measure the discrepancy between this bisector and the point of intersection of the first two bisectors at *p* to hundredths of feet. Also measure the angular discrepancy; it should not exceed two minutes. See accompanying sample notes, page 58.

Hints and Precautions.—(1) In setting the tacks on stakes *o*, *m*, and *n*, it is convenient to leave them projecting a little so that the cord may be tied in place. (2) To estimate the bisector of angle *B*, stand on the point and extend the hands toward *A* and *C* respectively. Now, with the eyes closed, bring the hands together. The point of meeting will be approximately on the bisector. (3) Observe all precautions



SAMPLE NOTES 14.

mentioned under Problems 12 and 13 that apply to this problem.

Practical Applications.—The method here used for determining the point of intersection of two lines is used in practice whenever it is necessary to find the meeting point of two lines produced at an angle with each other. For example, to measure the angles between the walls of a building it is necessary to construct lines parallel to each wall and to find their point of intersection over which the transit is to be set to measure the angle. (See Problem 15.)

Problem 15. Measurement of Angle when Transit Can Not be Set at Vertex

Object.—To measure the angle between two walls of a building.

Procedure.—(1) Drive and tack stake *a* (Fig. 12) at a convenient distance from the wall. Determine the perpendicular distance from the wall by holding a point of the tape

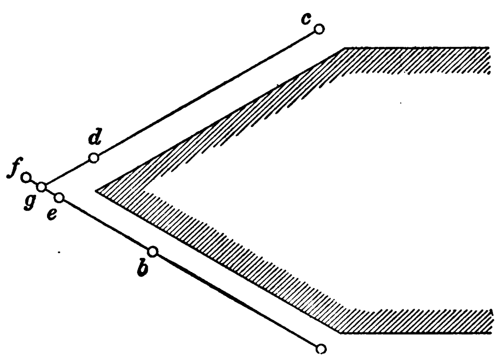


Fig. 12.—Angle Between Walls of Building.

on the tack and swinging the end through an arc, varying the radius until the arc becomes tangent to the wall. (2) Drive and tack another stake *b* near the other end of the wall and in such position that, when the tack is used as a center with the same radius as at *a*, the arc will again be tangent to the wall. The line *ab* will be parallel to the wall.

(3) Set stakes *c* and *d* in the same manner, making *cd* parallel to the other wall. (4) Set the transit at *a* and sight on *b*, then set and tack two stakes, *e* and *f*, on the line and on opposite sides of line *cd* prolonged. Stretch a cord between the points marked on *e* and *f*. (5) Set the transit on *c*, sight at *d*, and set point *g* on line and under the cord. (6) Move the transit to the intersection of the two lines, and measure the angle *agc*, which is equal to the angle between the walls.

Hints and Precautions.—(1) Some well-defined straight line of the wall should be selected as the working line. (2) Set the transit at *a* and *c* to prolong the lines to *g* instead of at *d* and *b*, to avoid any error that might result from plunging the telescope.

Practical Applications.—This problem is of frequent occurrence in structural work. It is necessary to know accurately the angle between two walls wholly or partly in place in order to detail the structural fabrics.

Problem 16. Prolongation of Line Past Obstacle (Offset Space Limited)

Object.—To prolong a line, determined by two fixed points, past an obstacle when the conditions are such as to limit the lengths of the offsets.

Procedure.—(1) Let *A* and *B* (Fig. 13) be two points on the same side of an obstacle. Set up the transit at *A*, sight on

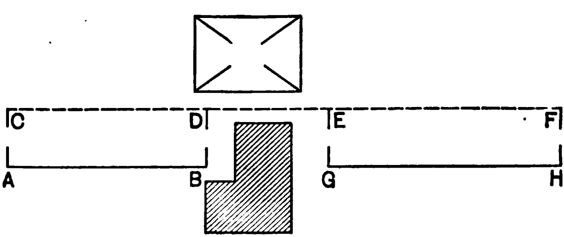


Fig. 13.

B, and turn off a right angle. Set and tack a stake at *C* on this perpendicular at a convenient distance from *A*, say 20 ft. (2) Set the transit at *B*, turn off 90° as before, and set another point *D* at the same distance from *AB*. (3) Set the transit on *D*, backsight on *C*, and by method of double sighting (Problem 13) set two stakes *E* and *F* in advance, one just beyond the obstacle and one at a considerable distance in advance. (4) From *E* and *F* offset to points *G* and *H*, using the same offset distance as for setting *C* and *D*. *G* and *H* are on the line *AB* prolonged, and from these points the line may be continued at will. (5) If the obstacle is imaginary, set the instrument on *A* and locate *G* and *H* directly for a check. Measure and record the errors at *G* and *H*.

Hints and Precautions.—(1) The lengths of offsets should be measured very carefully. (2) If the instrument is in good adjustment the points *E* and *F* may be set by a single reversal, or if a clear sight can be obtained the points may be set without reversal with the transit at *C*.

Practical Applications.—This problem is of very frequent occurrence in land and highway surveys. It is necessary within confined limits to offset in order to prolong a line which is obstructed by fences, hedges, brush, etc. The shortness of the offset distance necessitates taking two offsets and measuring them very carefully, because turning a right angle from the short backsight *DB* will not give an accurate long foresight. A short backsight should never be depended upon to give an accurate foresight.

Problem 17. Determination of an Inaccessible Distance

Object.—To obtain the distance between two points on opposite sides of a river.

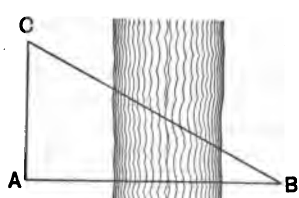


Fig. 14.

Procedure. *First Method.*—(1) Set the transit on *A* (Fig. 14), sight on *B*. Turn off an angle of 90°, and set *C* at a distance nearly as great as *AB*. (2) Measure angle *ACB* and length *AC*. Then $AB = AC \tan ACB$.

Second Method.—(1) Set the transit on A (Fig. 15), sight on B. Turn off 90°, set C at a distance estimated to be nearly equal to AB, and measure AC. (2) Set transit at C, sight on B, and turn off 90° to set D which must also be on line AB.

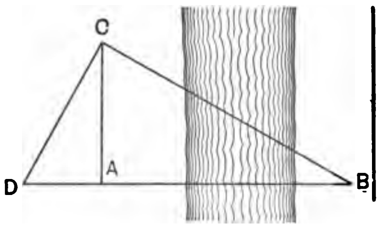


FIG. 15.

(For method of locating D at intersection of lines see Problems 14 and 15.) (3) Chain the distance AD. (4) Triangles ABC and ACD are similar. Hence $AB = \frac{\overline{AC}^2}{AD}$. If the river is imaginary, measure the distance AB for a check.

Hints and Precautions.—(1) The distance AC in each case should equal at least one-half of AB. (2) If good results are expected, distances must be carefully measured.

Practical Applications.—This problem is of use in measuring the distance between any two intervisible points where the distance can not be directly chained. The second method permits a solution when trigonometric tables are not at hand.

PROBLEM 18. ANGLES BY REPETITION

Object.—To obtain a more accurate determination of the angles between various stations about a point than would be possible by a single measurement. See Sample Notes 18.

Procedure.—(1) Set the transit very carefully over the point. (2) Set the A vernier at zero, read the B vernier, and record the readings. (3) With the telescope in its normal position, measure one of the angles in a clockwise direction, and record both vernier readings to the smallest reading of the vernier. (4) Leaving the upper motion clamped, again set on the first point and again measure the angle in a clockwise direction (thus doubling the angle). (5) Continue until six repetitions have been secured. Record both vernier read-

Prob. 18. Angles About △ 47 By Repetition. 8.

Sta. From-To	Direction	Bubble	Ver. A.	Ver. B.	Ver. Mean.	Single Ang.	Mean	Adj. Ang.	Notes.
50	↻	Down	0°00'00"	180°00'30"	0°00'15"				B. Hobbs, ⊼ G. Whit,
			89°23'	269°23'					2hrs. August 10, 1913.
53			536°21'00"	716°21'00"	536°21'00"	89°23'27"			Very Hot. Berger Transit No. 8.
50	↺	Up	180°00'00"	0°00'00"	0°00'00"				One Minute Vernier Estimated to 30." Six Repetitions.
53			716°21'30"	536°22'00"	536°21'45"	89°23'37"	89°23'32"	89°23'37"	
53	↻	Down	0°00'00"	180°00'00"	0°00'00"				
			51°44'	231°44'					
48			310°21'30"	490°21'00"	310°21'15"	51°43'33"			
53	↺	Up	179°59'30"	0°00'00"	359°59'45"				
48			490°21'30"	310°21'30"	310°21'30"	51°43'37"	51°43'35"	51°43'40"	
48	↻	Down	0°00'00"	180°00'00"	0°00'00"				
			218°53'	398°53'					
50			1313°16'00"	1493°15'30"	1313°15'45"	218°52'37"			
48	↺	Up	180°00'30"	0°00'00"	0°00'15"				
50			1493°16'30"	1313°16'00"	1313°16'15"	218°52'40"	218°52'39"	218°52'45"	
							359°59'46"	360°00'00"	ck.

SAMPLE NOTES 18.

ings and the total angle turned. (6) In a like manner, setting the B vernier at zero, measure the explement of the angle in a *counter-clockwise* direction with the *bubble down*, but read the horizontal circle as though the *angle itself* had been measured clockwise. (7) Go through the same process for all other angles about the point. (8) Compute the value of each of the angles for each direction turned and compare with the single measurement. (9) Find the mean of each of these sets of single angles. For a transit reading to single minutes the *total error should not exceed* $10''\sqrt{\textit{number of angles}}$. (10) Adjust the angles so that their sum shall equal 360° by distributing the error equally among the mean values.

Hints and Precautions.—(1) Level the transit very carefully before each repetition, but do not disturb the leveling screws while a measurement is being made. (2) The mean of each set of single angles should furnish a value free from instrumental errors. The station adjustment is an attempt to distribute the accidental errors so that the condition, that there are 360° about a point, shall be fulfilled. (3) Do not become confused when calculating the total angle turned. Observe how the horizontal limb is graduated, and do not omit 360°. (4) The instrument should be handled very carefully. When turning on the lower motion, the hands should be in contact with the *lower* plate (not the alidade), and when making an exact setting on a point, the last movement of the tangent-screw should be *clockwise* or against the opposing spring. (5) After each repetition the instrument should be turned on its lower motion in a direction *opposite* to that of the measurement. (6) The single measurement is taken as a check on the number of repetitions. It should agree closely with the mean value.

Practical Applications.—This method is used in triangulation work. The number of sets of readings and the number of repetitions in each set observed depend upon the desired accuracy.

Problem 19. Laying off Angle by Repetition

Object.—To lay off a given horizontal angle more accurately than is possible with a single setting of the vernier.

Procedure.—(1) Drive and tack two stakes about 500 ft. apart. (2) Set the transit carefully over one end of the line.

Sight at the point at the other end, and lay off the angle. (3) Set a stake on the line of sight about 500 ft. from the instrument (distance by pacing), and carefully set a tack. (4) By repetition measure the angle laid off, as in the previous problem, making six repetitions in each direction. (5) Find the difference between the angle laid off and the required angle, and by trigonometry compute the linear distance that the tack must be moved perpendicular to the line of sight. (6) Set the tack accordingly.

Hints and Precautions.—(1) The angle laid off must be increased or decreased by the difference between it and the required angle. This difference is too small to lay off with the transit, but the length of its arc can be computed. For very small angles the length of this arc is equal to the radius times the sine of the angle of error.

Practical Applications.—This method is of use in laying out large buildings or valuable city lots. With a 01′ transit an error of 30″ in a single reading might easily occur; in 300 ft. this would amount to approximately ½ in.

Problem 20. Deflection-angle Traverse of Field with Transit

Object.—To determine by the deflection-angle method the necessary data for plotting traverse and obtaining the area inclosed. (The data may be used in Problems 73, 84, 85, and 86, pages 183, 206, 211, and 213.)

Procedure.—(1) Stake out an irregular field containing an area of an acre or more. (2) Measure the sides carefully with the steel tape and chaining pins (see Problems 2 and 3), recording the distances to the nearest 0.01 ft. (3) Set the transit over one station and with the *A* vernier set at 180°, sight upon the preceding station. (4) Read and record the magnetic bearing of this backsight. (5) Turn the transit in azimuth on the upper motion to sight the next forward point. The vernier reading will now indicate the amount of the deflection angle. Observe whether the deflection is right or left from the prolongation of the preceding line, and record *R* or *L* accordingly, after the value of the angle in the deflection-angle column of the notes. Also read and record the magnetic bearing of the forward line. (6) Before the transit is moved, calculate the

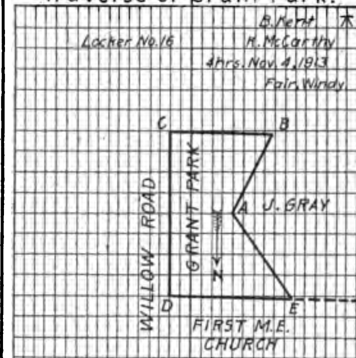
Prob. 20 Deflection Angle

Sigas, N.B.

Sta. At–To		Dist. —	Def. Ang.	Mag. Bear.	Ang. of Bear.	Cal. Bear.
A	E		57°54′R	N 28° W	58° R	
	B	507.65		S 30° W		S 29°37′W
B	A		113°38′L	N 29½° E	113½° L	
	C	784.68		S 84° E		S 84°00′E
C	B		98°15′L	N 84° W	98¼° L	
	D	994.60		N 2¼° W		N 2°15′W
D	C		88°19′L	S 2° E	88¼° L	
	E	739.72		S 89½° W		S 89°26′W
E	D		117°43′L	N 89½° E		
	A	526.00		S 28¼° E	117¾° L	S 28°17′E
			359°59′			

9\.

Traverse of Grant Park.



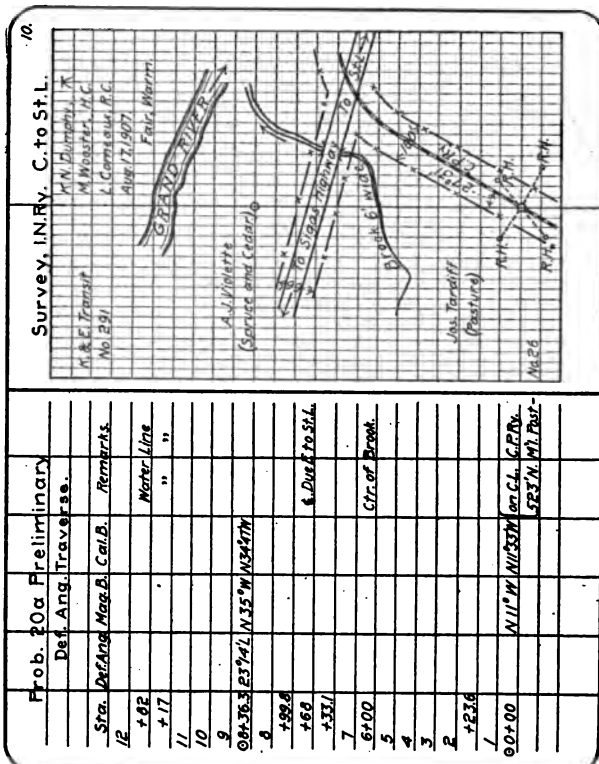
Note: Stakes set at corners, tacked and marked as shown in sketch.

SAMPLE NOTES 20.

deflection angle from the magnetic bearings taken at the station, and compare with the deflection angle indicated on the horizontal circle, as a rough check upon the amount and deflection of the angle. (7) Repeat the process at each station until the fieldwork is completed. (8) Add the deflection angles, assuming those to the right as positive and those to the left as negative. The algebraic sum should be 360° for any closed traverse. The *angular error of closure should not exceed* $30'' \sqrt{\textit{Number of Sides}}$. (9) Select and assume as correct the magnetic bearing of some course whose back and forward bearings have the same angular value, and from this and the successive deflection angles, calculate the bearing of each of the other courses. See Sample Notes 20.

Hints and Precautions.—(1) Do not forget to read the magnetic bearing both forward and back from each station. Angles computed from these bearings, read from the same station, will contain no error due to local attraction (unless caused by passing disturbance), as the needle has the same declination no matter in which direction the transit is sighted. (2) When reading the deflection angle on the horizontal circle, be sure to read from the circle which is graduated from 0° in the direction of the deflection. (3) If the instrument is in good adjustment, the *A* vernier may be set on 0° and the backsight taken with the telescope in the reversed position (bubble up). When the telescope is again plunged, the line of sight will lie on the line of backsight prolonged, and the vernier will be in a position to be read without the observer moving about the transit. (4) Hubs should be driven well into the ground so that they will not be easily moved. (5) A witness stake is usually driven at one side and leaning over each hub. The stake should be marked with the station number on the side next to the hub. Points likely to be disturbed before they are needed should be tied to nearby permanent objects.

Practical Applications.—The deflection-angle method is one of the two most common methods of transit traversing either for closed or continuous traverses. It is particularly well adapted to railroad location and similar work. On such surveys stations are established every hundred feet and numbered consecutively from the beginning of the line, "Sta. 103" indicating that the point is 10,300 ft. from the point of beginning. Intermediate stations are indicated by plusses.



Sample Notes 20(a).

Thus, "Sta. 103 + 57" represents a point 10,357 ft. from the point of beginning. Such notes are usually begun at the bottom of the page instead of the top, and the center line of the right-hand page represents the transit line. Sample Notes 20 (*a*) show a portion of the survey of this kind.

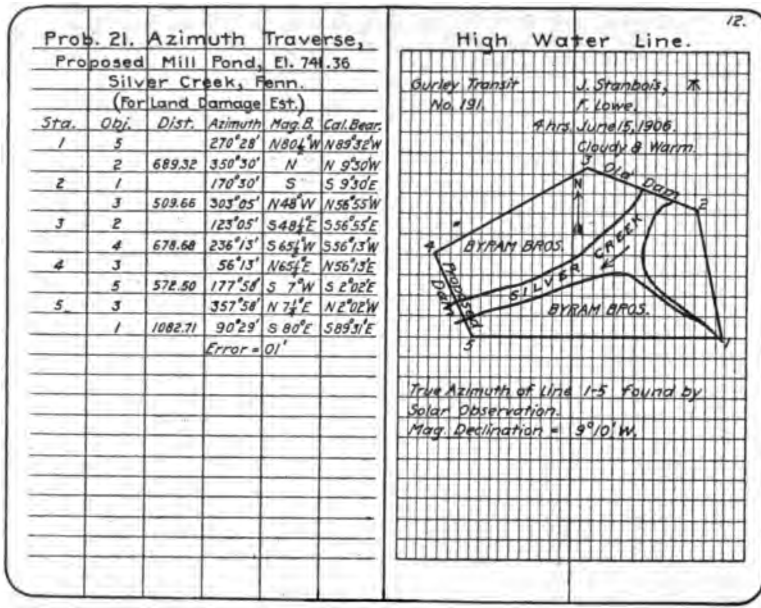
Problem 21. Azimuth Traverse with Transit

Object.—To collect data sufficient for plotting and determining the area of a field by the azimuth method. (The data may be used in Problems 73, 84, 85, and 86, pages 183, 206, 211 and 213.) See Sample Notes 21, page 70.

Procedure.—(1) Stake out the field and measure the sides as in Problem 20. (2) Set the transit over one station. Set the *A* vernier* at 0° and turn on the lower motion to sight along the meridian either north or south according as azimuths are to be reckoned from north or south. Clamp in this position. (3) Unclamp the upper motion, and turn the instrument to sight the next point forward. The angle turned off in a clockwise direction and read from the *A* vernier is the azimuth of the forward line. (4) Read and record the magnetic bearing. (5) From the azimuth, calculate the bearing of the line, and check with the magnetic bearing. Compute the back azimuth of the line by adding 180° to the forward azimuth. (6) Set the instrument on the next forward point with the vernier set on the back azimuth of the line, and backsight on the point previously occupied. Now turn upon the upper motion to sight the next point in advance; the azimuth of the forward line will be indicated by vernier *A*. (7) Proceed in this manner until all of the stations have been occupied. (8) Note the angular error of closure, which should not exceed $30'' \sqrt{\textit{Number of Sides}}$.

Hints and Precautions.—(1) Since the traverse forms a closed field, a check will be obtained if the back azimuth is read from the first point occupied. This should be 180° more or less than the forward azimuth of the same line obtained from the last station occupied. (2) Each time an azimuth is determined, its calculated bearing should be com-

* Either vernier may be used, but with the telescope in the direct position (bubble down) it will be more convenient with most transits to read the *A* vernier.


12.

Prob. 21. Azimuth Traverse, High Water Line.

Proposed Mill Pond, El. 741.36
Silver Creek, Penn.
(For Land Damage Est.)

Sta.	Obj.	Dist.	Azimuth	Mag. B.	Cal. Bear.
1	5		270°28'	N80¼°W	N89°32'W
	2	689.32	350°30'	N	N 9°30'W
2	1		170°30'	S	S 9°30'E
	3	509.66	303°05'	N48°W	N56°55'W
3	2		123°05'	S48¼°E	S56°55'E
	4	678.68	236°13'	S65¼°W	S56°13'W
4	3		56°13'	N65¼°E	N56°13'E
	5	572.50	177°58'	S 7°W	S 2°02'E
5	3		357°58'	N 7¼°E	N2°02'W
	1	1082.71	90°29'	S 80°E	S89°31'E
			Error = 01'		

SAMPLE NOTES 21.

pared with the magnetic bearing of the course. (3) If the azimuth of any line of the traverse is known, the transit may be set over one end of this line and the vernier at once set to read the proper azimuth. The transit will be oriented when its line of sight lies along the line, the azimuth of which is indicated by the vernier setting. The azimuth of the initial line may be arbitrarily assumed, but for facility in checking, the azimuth should be so chosen that the calculated and magnetic bearings will agree very closely. (4) The transitman may, instead of increasing the horizontal reading by 180° for each backsight, simply use the other vernier which, if there has been no slip between the plates in carrying the transit from one station to the next, will record the back azimuth of the line. This will necessitate plunging the telescope for each backsight. If the upper motion is left clamped after each foresight until the corresponding backsight has been taken in the manner just described and both foresight and backsight from any point are taken with the telescope in the same position (bubble up or down) and with the same vernier, neither compensating error due to imperfect vernier settings nor cumulative error due to the line of sights not being perpendicular to the horizontal axis will be introduced. Thus, this method has some advantage over the one described in the main part of the problem. If this method is used, the transitman should always read the vernier before a backsight is taken to make sure that no slip has occurred. As an insurance against using the wrong vernier, or having the telescope in the wrong position, the note keeper may mark the station numbers in the notes alternately *A* and *B*, or he may note that at odd numbered stations azimuths are read from vernier *A* with the bubble down and at even numbered stations from vernier *B* with the bubble up.

Practical Applications.—Perhaps more traverse lines are run by the azimuth method than by any other. Its advantage is that the direction of any line is indicated by the simple statement of one angle. This fact makes the method especially convenient for topographic surveying where many angles are taken about one point. The bearing of a line can be calculated at a glance, and thus a check between the magnetic and calculated bearings can be obtained much more readily than when deflection angles are used.

Problem 22. Altitude with Transit

Object.—To determine the height of a building above the water-table.

Method 1. Procedure.—(1) Set the instrument at *A* (Fig. 16), a distance from the building about equal to its height. Level the telescope, and note the position of the horizontal hair on building; mark the point sighted, and measure the distance h_1 above or below the water-table. (2) Determine the index error of the vertical circle. Sight on the

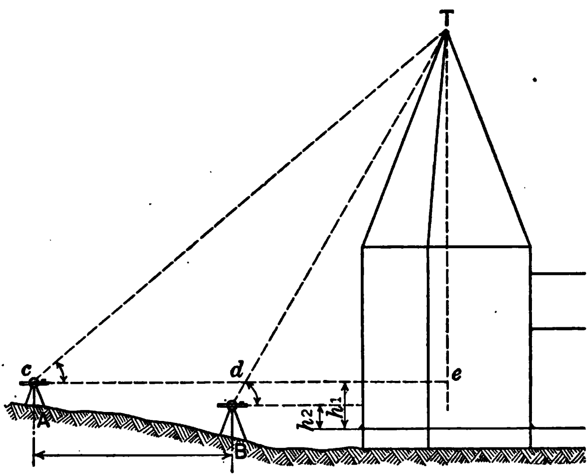


Fig. 16.

high point *T* of the building, and read and record the vertical angle indicated on the vertical circle. (3) Plunge the telescope, and set point *B* in the same vertical plane with *A* and *T*. (4) Set the transit on *B*, and measure h_2 and the verticle angle to point *T*, as at *A*. (5) Measure the horizontal distance *AB*. (6) Calculate the difference in elevation between *T* and the water-table by solving for the lengths of the lines *cd*, *dT*, and *Te*.

Method 2. Procedure.—(1) Set the transit at *A* (Fig. 17) in a convenient position and at a distance from the building roughly equal to its height. (2) Note the index error of the vertical circle. Set the horizontal vernier *A* at 0°, sight at the high point T, and read the vertical angle. (3) Depress the telescope to a horizontal position, mark the point a, and measure the distance h above or below the water-table. (4) Now turn off 90° on the horizontal circle, and set point B on line 100 ft. from A. (5) Set the transit at B, and with horizontal vernier at 0°, sight at T. Turn on the upper hori-

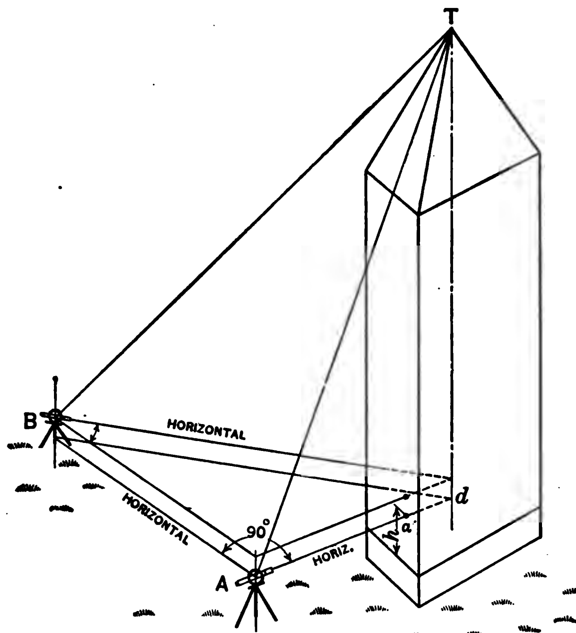


FIG. 17.

zontal motion to A, thus measuring the horizontal angle B. (6) Solve the two right triangles BAd and TdA for the height dT. $dT + h = H$ the required altitude.

Hints and Precautions.—(1) The index error may be read either before or after observing a vertical angle, but should be determined with the *line of sight in the same vertical plane as the point to which the vertical angle is taken, i.e.*, with the same horizontal vernier reading. (2) In case the transit used is provided with a movable vertical vernier controlled by a tangent-screw, the index error should be eliminated by making the vernier read zero when the telescope is level.

Practical Applications.—Although a problem of this kind is not of frequent occurrence in the work of the engineer, it is sometimes desirable to obtain altitudes which can not be measured directly, as in a topographic survey of mountainous country.

PROBLEM 23. DETAILS WITH TRANSIT AND TAPE

Object.—To obtain sufficient data for plotting a detailed map of a portion of the campus. (The data may be used in Problem 88, page 225.)

Procedure.—(1) Run a closed traverse by the azimuth method (Problem 21) through the area to be mapped, so locating the lines and corners of the traverse that linear and angular measurements necessary to fix the position of details with respect to the traverse lines may be taken with the least labor. (2) Obtain the location of buildings, streets, walks, hydrants, fences, etc., by the several methods for locating such details from a traverse line. (3) Sketch indefinite details such as trees, streams, shrubbery, etc., without measurements other than by pacing.

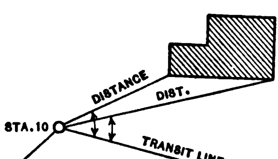


FIG. 18.

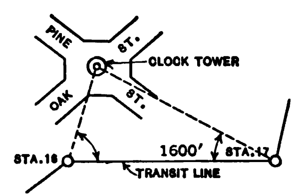


FIG. 19.

Hints and Precautions.—(1) Details may be taken as the traverse is run. (2) Note the angular error of closure of the traverse. If this exceeds $30''\sqrt{\textit{Number of Sides}}$ re-run the traverse until the mistake is discovered. (3) Reference the hubs so that if discrepancies are later discovered the points may again be occupied by the transit. (4) Details may be located with the transit by an azimuth and distance from one point (Fig. 18), or by azimuths from two points (Fig. 19). Details may be located with the tape by distances from two transit points (Fig. 20), or by perpendicular offsets from the transit line and distances along the line (Fig. 21). In addition to the foregoing, buildings may be readily located when not too far from the transit line by range ties, swing offsets, or simple ties (Fig. 22). No matter how many points of an

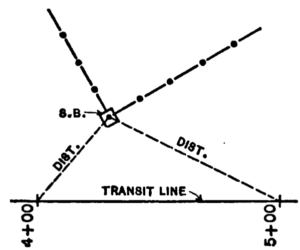


FIG. 20.

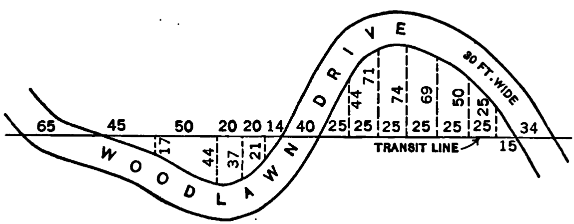


FIG. 21.

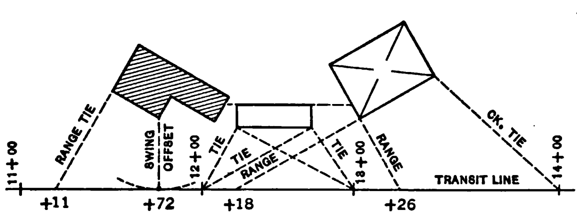


FIG. 22.

object have been located, its dimensions should be obtained by direct measurement where such is possible. This applies particularly to the length of land lines, streets, and sides of buildings. The location of important details should be checked, preferably by some other method than that used in the first location. No set rules can be laid down as to how details shall be located, for conditions are too varied; but the surveyor should endeavor to choose that method which will give the required accuracy with the least time and labor. (5) Sketches should be made approximately to scale, and should not be so crowded with measurements as to become confusing. Many measurements can be tabulated on the left-hand page and referred to on the sketch by a single letter or number.

Practical Applications.—The methods of this problem are those in general use where a considerable degree of precision is required and the details are definite in character, as in city surveying or surveys for large-scale maps.

CHAPTER IV

LEVEL PROBLEMS

Problem 24. Adjustments of the Wye Level*

(1) To Make the Horizontal Cross-hair Lie in a Plane Perpendicular to the Vertical Axis.

Test.—Sight at a clearly defined point about 150 ft. distant, and turn the instrument slowly about its vertical axis so that the point will appear to move from one end of the horizontal cross-hair to the other. If the point moves continually on the hair, it lies in a plane perpendicular to the vertical axis.

Correction.—If the point fails to follow the hair, loosen two adjacent reticule screws slightly, and turn the reticule in the barrel of the telescope until the point appears to traverse the entire length of the hair when the instrument is revolved. Tighten the *same* screws.

(2) To Make the Line of Sight Coincide with the Axis of the Wyes.

Test.—Unclamp the wyes, and sight the intersection of the cross-hairs upon a well-defined point 150 to 200 ft. from the instrument. Clamp the vertical axis. Rotate the telescope 180° in the wyes (bubble up). If in adjustment, the intersection of the cross-hairs will remain unchanged.

Correction.—If not, adjust the reticule until the intersection takes a position *midway* between the original point and the point sighted in the rotated position. Loosen only one screw at a time and tighten the opposite screw.

(3) To Make the Axis of the Bubble-tube Lie in the Same Plane with the Axis of the Wyes.

Test.—Bring the bubble to the center of the tube by means of the foot-screws. Rotate the telescope in the wyes through a small angle (perhaps 10°). If in adjustment, the bubble will remain in the center of the tube.

* In describing the adjustments of the wye level it is assumed that the optical axis coincides with the axis of the wyes and that the objective slide is permanently fixed as far as lateral motion is concerned. For level with adjustable objective slide see page 78. Also see Note, page 47.

Correction.—If the bubble moves, bring it back to the center of the tube by means of the azimuth screws.

(4) To Make the Axis of the Bubble-tube Parallel to the Axis of the Wyes.

Test.—Level the instrument, and clamp the telescope over one pair of foot-screws. Loosen the wye clips, and very carefully turn the telescope end for end in the wyes. If the bubble is in adjustment, its position will remain unchanged.

Correction.—If the bubble moves from the center of the tube, bring it *halfway* back with the altitude screws. Level up the instrument, and repeat the test to insure perfect adjustment.

(5) To Make the Axis of the Bubble-tube Perpendicular to the Vertical Axis.

Test.—Level the instrument, as well as its condition will permit, over both pairs of foot-screws; then bring the bubble carefully to the center over one pair. Revolve the level 180° about its vertical axis. If in adjustment, the bubble will retain its position.

Correction.—If the bubble moves away from the center of the tube, bring it back *one-half* of the displacement with the wye nuts. Level up, and repeat the test to insure perfect adjustment.

Level with Adjustable Objective Slide.—(See Transit with Adjustable Objective Slide, page 51). The adjustments of a level having an adjustable objective slide are the same as those just described except that after Adjustment (2) for a distant point, a near point is sighted and the telescope revolved in the wyes as in Adjustment (2). If the intersection of the cross-hairs moves off the point the adjustable objective ring is moved by its controlling screws until the cross-hairs appear to traverse over one-half the apparent error. The relation between this adjustment and the cross-hair adjustment is such that each must be repeated until both are found to need no further correction.

To Center Eyepiece Slide.—See Corresponding Adjustment of the Transit, page 52.

Hints and Precautions.—(1) The test of every adjustment should be repeated after the correction has been attempted, in order to insure perfection of adjustment. (2) Adjustment (4) fails if the telescope rings are worn or are not of equal size. In this case the wye level must be tested as a dumpy level

PLATE V.—ENGINEERS' WYE LEVEL (ERECTING EYEPIECE).
(From a photograph furnished by C. L. Berger & Sons.)

(*Facing page 78.*)

and adjusted as such, by the peg adjustment. (See Adjustments of the Dumpy Level, Problem 25.) (3) The adjusting screws should be brought to a snug bearing, but care should be taken not to strain the metal. (4) The azimuth and altitude adjustments of the bubble depend on each other. For this reason each should be repeated after the other has been made, until no further adjustment of either is necessary.

Problem 25. Adjustments of the Dumpy Level

(1) To Make the Horizontal Cross-hair Lie in a Plane Perpendicular to the Vertical Axis.

The method of procedure is identical with that of Adjustment (1) of the wye level.

(2) To Make the Axis of the Bubble-tube Perpendicular to the Vertical Axis.

The method of procedure is identical with that of Adjustment (5) of the wye level, except that the correction is made with the altitude screws at the end of the bubble-tube instead of the wye-nuts.

(3) To Make the Line of Sight Parallel to the Axis of the Bubble-tube (Two-peg Method).

Test.—Set two stakes on approximately level ground about 300 ft. apart. Set the instrument in such position that the

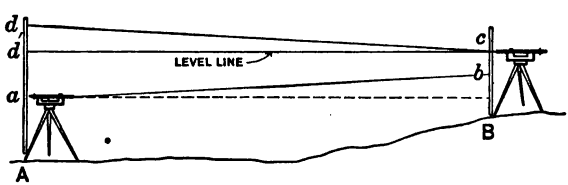


Fig. 23.—The Two-peg Test.

eyepiece is ½ in. or less in front of the rod held on one stake, as at *A* (Fig. 23). Take a rod reading *a* on *A*, sighting through the objective end of the telescope and centering the small circle of rod seen through the eyepiece. Take a rod reading *b* on *B*. Move the instrument to *B*, set up as before, and take rod readings *c* and *d*. The correct rod reading at *A* for a level line with the instrument in its last position at *B* will be

$d' = \frac{c + (a - b) + (d - c)}{2}$.* If the line of sight is in adjustment, d will equal d'. The mean of the two differences in elevation found from the rod readings taken from each position of the instrument will represent the true difference in elevation, since the line of sight will be inclined as much in one position of the instrument as in the other.

Correction.—If not in adjustment, move the cross-hair ring vertically until the line of sight cuts the rod at d', which is the rod reading for a level line.

Hints and Precautions.—(1) The screws controlling lateral movement of the reticule should not be moved during the peg test. After Adjustment (3) has been completed, the test for Adjustment (1) should be repeated. (2) Each test should be repeated after the correction has been attempted in order to insure perfect adjustment. (3) Undue strain should not be brought upon the adjusting screws. (4) If a tape is at hand or if the instrument is equipped with stadia-hairs, the true difference in elevation between the points A and B may be found by setting the instrument at the mid-point of the line AB. With the level in this position the error in rod reading, due to line of sight not being horizontal, will be the same at A as at B. The difference in elevation will, then, be the difference between rod readings. The correction is made by setting up the level at one peg and bringing the line of sight to the rod reading for a level line on the other peg as previously explained. (5) Readings should be observed to thousandths of feet.

Problem 26. Differential Leveling with Engineers' Level and Self-reading Rod

Object.—To determine the elevations of points in an assigned level circuit. See Sample Notes 26, page 83.

Procedure.—(1) Set up the level at a convenient distance from a *bench-mark* whose elevation is known or assumed, and along the general direction of the course. The distance from the initial bench-mark to the instrument should not

* For exact adjustment this rod reading should be corrected by adding the effect of curvature and refraction. This correction in feet is approximately $0.02\ M^2$ in which M is the distance from instrument to rod in units of 1000 ft. For a 300-ft. sight the correction amounts to a little more than 0.002 ft., a negligible quantity in ordinary differential leveling but one which must be considered in the adjustment of precise levels.

PLATE VI.—ENGINEERS' DUMPY LEVEL (INVERTING EYEPIECE).
(From a photograph furnished by C. L. Berger & Sons.)

(*Facing page* 80.)

be greater than 300 ft., and the position of the level should be such that the line of sight will not only strike the rod when held at the initial point but will also be favorable for a sight to a *turning-point* farther along the line. (2) With bubble centered read the rod (held on the initial bench-mark by the rodman) to 0.01 ft. and estimate to 0.001 ft. (3) Record the reading in the *backsight* (*B.S.*) column of the notes in line with the number and description of the bench-mark. (4) Center the bubble, and again read the rod as a check. (5) If the level is equipped with stadia-hairs, observe and record the intercept on the rod; if not, record the distance from bench-mark to level as paced by the rodman as he goes forward. (6) Calculate the *height of instrument* by *adding* the observed backsight to the elevation of the point sighted, and record in the *H.I.* column of the notes. (7) Take a *foresight* to a solid point (turning-point) chosen by the rodman, a convenient distance in advance of the instrument. Observe the same precautions as when reading the rod at the initial point. (8) The turning-point is designated in the station column of the notes by the initials *T.P.* On a new line record the number of the turning-point, its description if in the future it might be used as a bench-mark, the foresight reading, the elevation of the turning-point determined by subtracting the foresight reading from the height of instrument, and the distance from level to turning-point. (9) Move the instrument forward, and take a backsight in the same manner as at the first point occupied. Record the rod-reading and distance in their columns *in the same line with other data concerning the turning-point on which sight is taken.* (10) Continue in the same manner around the circuit, except at bench-marks where total lengths of backsights and foresights should be balanced.

Hints and Precautions.—(1) Select solid ground for instrument stations. Be sure to plant the tripod feet firmly in the soil. Spread the tripod well, especially if the wind is strong. (2) Remember, in leveling up, that the bubble will travel in the same direction as the left thumb. (3) Do not try to get the bubble in the center over one pair of leveling screws before bringing it approximately to the center over the other pair. It is a waste of time to center the bubble exactly before the rod has been sighted. (4) If one pair of leveling screws turn hard, loosen the other pair a little. Finish leveling up with all foot-screws bearing solidly but not so tightly as to spring the plate. (5) Test for parallax as follows: Slightly move the head from side to side while looking through the telescope with the instrument focused on the rod. If parallax exists, the cross-hairs will appear to move on the face of the rod. To remove this condition, first hold a white sheet of paper 2 or 3 in. in front of the instrument and focus the eyepiece until the cross-hairs appear sharp and distinct, then re-focus the objective carefully upon the rod. Once adjusted the focus of the eyepiece should remain perfect, but the objective focus must be changed for each different distance. (6) Look at the bubble just before taking each reading, and glance at it again just after, to be sure that it has not moved. The observer should stand in such a position that this will be possible without moving his feet. (7) Be sure the rod is held vertical while a sight is being taken; keep the foot of the rod free from dirt. (8) When the Philadelphia rod or similar rod is extended (long rod), it should be firmly clamped in the proper position; and between rod readings the rodman should examine it to see that no slip has occurred. Do not let the "long rod" down "on the run." (9) Slowly *wave the rod* toward and from the instrument when the "long rod" is used, and take the least reading on the rod. (10) Remember that the reading is recorded opposite the station number of the station on which the rod is held and has nothing to do with the instrument station. (11) Backsights are always taken upon points of *known* elevation; foresights, on points of *unknown* elevation. (12) Check all calculations by showing that the difference between the sum of the backsights and the sum of the foresights equals the difference in elevation between the initial and terminal stations. (13) Give a clear and concise description of each bench-mark and its location, on the right-hand page in line with numerical notes concerning that bench-mark. (14) Make the sum of backsight distances between bench-marks equal the sum of foresight distances as nearly as possible, to eliminate the effect of lack of adjustment of the instrument and curvature of the earth. (15) There should be a definite, well-understood system of signals between the instrumentman and rodman. (See Art. 14, page 11.) (16) The rodman should choose the position of the turning-point with an eye to simplicity of field operations. The turning-point may be an iron plug

13.

Prob. 26. Differential Levels. Self-Reading Rod. | **Eng. Hall to Ag. Bldg & Return.**

Berger Dumpy No. 114, Locker No. 30. J.K. Pratt, ⊼ C.P. Kidd, Rod
2 hrs. Oct. 21, 1912.
Cold & Cloudy.

Sta.	B.S.	H.I.	F.S.	Elev.	Dist. B.S.	Dist. F.S.	
B.M. 1	6.102	419.212		413.11	97		U.S.G.S. B.M. Cor. Eng. Hall.
T.P. 1	9.842	425.858	3.196	416.016	96	+11 108	Top Hydrant Front Σ. N. House.
T.P. 2	11.276	432.950	4.184	421.674	85	+15 100	Plug.
T.P. 3	3.616	435.654	0.912	432.038	104	−6 64	N.E. Cor. Lower Step Fellows Hall.
B.M. 2	2.256	431.796	6.114	429.540	84	−2 108	Concrete Monument S.W. Cor. Big Barn.
T.P. 4	3.387	423.535	11.648	420.148	44	+10 96	Plug.
T.P. 5	1.110	414.958	9.687	413.848	115	+42 76	Plug.
B.M. 3	5.302	415.887	4.373	410.585	96	0 73	Copper Bolt in Sidewalk Front of Ag. Bldg.
T.P. 6	10.415	423.655	2.647	413.240	80	−5 91	S.E. Cor. Brick Pier at Observatory.
T.P. 7	6.788	428.330	2.113	421.542	93	−5 80	Plug.
T.P. 8	0.669	423.837	5.162	423.168	75	−15 82	Plug.
B.M. 1			10.768	413.069		+2 93	U.S.G.S. B.M.
	60.763		60.804	413.110	969	971	ck.
			60.763	413.069			
			0.041 =	0.041	ck.		

Error = $0.04\sqrt{\text{dist. in miles}}$

Distances Expressed in Paces.
37 Paces = 100 ft.

Sample Notes 26.

driven in the earth and removed by the rodman as soon as backsight and foresight have been taken, or it may be on any solid object such as a rock, curb, rail, or root of tree. Permanent turning points should be marked and described in the notes.

Practical Applications.—This method of differential leveling is employed on nearly all work in which the relative elevation of points is desired and extreme precision is not necessary. With careful observers and a good instrument the error should not exceed $0.05\ ft. \times \sqrt{distance\ in\ miles}$. Such a degree of precision is maintained by the primary levels of the U. S. Geological Survey. If the allowable error is greater than this, readings to the nearest 0.01 ft. will be sufficiently exact and lengths of sights need not be observed.

Problem 27. Differential Leveling with Engineers' Level and Target Rod

Object.—To determine the elevation or difference in elevation of points assigned.

Procedure.—The procedure differs from Problem 26 only in that the rodman sets the target as directed by the instrumentman. Both men read the rod from the attached vernier to the nearest 0.001 ft. The rodman should record the distances and rod readings in a book for that purpose. The field notes will be kept by the leveler as explained in the preceding problem. Compare the results of this method with those of the preceding problem. Note the relative errors of closure of the circuits and the time required for each method per set-up of the instrument.

Hints and Precautions.—In addition to those for Problem 26, (1) The leveler should make his signals easily distinguishable, holding his hand well up to raise the target and well down to lower the target. It is not necessary to wave the hand. (2) The rodman should move the target rapidly at first until the opposite signal is given by the instrumentman; he should then move it slowly until the instrumentman indicates that it is in the proper position by the "all right" signal (extending arms horizontally). (3) After the target is clamped, the rod should be waved slowly toward and from the instrumentman, particularly when the "long rod" is used. If the horizontal line on the target appears above the horizontal hair, the target should be lowered. (4) When using the "long rod" the target must be clamped to read exactly the reading of the vernier on the back of the rod when the rod is short.

Practical Applications.—The target rod is usually used on surveys where a considerable degree of precision is required. However, it is possible with proper instrumental equipment and care to run levels with the self-reading rod with about the same precision and at a less cost. Mistakes are less likely to occur with the target rod, and for this reason it may well be used on lines of levels that are not to be checked by running in both directions or by a complete circuit.

Problem 28. Profile Leveling for a Railroad

Object.—To determine the elevations necessary for plotting the profile of a line. (The data of this problem may be used in Problem 81, page 202.) See Sample Notes 28, page 86.

Procedure.—(It is assumed that the line has been laid out and stakes, marked with consecutive station numbers, driven every 100 ft.) (1) If no bench-mark is given, select some permanent point as a bench-mark, assuming an elevation such that no station will fall below the datum plane. (2) Set up the instrument near the line in such position that it will be possible to read the rod, not only on the bench-mark, but also at all stations and necessary intermediate points between the bench-mark and the instrument. (3) Obtain the height of instrument by backsighting upon the rod held upon the bench-mark, reading the rod to the nearest 0.01 ft. (4) Obtain elevations of stations and necessary plus points on the line by foresights, reading to the nearest 0.1 ft. *with the rod held on the ground.* (Plus distances may be obtained by pacing, or by measuring with the rod when the distance is short.) (5) In this manner determine ground elevations as far along the line as is convenient from the position of the instrument. (6) Establish a turning-point as described in Problem 26. Take a foresight upon this point to the nearest 0.01 ft., and move the instrument forward to another convenient point some distance in advance of the turning-point. (7) Take a backsight and determine the new height of instrument. Proceed with the determination of ground elevations as before. (8) Continue the process to the end of the line. (9)

Prob. 28. Profile Levels — Cox Brook to Big Forks.

I.N.R. Location. 20.

Sta.	B.S.	H.I.	I.F.S.	F.S.	Elev.	
						Buff Dumpy Level. J.C. Brown, ⊼ F. Graham, Rod. Sept. 16, 1907. Fair.
B.M. 28	1.56	565.87			564.31	On Spruce Root 50 ft. Lt. Sta. 605.
605			0.7		565.2	
606			2.9		563.0	
607			3.5		562.4	
608			6.7		559.2	
609			11.9		554.0	
T.P.	0.41	554.65		11.63	554.24	On Stone
609+50			3.2		551.5	
610			8.6		546.1	
+40			9.0		545.7	Bank Cox Brook.
+50			12.2		542.5	Ctr. " " Water 1.5 ft. deep.
+65			9.3		545.4	Bank " "
611			8.4		546.3	
612			7.1		547.6	
613			8.4		546.3	
614			7.0		547.7	
615			4.1		550.6	
T.P.	8.02	559.94		2.73	551.92	On Plug.
616			9.7		550.2	
+40			6.3		553.6	Ctr. Highway to St. Leonards.
	9.99		564.31	14.36		
			559.94	9.99		
			4.37 =	4.37	ck.	

SAMPLE NOTES 28.

Here establish another bench-mark. (10) Run a line of check (differential) levels back to the initial bench-mark.

Hints and Precautions.—(1) Read the rod carefully to the nearest 0.01 ft. on bench-marks and turning-points and roughly and quickly to the nearest 0.1 ft. on the ground points. (2) To be consistent, ground elevations should be recorded only to the nearest 0.1 ft. (3) Rod readings should be taken at all full stations and such other points (plus stations) as are necessary to obtain a sufficiently accurate profile. These plus stations in general will be at points where the slope of the ground changes noticeably and at highways, railroads, and streams. (4) Bench-marks should be established every 1500 or 2000 ft. if the line is long. These should be placed at some distance to one side of the line, in such positions that they are not likely to be disturbed during construction. All bench-marks should be well described in the notes. (5) It is customary to mark the number and elevation on each bench-mark at the time it is established. It is important, therefore, that the computations for turning-points be performed as the work progresses, and each page of notes should be checked as soon as it is filled. The accompanying sample notes for a portion of a line of profile levels shows the computations for this check at the foot of the page.

Practical Applications.—Profile levels are run for railroads, sewers, pipe-lines, canals, and highways. Data for the elevations of contours on large scale topographic maps are often obtained by this method slightly modified.

Problem 29. Profile Leveling for a Pipe-line

Object.—To prepare a line for proposed sewer or water main for excavators. It is supposed that the line has already been run and that center stakes marked with station and plus have been set every 25 or 50 ft. (Data may be used in Problem 82, page 205.)

Procedure.—(1) Opposite each stake on the line and far enough from the line to insure its not being disturbed by the excavation, drive a short peg (or spike) flush with the ground, and beside this peg drive a stake marked with the station number of the center stake and the offset of peg from center stake. (2) Start from a bench-mark as in Problem 28, and take profile readings on the ground pegs to the nearest 0.01 ft.

Complete the level work as in Problem 28. (3) Roughly plot the profile, fix the grade of the bottom of the trench, and determine the amount of cut at each station. (4) Mark the cut in plain figures to the nearest ⅛ in. upon the side stakes, with waterproof kiel.

Hints and Precautions.—(1) Take rod readings upon the turning-points with greater care than upon the ground pegs. (2) The form of notes will be similar to Sample Notes 28, page 86, except that columns must be added for offsets of pegs from center line, grade elevations, and cuts. (3) All stakes should be marked to read down the stake. Center stakes should be driven with the marked side toward the beginning of the line. Side stakes should be driven side to the line in order that they will not be confused with center stakes. On the back or the side farthest from the line should appear the station number and offset; on the front or side nearest the line, the cut. (4) In paved streets or hard roads it is impossible to drive stakes or ground pegs, and 6-in. spikes are used instead, the spikes being driven flush with the road surface. In order that they may be found without difficulty, their position with respect to more prominent objects is carefully noted. (5) The purpose of the ground pegs is to furnish grade and line (after the center stakes are removed) to ditchers and pipers. (6) Observe the hints and precautions of Problem 28.

PROBLEM 30. SETTING SLOPE-STAKES. CROSS-SECTIONS

Object.—To prepare a proposed railroad for grading, and to obtain data for calculating earthwork. (The data may be used in Problem 80, page 199.) See Sample Notes 30, page 90.

Procedure.—(1) From the level notes of Problem 28 plot a profile and fix a grade (Problem 80) such that the amount of cut will approximately balance the amount of fill. (2) Drive short pegs flush with the ground beside the center stakes; run profile levels over the line, checking the elevations obtained with those of Problem 28; and mark on the back of each center stake the cut or fill at that point as *C* 3.9 or *F* 4.7. (3) Opposite each center stake at right angles to and on both sides of the line set slope-stakes at points where the side slope of the cut or fill will intersect the surface of the ground. These stakes should be driven side to the line, leaning toward the center line if in cut and away from the center line if in fill. They should be marked on the side facing the center line with the cut or fill and distance from the center stake, as C 6.2/19.3; on the side farthest from the center line with the station number of the center stake and whether on the right or left, as L 17 + 00. The numbers should read down the stake. (4) Drive ground pegs at "grade points" or points where the grade passes from cut to fill, and mark the position of such pegs by stakes marked "grade." Grade pegs should be driven at the center and on either side at a distance of one-half the width of the roadbed from the center. (5) Record all data as shown in sample notes.

Hints and Precautions.—(1) The distance from center to slope-stake may be found by the expression $D = \frac{W}{2} + SC$, in which W is the width of roadbed, S the rate of slope, and C the cut or fill at the slope-stake. Except on level ground, since the cut or fill will not remain constant, the distance D must be found by trial. The rodman, knowing the cut or fill at the center, holds the rod in the estimated position of the slope-stake, the distance from the center being measured with a metallic tape. The leveler reads the rod, and calculates the cut or fill and the distance D for such cut or fill. If this is not the measured distance, the rodman again estimates the position of the slope-stake. Thus the process is repeated until the measured and the computed distances agree. (2) It is necessary that the leveler be able to calculate rapidly the cut or fill. This may be done as follows: Calculate the rod reading for grade (*grade-rod*) at the station to be cross-sectioned. If the elevation of grade is *less* than the height of instrument, the cut or fill at any point will be the *difference* between the grade-rod and the ground-rod at that point; if the elevation of grade is *greater* than the height of instrument, the *sum* of the grade-rod and ground-rod will give the fill at that point. That these statements are true may be easily shown by sketches. (3) The width of roadbed is somewhat greater in cut than in fill on account of side ditches. The roadbed of the average American single-track railway is perhaps 22 ft. wide in cut and 16 ft. wide in fill. (4) The slope of the cut or fill will depend upon the material. For earth a slope of 1½ (horizontal) to 1 (vertical) is commonly used in practice; for soft clay, 3 to 1; for solid rock, ¼ to 1 or less depending upon its structure. (5) As an example, suppose that a cross-section is to be taken

Prob. 30. Cross-Sections.						I.N.R. Final Location.				21.
Cox Brook to Big Forks						June 4, 1908.			F.F. Smith.	⊼
Road Bed 20 ft in Cut, 16 ft. in Fill						Cloudy.			J. Richie.	Rod
Slope 1½ : 1									O. Byram.	Tape
Sta.	B.S.	H.I.	F.S.	Elev.	Grade	Left.	Ctr.	Right.	Remarks	
B.M.28	2.67	566.98		(564.31)					50 ft. lt. Sta. 605 + 10	
605			1.9	565.1	556.00	C8.6 / 22.9	C9.1	F11.2 / 24.8	Gravel in this Hill	
606			4.0	563.0	555.60	C5.0 / 17.5	C7.4	C8.4 / 22.6		
607			4.5	562.5	555.20	C4.5 / 16.8	C7.3	C8.0 / 22.0		
608			8.0	559.0	554.80	C1.8 / 12.7	C4.2	C5.1 / 17.7		
+25			9.2	557.8	554.70	0.0 / 10.0	C3.1	C2.6 / 13.9		
T.P.	1.94	557.19	(11.73)	(555.25)					On Plug.	
+90			2.8	554.4	554.36	F1.8 / 10.7	0.0	C2.4 / 13.6		
609			3.4	553.8	554.40	F3.2 / 12.8	F0.6 0.0 / 4.0	C1.0 / 11.5		
+20			5.6	551.6	554.32	F3.6 / 13.4	F2.7	0.0 / 10.0		
610			11.1	546.1	554.00	F8.2 / 20.3	F7.9	F5.3 / 15.9		
+40			11.2	546.0	553.84	F8.0 / 20.0	F7.8	F7.8 / 19.7	Top of Bank, Cox Brook	
+45			14.6	542.6	553.82	F11.2 / 24.8	F11.2	F11.2 / 24.8	In Brook	
+60			14.5	542.7	553.76	F11.1 / 24.7	F11.1	F11.1 / 24.7	" "	
+65			11.6	545.6	553.74	F8.2 / 20.3	F8.1	F8.0 / 20.0	Top of Bank.	
611			10.9	546.3	553.60	F7.8 / 19.7	F7.3	F7.2 / 18.8		
612			9.7	547.5	553.20	F7.6 / 19.4	F5.7	F3.1 / 12.7		
613			10.8	546.4	552.80	F7.6 / 19.0	F6.4	F2.0 / 11.0		
T.P.	11.96	559.69	(9.46)	(547.73)					On Stump.	
	16.57	564.31	21.19							
		559.69	16.57							
		4.62	= 4.62	ck.						

Sample Notes 30.

of an earth cut with 1½ to 1 slope. The roadbed is 20 ft. wide; the grade-rod is computed to be 12.0 ft. A slope-stake is to be set on the uphill side of the line. The rodman estimates the stake will be 19.0 ft. from the center. The leveler reads 5.4 ft. on the rod, which gives a cut of 12.0–5.4 equals 6.6 ft. and computes a distance out of $\frac{3}{2} \times 6.6 + 10.0$ equals 19.9 ft. Since the ground is rising it is evident that at a distance out of 19.9 ft. there would be a still greater cut. The rodman, realizing this, estimates the point at 22.0 ft. The rod reading at this point is 4.0 ft., the corresponding cut 8.0 ft., and the computed distance 22.0 ft. Thus the position of the slope-stake is determined. (6) In a party of two the tape may be held at the center by a knife stuck in the stake. (7) All hints and precautions of Problem 28, page 85, should be observed.

Practical Applications.—This method is essentially the same as that used on railroads, canals, and highways. For short sights, as when the cut or fill is small, the hand-level may well be used. The use of the Ward tape and tape-rod make greater speed possible. After setting the tape-rod to read the cut or fill at the center stake, other cuts or fills may be read directly from the rod. The Ward tape is a metallic tape graduated on one side in feet and tenths, and on the other side corresponding to the distance out, are amounts of cut or fill for a particular width of roadbed. The tapes are made for various slopes and widths of roadbed. Thus, with these accessories no computations are necessary.

PROBLEM 31. BALLAST GRADES FOR RAILROAD

Object.—To set grade stakes for ballast at changes of grade and full stations. (It is assumed that the road is finished to sub-grade.)

Procedure.—(1) From the elevations of sub-grade calculate the elevations of base or top of rail for the specified thickness of ballast beneath the tie. (2) Run a line of levels from some point of known elevation, drive a stake to the calculated grade (nearest 0.01 ft.) opposite each station and change of grade, and far enough from the center (6 or 7 ft.) so that it will not be disturbed. Mark the top of each stake with kiel to show that it is a grade point. (3) Where there are no curves and no changes of grade, "shoot in" the grade for distances as great as the condition of the atmosphere and the magnifying power of the telescope will allow. (4) Check the notes at each reading, and check the accuracy of the line of levels at every bench-mark.

Hints and Precautions.—(1) To "shoot in" a grade line proceed as follows: Drive a stake to grade at the end of the line near the instrument. Run a line of differential levels the desired distance, and set another stake to grade. With the instrument close to the stake last set, read the rod by looking through the objective end of the telescope and record the reading. With the rod held at the stake first set, elevate or depress the telescope until the horizontal hair cuts the rod at a similar reading. If possible, note the position of the horizontal hair on some prominent mark of the landscape. The line of sight is now a uniform distance above the grade line, and intermediate grade stakes may be set with the same rod reading. Check the line of sight frequently by observing the position of the horizontal hair on the reference mark. (2) If a target rod is used in "shooting" grade, it will be convenient to set the target at the constant reading. If the rod has no target, a card fastened with a rubber band will answer the purpose. (3) If a level is used in "shooting" grade, the telescope should be elevated or depressed by a single pair of foot-screws; in order that the horizontal hair may remain truly horizontal, this pair of screws should be brought in line with the distant stake. (4) Observe the precautions stated in Problem 28. (5) It frequently happens that stakes can not be driven to grade; it is customary to mark very plainly the position of grade on the sides of these stakes and to omit marking their tops.

Problem 32. Levels for Determining Quantities of Excavation for Grading Street

Object.—To determine the elevations of points necessary to plot the profile and cross-sections of a proposed city street. (The data may be used to Problem 83, page 205.)

Procedure.—(1) Drive stakes on a line 50 ft. apart to represent the center line of the street. Mark each with its proper station number beginning at 0 (50-ft. points are marked + 50). (2) Drive stakes opposite each of these on both sides of the center line at such distances from the center as 20, 35, and 40 ft. to mark right and left curb lines, right and left sidewalk lines, and right and left property lines. (3) Follow the

22.

Prob. 32. Levels for Profile and Cross-Sections. Rochester St. Extension.

B. & L. Transit No. 175. P. F. Davids ⊼
Phila. Rod. S. Marion, Rod
Sept. 6, 1913
Fair.

Sta.	B. S.	H. I.	F. S.	Elev.				
	L. Prop.	L. Sidewk	L. Curb	Ctr.	R. Curb	R. Sidewlk	R. Prop.	
4			3.37/740.28	3.48/740.17	3.59/740.06			On Ctr. Line Seavey St. (Paved).
+80			3.84/739.81	3.97/739.68	4.05/739.60			In Gutter.
+80			3.11/740.54	3.21/740.44	3.33/740.32			On Curb.
+62.5		3.12/740.53				3.44/740.21		Ctr. Sidewalk on Seavey St.
+50	3.2/740.5	3.8/739.9	4.3/739.4	4.9/738.8	5.6/738.1	5.7/738.0	5.8/737.9	On Ground.
T. P.	8.48	743.65	10.21	735.17				Spike in Post 50 ft. R. 3+00.
3	6.8/738.6	7.0/738.4	7.4/738.0	8.4/737.0	9.2/736.2	9.6/735.8	9.7/735.7	
+50	5.6/739.8	5.7/739.7	6.4/739.0	7.0/738.4	7.3/738.1	7.7/737.7	7.8/737.6	
2	4.1/741.3	4.3/741.1	4.8/740.6	5.1/740.3	6.1/739.3	6.4/739.0	6.5/738.9	
+50	2.5/742.9	2.6/742.8	3.2/742.2	4.1/741.3	4.4/741.0	5.0/740.4	5.2/740.2	
1	1.4/744.0	1.6/743.8	2.0/743.4	2.6/742.8	3.4/742.0	3.9/741.5	4.3/741.1	
+50	0.8/744.6	0.9/744.5	1.3/744.1	2.0/743.4	2.8/742.6	3.2/742.2	3.4/742.0	On Ground.
+37.5		1.42/743.96				1.78/743.60		Ctr. Concr. Sidewalk Lamb St.
+20			1.46/743.92		1.66/743.72			On Curb Lamb St.
+20			0.71/744.67	0.81/744.57	0.92/744.46			In Gutter on Lamb St.
0			1.22/744.16	1.31/744.07	1.41/743.97			On Ctr. Line Lamb St. (Paved)
B. M.	1.27	745.38		744.11				Point of Frog, P. & W. St. Car Line.

SAMPLE NOTES 32.

same method of procedure as in Problem 28. Keep the notes as shown in the accompanying sample notes, page 93. The figures above the short diagonal lines are ground-rod readings; those below are the elevations of the points.

Hints and Precautions.—(1) All hints and precautions mentioned in Problem 28, p. 87, apply to this problem. (2) Elevations of the points of intersection of the sidewalk, curb, and center lines of the proposed street with corresponding lines of existing paved streets or with bridges and railways should be determined with greater accuracy than ground elevations, as shown in the sample notes. (3) If the ground is rough or irregular at any section it will be necessary to obtain the elevations of extra ground points with their distances from the center stake, for otherwise the plotted cross-sections will not represent the true contour of the ground and cross-sectional areas can not be computed with exactness. If there are appreciable irregularities between sections where stakes have been set, it will be necessary to take extra sections, but no stakes need be set. (4) If the survey is over an existing street the ground may be so hard as to make the driving of stakes well-nigh impossible. In such case 6-in. wire nails may well be used. They should be driven flush with the ground, and through a piece of tough paper so that they may be found readily. (5) It is common practice to set the stakes for sidewalk and curb on offset lines so that a re-survey will not be necessary after excavation. When set in this manner each stake is marked with its offset distance. (6) After grade lines have been determined (Problem 83, page 205) the cut or fill is marked on each stake.

Problem 33. Reciprocal Leveling

Object.—To determine accurately the difference in elevation between two points (*B.M. a* and *B.M. b*) on opposite sides of a wide stream or ravine. See Sample Notes 33.

Procedure.—(1) Set up the level in such a position that rod readings may be taken upon each bench-mark. (This, of course, necessitates the instrument being much closer to one point than to the other.) Carefully take a series of five consecutive readings upon *B.M. a*. The mean of these is to be used as a backsight. (2) Take ten careful readings upon *B.M. b*, the distant point. The mean of these readings is to

Prob. 33. Reciprocal Leveling
Across Sun River at Leno.

Obs. No.	B.S. on a	B.S. on b	F.S. on a	F.S. on b	Elev.
1	4.689	4.166			739.215
2	4.687	4.168			
3	4.686	4.169			
4	4.688	4.167			
5	4.690	4.166			
Mean	4.688	4.167			
1			1.030	7.822	
2			1.025	7.833	
3			1.018	7.825	
4			1.036	7.819	
5			1.022	7.826	
6			1.017	7.837	
7			1.025	7.822	
8			1.023	7.830	
9			1.031	7.818	
10			1.017	7.815	
Mean			1.024	7.825	
	7.825	4.167			
	4.688	1.024			
	3.137	3.143	Diff. El. =	3.140	736.075

23.

Gurley 20" Wye — P.A. Charles ⩚
2-Phila. Rods. — N.L. Christy, Rod.
R.H. Evans.
June 13, 1913.
Fair.

B.M.a. Iron Pipe, U.S.R.S.

Note: B.S. distances to B.M.a & B.M.b were 100 ft.

B.M.b. Paint on Ledge marked U.S.R.S. B.M.

SAMPLE NOTES 33.

be used as a foresight. (3) Now set up the level on the opposite side of the stream in such position that the distances from the instrument to *a* and *b* are respectively the same as the distances to *b* and *a* from the former position of the instrument. Take a series of readings on the near and distant points as before. (4) The difference between the mean of the backsight readings on *b* and the mean of the foresight readings on *a* from this series will also give a difference in elevation between the two points. (5) The mean of the two differences in elevation secured from the two settings of the instrument should be the true difference. (6) If the stream or ravine is imaginary, run a line of differential levels between the two points and note the discrepancy.

Hints and Precautions.—(1) Be sure that the bubble is exactly in the center at the time of each reading. The effect of bubble displacement will be particularly great on long distance readings. For distant sights the bubble should be moved and reset after each observation. (2) If the instrument can be set up near the bench-mark used as a backsight, only one observation need be taken to that point. (3) If the rod has no target, a white card may be used as such for long sights. The target should be moved and reset for each observation.

Practical Applications.—The method of reciprocal leveling is used to transfer accurately a line of levels across a river or deep ravine. On account of the curvature of the earth and lack of perfect adjustment in the instrument a considerable error is likely to be introduced if the ordinary method of differential leveling is used. This is due to the difference in length between backsight and foresight.

Problem 34. Test of Accuracy of Setting Level Target

Object.—To determine the probable error of setting the level target at distances of 100, 300, and 600 ft. from the instrument, and to determine what length of sight will give best results in running a line of levels.

Procedure.—(1) Set the level in position to permit a 600-ft. sight. Drive stakes solidly at 100, 300, and 600 ft. from the instrument (distances by pacing). (2) Take a series of ten consecutive rod readings upon each stake, reading the target

24.

Prob. 34. Accuracy of Setting Level Target at 100, 300 & 600 ft.

	At 100 ft.			At 300 ft.	
Rod	V	V^2	Rod	V	V^2
4.675	0.000	0.000000	3.178	0.003	0.000009
4.675	0	0	3.175	0	0
4.676	1	1	3.174	1	1
4.674	1	1	3.176	1	1
4.675	0	0	3.176	1	1
4.676	1	1	3.173	2	4
4.676	1	1	3.174	1	1
4.673	2	4	3.177	2	4
4.676	1	1	3.172	3	9
4.674	1	1	3.175	0	0
4.675	ΣV^2 =	0.000010	3.175	ΣV^2 =	0.000030
	At 600 ft.				
2.467	0.002	0.000004	E_s (100 ft.) = ±0.00071		
2.460	5	25	E_s (300 ft.) = ±0.00123		
2.469	4	16	E_s (600 ft.) = ±0.00262		
2.465	0	0	E_m (100 ft.) = ±0.00023		
2.471	6	36	E_m (300 ft.) = ±0.00039		
2.461	4	16	E_m (600 ft.) = ±0.00083		
2.463	2	4			
2.466	1	1			
2.460	5	25			
2.468	3	9			
2.465	ΣV^2 =	0.000136			

Gurley 20" Wye No. 89. J. Craig, ⊼
Philadelphia Rod. L. Jacobs, Rod.
Nov. 11, 1913.
Cloudy, Moderate.
No Wind.

Probable Error in Running 600 ft. with
Single Obs. & 100 ft. Sights = $\pm\sqrt{6} \times 0.00071 = \pm 0.00174$
" " & 300 ft. " = $\pm\sqrt{2} \times 0.00123 = \pm 0.00174$
" " & 600 ft. " = ± 0.00262

SAMPLE NOTES 34.

vernier to the nearest 0.001 ft. Center the bubble, and reset the target at each observation. (3) Compute the mean rod reading for each distance. (4) Record in the column for residuals (Sample Notes 34, page 97) the difference between each rod reading and the mean of all the rod readings for each distance. (5) Compute the probable errors of each set of observations by the following formulas:

Probable error of a single observation, $E_s = \pm\ 0.6745\sqrt{\frac{\Sigma v^2}{n - 1}}$

Probable error of the mean of all the observations, $E_m = \pm\ \frac{E_s}{\sqrt{n}}$

in which Σv^2 is the sum of the squares of all residuals and n is the number of observations. (6) From the probable error of a single observation at 100, 300, and 600 ft., compute the probable error in running a line of levels of any given length when the sights are in one case all 100 ft. long, in a second case all 300 ft. long, and in a third case all 600 ft. long.

Hints and Precautions.—(1) Be sure that the bubble is in the center of the tube for each reading even though you may think that the level has not moved. (2) The rodman should move the target several inches between each setting and reset it without prejudice, as directed by the instrumentman. (3) Note carefully the effect of distances or length of sight upon the precision of rod readings. In considering the effect of distance upon the accuracy of a line of levels, it must be remembered that three times as many 100-ft. sights are necessary as 300-ft. sights, and that the probable error of a line of levels varies as the square root of the number of set-ups.

Practical Applications.—This problem should give the student not only an idea of his own reliability at target setting with the particular instrument he is using, but also the effect of length of sight upon the probable accuracy of target setting. The elements affecting the possible accuracy are magnifying power of telescope, sensitiveness of bubble, size of cross-hairs, power of observation of the leveler, and condition of the atmosphere.

Problem 35. Magnifying Power of Telescope

Object.—To determine the number of diameters an object viewed through the telescope is magnified.

Procedure.—(1) Sight at the rod held erect about 15 ft.

in front of the instrument. (2) With both eyes open turn the instrument until the images, as seen by the naked eye and as seen through the telescope, appear to fall one upon the other. (3) Compare 0.1 ft. on the rod as seen through the telescope with a space as seen with the naked eye. The number of tenths apparently covered on the unmagnified image is the magnifying power of the telescope.

Hints and Precautions.—(1) Some practice will probably be necessary before the student will be able to see both images at the same time. First sight the image through the telescope; then still keeping the image in sight, look at the rod with the other eye. After a little practice both images will appear distinct. Turning the level slightly, if necessary, will cause the unmagnified image to fall upon the magnified image. For observation select the tenth on the magnified image which is located wholly in the field of vision. Observe the reading of the upper line of this tenth on the unmagnified image, and then observe the lower. The difference of these readings in tenths of feet will be the magnifying power.

Practical Applications.—This is a convenient, quick, and sufficiently accurate method of determining the magnifying power of any telescope for the purpose of appraising or comparing instruments. For the engineers' transit the magnifying power should be from 20 to 30 diam. depending upon the least count of the vernier, and the sensitiveness of the bubble-tube; for the level, 30 to 40 diam.

Problem 36. Radius of Curvature of Bubble-tube

Object.—To determine, in the field, without the use of special apparatus, the radius of the curvature of the bubble-tube of transit or level.

Procedure.—(1) Hold the rod on a solid point 300 ft. from the instrument. With one end of the bubble at a division near the end of the tube, take a careful rod reading to the nearest 0.001 ft. Note the exact position of each end of the bubble. (2) Adjust the foot-screws until the other end of the bubble falls near the other end of the tube. Take another rod reading, and measure the exact distance traversed by each end of the bubble. (3) Determine the bubble movement (this should be expressed to the nearest 0.001 ft.) and the difference between the two target readings or target movement.

(4) In this manner obtain a series of five bubble movements and their corresponding target movements. (5) Compute the radius of curvature by the following formula: $R = \frac{b}{t} D$; in which R is the radius of curvature, D is the distance from the instrument to the rod, b is the mean of the five bubble movements, and t is the mean of the five target movements. (6) Compute the value of one division of the bubble-tube in seconds of arc.

Hints and Precautions.—(1) Observe the bubble and target movements very carefully, as they are very small in comparison to R and D. (2) One division on the bubble tube of most instruments is equal to 1/10 in.

Practical Applications.—This is a quick and sufficiently accurate means of testing the delicacy of the bubble of a transit or level, as the delicacy is directly proportional to the radius of curvature, or inversely proportional to the value of one division expressed in arc. Thus one is enabled to judge the reliability of an instrument in so far as it is affected by the sensitiveness of the bubble.

In a well-balanced instrument the relation between the sensitiveness of the bubble and the magnifying power should be such that for the slightest perceptible movement of the bubble there will be a perceptible movement of the line of sight. Neglecting the effect of size of cross-hairs and assuming that the image is well illuminated, this condition is approximately approached when $M = 400/S$ in which M is the magnifying power of the telescope and S the value in seconds of arc of one division (1/10 in.) of the bubble-tube.

Problem 37. Differential Levels—Double-rodded Line

Object.—To run a line of levels with two sets of turning-points in such manner as to furnish two independent heights of instrument for each set-up of the level.

Procedure.—The method of procedure is similar to that of Problem 26 or 27 (p. 80–85), except that at each set-up of the level backsights and foresights are taken on two separate lines of turning-points, the order of observing being backsight, foresight, foresight, backsight. Two separate sets of level notes are kept as shown in Sample Notes 37. Usually

26.

Prob. 37. Levels, Dixfield to Peru.

Double Rodded Line

Along P. & R.F. Ry.

Sta.	B.S.	H.I.	F.S.	Elev.
$B.M._1$	5.241	532.871		527.63
$B.M._1$	5.239	532.869		
$TP_1 H$	6.943	535.898	3.916	528.955
$TP_1 L$	7.897	535.893	4.873	527.996
$TP_2 H$	8.337	541.804	2.431	533.467
$TP_2 L$	9.746	541.797	3.842	532.051
$TP_3 H$	5.173	541.508	5.469	536.335
$TP_3 L$	7.549	541.504	7.842	733.955
$TP_4 H$	3.411	536.731	8.188	533.320
$BM_2 L$	4.963	536.725	9.742	531.762
$TP_5 H$	2.344	531.837	7.238	529.493
$TP_5 L$	5.729	531.830	10.624	526.101
$BM_3 H$	7.004	731.043	7.798	724.039
$TP_6 L$	8.021	731.039	8.812	723.018
	87.597		80.775	

Gurley Wye Level. F.K. Burton, ⊼

2- Phila. Rods. J.J. Hamel, Recorder.

Lowe & Smith, Rods.

July 9, 1908.

U.S.G.S. in Culvert Fair & Warm.

800 ft. S. of Mile Post 13.

	731.043	731.039
	727.63	727.63
87.597	3.413	3.409
80.775	3.409	
6.822 =	6.822	ck.

Spk. in Tel. Pole at Road to Abbotts Mills.

d.h. in Culvert at Alder Brook.

SAMPLE NOTES 37.

two rodmen are employed, each rodman remaining at the two corresponding turning-points until all sights to these points have been taken.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 26 and 27. (2) Upon the conclusion of observations at any set-up the instrumentman should *immediately* calculate the height of instrument for each line. If there has been no mistake in observing, these heights should agree very closely at the beginning of the line, but following the law of compensating errors, may differ by a considerable amount before the end of the line is reached. In ordinary leveling where lengths of sights are less than 300 ft., reasonable care is taken to prevent unequal heating of the level, and the rod is read to thousandths of feet, the difference between any two corresponding heights of instrument should not vary from the difference between the two preceding heights of instrument by more than 0.004 ft. In case the notes indicate a greater variation than is allowable, without moving the level, a new set of readings should be taken and the first set should be discarded. (3) The reason for taking sights in the order stated above is to partially produce conditions likely to be met in running two lines of levels in opposite directions. If both backsights were taken before the foresights or if each backsight were taken before its corresponding foresight, constant errors, such as settling of the instrument, might go undetected since they would affect the heights of instrument in one line as much and in the same direction as those in the other line. It will be seen that the method here used is such that, while the error may be the same in each line, it will in one case be positive and in the other case negative, and for this reason the heights of instrument will tend to diverge. It should be noted, however, that such a course of procedure will not eliminate errors due to unequal backsight and foresight distances. (4) It is best to have the turning-points of one line a foot or more above the corresponding turning-points of the other line, for this will lessen the possibility of mistakes of 1 ft. being made in two corresponding rod readings. It is evident that such mistakes would still make the heights of instrument check. The turning-points of one line may be distinguished from those of the other by the subscripts H and L (meaning high and low) as shown in the sample notes. (5) Backsight and foresight distances should balance between bench-marks, but very frequently where the line passes through smooth country or along a railroad these distances are not recorded in the notes because approximately constant lengths of sight are maintained throughout the length of the line. All bench-marks should be used as turning-points.

Practical Applications.—The double-rodded line is used principally on courses that do not form a complete circuit or do not close upon some point of known elevation. It is more economical than two lines of ordinary differential levels; it can not be said, however, that in point of accuracy it is the equal of two single-rodded lines run in opposite directions over the same course. Its principal advantage over a single line lies in the greatly increased probability of the resulting elevations being free from large errors or mistakes. The U. S. Geological Survey has used it extensively on short branch lines for the purpose of furnishing vertical control to topographers, and formerly it was used extensively on lines of precise levels.

CHAPTER V

PLANE-TABLE AND SEXTANT

PROBLEM 38. ADJUSTMENTS OF THE PLANE-TABLE

Note.—There are two quite distinct types of plane-table alidades in use. The telescope tube of one type (see Plate VII) is rigidly attached to or is an integral part of the horizontal axis, as with the engineers' transit. The telescope tube of the other type is fastened into a cylindrical sleeve which is an integral part of the horizontal axis; the telescope may be turned about its axis in the sleeve much as may the telescope of the wye level in its wyes, and on the telescope tube are turned two shoulders, perhaps 5 in. apart, upon which rests a striding level which may be removed at will. The telescope of the former type may be equipped with striding level as just described or may have an attached bubble-tube, like the transit telescope.

With either type the vertical vernier may be fixed or movable or there may be an auxiliary bubble-tube attached to the vernier arm, as with the transit; and the levels for controlling the table may be mounted on the base-plate or straight-edge of the alidade or they may be mounted with the compass on a smaller plate.

Adjustments.—The relations that should exist in a *perfectly* adjusted instrument are: (1) that the vertical cross-hair lie in a plane perpendicular to the horizontal axis, (2) that the axes of the control bubble-tubes be parallel to the plane of the plate, (3) that the line of sight revolve in a plane, (4) that the horizontal axis lie in a plane parallel to the plane of the plate, (5) that the line of sight coincide with the optical axis, (6) that the axis of the striding level or bubble-tube attached to the telescope be parallel to the line of sight; further, for convenience in measuring vertical angles, (7) if the vernier is fixed, that it should read zero when both straight-edge and line of sight are horizontal, or (8) if the vernier has an attached bubble-tube, that its axis should be

axis of the telescope bubble-tube fo
vernier.

the methods of performing the adju
ith those of corresponding adjustment
m 11, page 47) or of the wye level (
but in general the adjustments do not
such precision.

(1) is performed exactly as in the trans
(2) is like the plate-bubble adjustmen
that the plate on which the bubble-tu
rned end for end after its position h
plane-table board. It should be noted
not turned 180° about its axis.

(3) and (4) rarely, if ever, need att
have no provision for Adjustment (4).
(5) in the alidade with telescope in axia
y sighting at a point and turning the te
axis as in Adjustment (2) of the wye
care of Adjustment (3). In the alidac
ly attached to horizontal axis the adju
ned as explained under Hints and Preca

(6), if the level is attached to the telesc
the peg method as with the transit; if
like Adjustment (4) of the wye level
g level is lifted from its supporting sho
for end, instead of lifting the telescop
eversing it.

(7) and (8) are performed exactly
) and (8) of the transit.

recautions.—(1) If Problems 11 and 2
rmed, study them carefully. (2) In
s essential that vertical angles be mea
r this reason Adjustments (5), (6), an
rmed with considerable care.

Survey of Field with Plane-tab Radiation Method

make a survey of the assigned field
tion.

1) Choose some point in the field fron

parallel to the axis of the telescope bubble-tube for a zero reading of the vernier.

In principle the methods of performing the adjustments are identical with those of corresponding adjustments of the transit (Problem 11, page 47) or of the wye level (Problem 24, page 77), but in general the adjustments do not need to be made with such precision.

Adjustment (1) is performed exactly as in the transit.

Adjustment (2) is like the plate-bubble adjustment of the transit, except that the plate on which the bubble-tubes are mounted is turned end for end after its position has been marked on the plane-table board. It should be noted that the plane-table *is not* turned 180° about its axis.

Adjustments (3) and (4) rarely, if ever, need attention. Many alidades have no provision for Adjustment (4).

Adjustment (5) in the alidade with telescope in axial sleeve is performed by sighting at a point and turning the telescope 180° about *its* axis as in Adjustment (2) of the wye level. This also takes care of Adjustment (3). In the alidade with telescope rigidly attached to horizontal axis the adjustment may be performed as explained under Hints and Precautions (6), page 53.

Adjustment (6), if the level is attached to the telescope, is performed by the peg method as with the transit; if it is a striding level, like Adjustment (4) of the wye level except that the striding level is lifted from its supporting shoulders and turned end for end, instead of lifting the telescope from the wyes and reversing it.

Adjustments (7) and (8) are performed exactly as are Adjustments (7) and (8) of the transit.

Hints and Precautions.—(1) If Problems 11 and 24 have not been performed, study them carefully. (2) In plane-table work it is essential that vertical angles be measured accurately. For this reason Adjustments (5), (6), and (8) should be performed with considerable care.

Problem 39. Survey of Field with Plane-table by Radiation Method

Object.—To make a survey of the assigned field by the method of radiation.

Procedure.—(1) Choose some point in the field from which all corners are visible. Set up and clamp the plane-table, and choose some point *o* on the plane-table sheet such that no corner to be plotted will fall off the paper. This point on the paper represents the point occupied on the ground. (2) With compass or declinator draw the meridian. (3) Move the alidade with the straight-edge cutting this point until the line of sight strikes some corner such as *A* (Fig. 24).

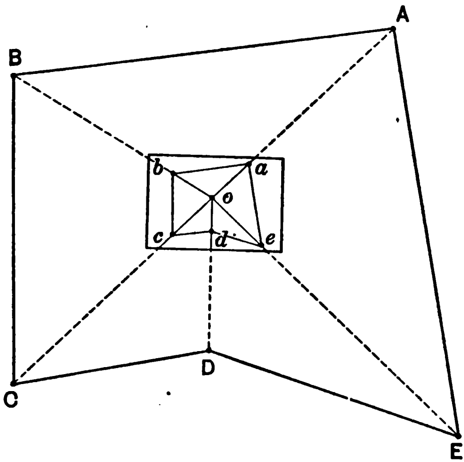


FIG. 24.—RADIATION WITH PLANE-TABLE.

(4) Draw a line of indefinite length along the straight-edge, and read the stadia intercept on the rod held by the rodman at *A*. (5) Move the alidade to one side, and scale the distance *oA* corresponding to the stadia reading, thus locating the point sighted. Letter the point so that it will not be confused with the points plotted later. (6) In like manner locate other corners *B*, *C*, *D*, etc. (7) Make a neat title containing the following information: name of survey, method used, members of party, date, and scale of the drawing.

Hints and Precautions.—(1) Set the plane-table low enough to permit taking sights without leaning against or placing the elbows on the board. (2) Be sure that the table is securely clamped. To guard against the possibility of undetected movement of the board sight at some prominent feature of the landscape as soon as the plane-table is in position, and draw a line. Before leaving the station, with the straight-edge along this line note whether or not the line of sight passes through the same point. (3) The stadia intercept can be calculated with greater facility if the lower stadia hair is made to intersect the stadia rod at some footmark. To avoid mistaking the middle hair for one of the stadia hairs, always note the half interval. (4) In most instruments the stadia interval factor is 100; the stadia interval may be tested as explained in Problem 58, page 151. (5) If the stadia intercept is greater than the length of the rod, note the interval cut by the lower and middle hairs, then plunging the telescope slightly, note the interval between the middle and upper hairs. Obviously the sum of these two readings is the stadia reading for that distance. (6) A stadia constant of 1 ± ft. is added to all distances determined from rod readings. The amount of this constant is usually determined by the maker and marked in the instrument box. It will be unnecessary to consider this constant on small scale work. (7) Keep the telescope as nearly level as is practicable. On large scale work vertical angles up to 3° may be used without any appreciable error in the horizontal distance, but when the vertical angle is greater than 3° a horizontal correction must be applied. If the scale is small no horizontal correction need be made unless the country is very rough. (8) Too much care can not be taken to keep the pencil well sharpened, and a soft pencil should never be used. Make fine lines and no more of them than are absolutely necessary. (9) Considerable time can be saved by sticking a pin in the paper at the point from which all sights radiate.

Practical Applications.—The plane-table is not intended for precise work, nor can it be used where the numerical values of angles are required; hence it is generally made use of in such classes of surveys as map making, preliminary surveys, and topographical work. The plane-table is especially useful in plotting indefinite details such as the banks of rivers, the shores of lakes, roads, woods, and parks. It is advantageous over transit surveying in that more work can be done at a less cost and in less time. Since the map is made in the field, it dispenses with field notes; consequently no useless data are collected, lengthy computations are avoided, and omissions in the map are prevented. On the other hand, it is disadvantageous because angles are not recorded, hence is of little value in calculating areas; because the table is cumbersome and awkward to handle, and the accessories are numerous; and because in wet weather good results can not be obtained. The problem as here given is of little practical value except as a means of acquainting the student with fundamental methods.

Problem 40. Plane-table Survey by Intersection

Object.—To make a plane-table survey of the same field assigned in the previous problem by method of intersection.

Procedure.—(1) Measure with a steel tape or by stadia a base-line *MN* (Fig. 25) 150 or 200 ft. long within or without

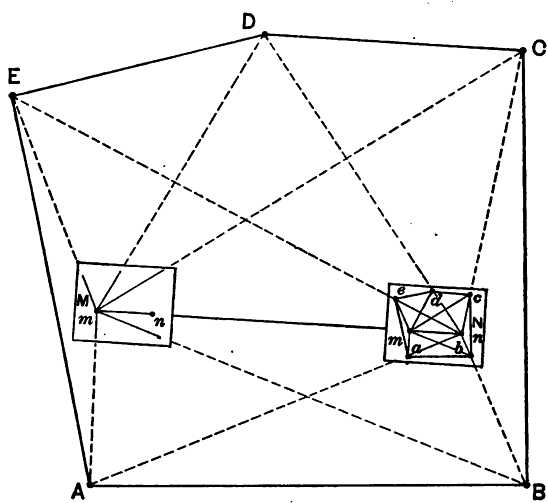


Fig. 25.—Intersection with Plane-table.

the field, so that when produced it will not pass near any corner of the field. (2) Plot the base-line *mn* on the plane-table sheet to the desired scale in position such that when the field is plotted all corners of the field will fall on the paper. (3) Set up over one end of the base-line *M*, and make the line *mn* on the paper coincide with the base-line *MN* on the ground (orient) by moving the board about the vertical axis of the plane-table until, with the straight-edge of the alidade along the line *mn*, the line of sight cuts *N*. (4) Clamp the table in this position, and move the alidade about *m* as a center, drawing radial lines of indefinite length toward *A*, *B*, *C*, etc. Letter these lines so that there may be no doubt as to their identity. (5) By means of the compass or declinator draw a meridian, correcting for declination if desired. (6) Move the table to *N* and orient as before, making the line *nm* on the paper lie in the same vertical plane with *NM* on the ground. (7) With the edge of the alidade passing through *n*, draw radial lines toward *A*, *B*, *C*, etc., to intersect the corresponding lines drawn from *m*. Each point of intersection is (from similar triangles) the plotted position of the corner toward which the line of sight was directed when the radial lines were constructed. (8) Check the meridian line previously drawn. (9) Mentally compare the method with that of the preceding problem. Make a neat title.

Hints and Precautions.—(1) Use the same care in setting up the plane-table as in the previous problem. When orienting the board, if the scale of the drawing is large, make sure that the point on the paper is approximately over the corresponding point on the ground. (2) To make sure that intersections will be definite, the angle between intersecting lines should not be less than 30° nor more than 150°; when laying out the base-line this should be borne in mind. If it is impossible to obtain a good intersection from the points occupied on the base-line, a third point previously located by intersection may be occupied for that purpose, and the work may be carried on as before. (3) Make the radial lines as light as can be easily distinguished and only long enough to insure intersections. (4) In small-scale work it is not essential that the point on the paper should be directly over the point on the ground because the error is negligible, but on large-scale work it is quite necessary.

Practical Applications.—The method of intersection is generally used in connection with other methods, principally in the location of inaccessible objects or as a check. It is very convenient at times to choose some well-defined landmark such as a church steeple, which can be seen from several different points to be occupied by the plane-table. If the point is plotted at the intersection of lines directed toward the object from any two stations, a line drawn in the same manner from every other station occupied should pass through this same point. Any failure indicates either that the board has not been properly oriented or that the point occupied has not been plotted in its proper position on the plane-table sheet.

On small or intermediate scale work this method is often used to locate points that are likely to be used as plane-table stations.

PROBLEM 41. SURVEY WITH PLANE-TABLE BY METHOD OF PROGRESSION OR TRAVERSING

Object.—To make a survey of the same field used in the previous problem by method of traversing or progression.

Procedure.—In this method, as the name implies, the plane-table is moved from one station to the next in order of location,

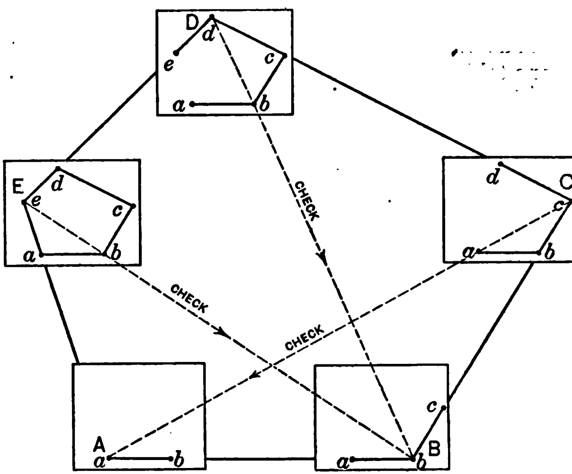


FIG. 26.—TRANSVERSE WITH PLANE-TABLE.

around the area to be mapped. The board is oriented at each point by sighting at the station previously occupied with the edge of the alidade passing through the plotted positions of the station occupied and the station sighted. (1) Set up over some corner *A* (Fig. 26). Sight to *B* and note the stadia distance. (2) Plot *b* as explained in Problem 39. (3) Draw a meridian with the compass. (4) Move the plane-table to *B*, and after orienting the board by sighting on *A*, plot the point *c* in the same manner as *b*. With the compass, check the meridian drawn while the first station was occupied. (5) Continue in the same manner around the entire field. (6) Calculate the error of closure. (7) Make a comparison of the three methods. Make a title.

Hints and Precautions.—(1) Choose the first point on the paper with due regard for the limits of the plane-table sheet. (2) If from *A* the two adjacent corners are plotted, there will be no necessity of occupying the last station of the traverse except as a check. (3) Observe the hints and precautions of Problem 39 that are applicable to this problem.

Practical Applications.—The method of progression is especially adapted to the survey of roads and streams and is used in combination with the methods taken up in the two previous problems. In topographic work, traverses are usually run between two points some distance apart which have been established by a more accurate method of surveying. The details are located from the various traverse points by the radiation method, and a check on the work may frequently be furnished by the method of intersection. Thus, except in very small surveys, one rarely has occasion to use one method without using them all.

PROBLEM 42. PLANE-TABLE SURVEY OF CAMPUS

Object.—Make a plane-table survey of a portion of the campus, locating all buildings, walks, trees, etc., by a combination of the methods of radiation, progression, and intersection.

Procedure.—(1) Set up at some point from which a considerable number of objects to be located may be seen. Locate these points by method of radiation (Problem 39, p. 105). (2) By progression (Problem 41) establish some other point from which more of the details to be mapped can be seen. (3) As a check, sight on some distant object which can be located by intersection (Problem 40, p. 108) from the next station occupied. (4) Set up over the traverse point located and continue as before. (5) In this manner make a closed traverse, locating from the traverse points all the points necessary to construct a complete map. If at any time there is any marked error made, as will be shown by the intersection of the various lines drawn to the distant point, the previous station should be re-occupied or the orientation of the plane-table tested. (6) Finish by making a neat title as explained in Problem 39.

Problem 43. Location of Plane-table by Resection

Object.—To locate plane-table station from a base-line only one end of which can be conveniently occupied. The base-line may be the line between two triangulation stations. It is supposed that it has already been plotted on the plane-table sheet.

Procedure.—(1) Set up the plane-table at one end of the base-line, orient the board by bringing the plotted base *ba* (Fig. 27) over the actual base *BA*. (2) Sight to the point to

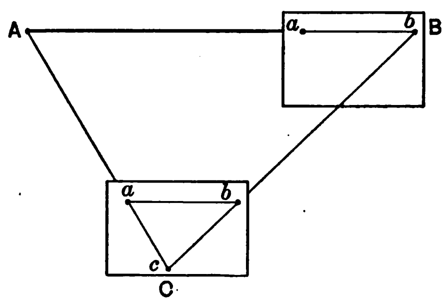


Fig. 27.—Resection.

be occupied, *C*, and draw a line of indefinite length. (3) Set up the plane-table at *C*, and orient the board by sighting at *B* as in traversing. (4) With the straight-edge passing through *a*, sight to *A* and draw a line to intersect *bc*. The point of intersection is the plotted position of the station occupied.

Hints and Precautions.—(1) It is evident that the strength of the determination of the point is greatest when the intersecting lines make an angle of 90° and decreases as the angle increases cr decreases from 90°. The solution is indeterminate for all points on the base-line or its prolongation. (2) On large-scale maps the position of the point *c* must be carefully estimated before orienting at *C*, and the estimated position of the point should be over or nearly over the actual point on the ground. On small-scale maps the plane-table may be roughly set over the station and the plotted point allowed to come where it will. The resulting error will be negligible.

Practical Applications.—This method of resection is used but little for actually locating stations. It may sometimes happen for one reason or another that it is impractical to locate the point by intersection or progression. The method is more often used as a check on other methods. As an example, suppose the station had been located by progression or traversing. A check would have been performed if on resecting through *a* to *A* the straight-edge passed through the point *c*. It follows that, if the point occupied is correctly plotted and the table is oriented, resecting lines from plotted points to their corresponding actual points will intersect at the plotted position of the station occupied. Fig. 26, page 110, illustrates the above method as a check on the traverse method.

Problem 44. The Three-point Problem with Plane-table (Mechanical Solution)

Object.—To plot the position of the station occupied by resection on three known points whose positions have already been located on the plane-table sheet.

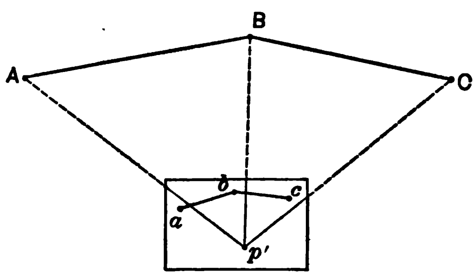


Fig. 28(*a*).

Procedure.—(1) Fasten a piece of tracing cloth over the plane-table sheet. (2) Assume any point *p'* on the tracing cloth. With the straight-edge passing through this point, sight at each of the known points, *A*, *B*, and *C* (Fig. 28(*a*)), and draw lines of indefinite length. (3) Remove the alidade, and shift the tracing cloth until each of the lines drawn to the three actual known points passes through the corresponding plotted points, *a*, *b*, and *c* (Fig. 28(*b*)). The new position of the intersection of the radial lines on the tracing cloth is the point sought or the plotted position of the station occupied.

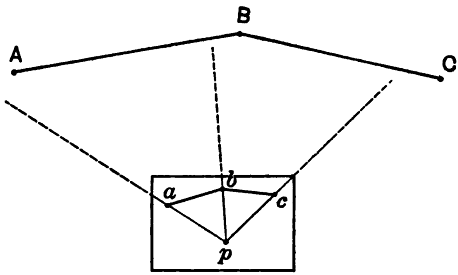


FIG. 28(*b*).

Prick through to the plane-table sheet. (4) Remove the tracing cloth, and with the straight-edge passing through *p* and one of the plotted points, turn the board until the line of sight strikes the corresponding actual point (Fig. 28(*c*)).

Hints and Precautions.—(1) Observe the hints and precautions of Problems 39, 40, and 41, that are applicable to this

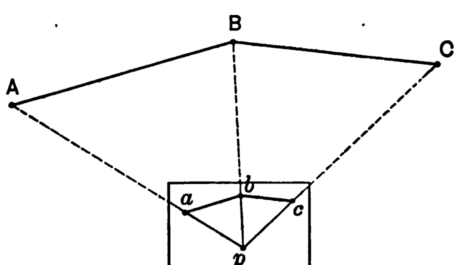


FIG. 28(*c*).

problem. (2) When the point sought is at or near a great circle passing through the three known points, the solution is indeterminate. (3) When the point sought is at or near the center of gravity of the great triangle formed by lines joining the known points the determination is strong.

Practical Applications.—On large surveys where the horizontal control is furnished by a system of triangulation and by accurate transit traverses, the three-point problem is particularly applicable because of the readiness with which known points can be plotted on the plane-table sheet and located in the field. By this method the position of each plane-table station is located independently; thus the error in the position of one station has no effect upon other stations. This is a decided advantage over the method of progression where the error in the position of one station must necessarily affect others. The tracing-cloth solution is not in general favor with experienced topographers; it has, however, the advantage of extreme simplicity over other methods.

PROBLEM 45. THE THREE-POINT PROBLEM WITH PLANE-TABLE (COAST SURVEY SOLUTION)

Object.—To plot the position of the station occupied by resection on three known points whose positions have already been located on the plane-table sheet.

Procedure.—(1) By means of the compass or declinator orient the table as nearly as possible. Resect through each of the plotted points from the corresponding actual points, and draw lines of indefinite length on the plane-table sheet. If the lines intersect at a common point, the table is correctly oriented and the plotted position of the station occupied is at that point of intersection. (2) If the table is not correctly oriented, the three lines will form a triangle, termed the "triangle of error" as in Illustration 29, p. 117. The position of the "point sought" p will depend upon the size and shape of the triangle of error and its position relative to the great circle passing through the three fixed points. (3) Estimate the position of the point sought as explained in the following article, and with the straight-edge through this point and one of the fixed points, turn the table until the corresponding actual point is sighted. The table should now be nearer perfect orientation. (4) Again resect from the three known points, and determine a new triangle of error. (5) Estimate the position of the point sought, and perform the trial orientation. (6) Continue in this manner until the resection lines meet at a point. The table is, of course, to be clamped after each trial orientation.

Position of Point Sought.—The following extract taken from *A Plane-table Manual,** by Mr. D. B. WAINWRIGHT, simplifies the estimation of the position of the point sought.

"*The relative positions of the three fixed points with reference to the new station* have an important bearing on the strength of its determination.

"In the following statement in regard to the different groupings of points met in practice, for the sake of brevity, the term "fixed points" will be understood to mean points already determined and plotted on the sheet; the "great triangle" referred to is one formed by the three fixed points; and the "great circle" is the circle passing through them.

"*When the new station is outside the great circle,* the strength for determination of a position will be weak when the middle point as seen from the new station is the farthest of the three and the angles are small. (See Illustration 29, Fig. 3.) If the new station is located outside the great circle, and some distance below it, the angles are small and the determination correspondingly weak.

"The determination increases in strength for given angles as the middle point approaches the new station (Fig. 1).

"When one angle is small or 0° (points in range), the determination will be strong, provided the two points making the small angle or range are not too near each other when compared to the distances to the new station and to the third point; provided also the angle to the third point is not too small (Fig. 2).

"*When the new station lies on or near the great circle,* its position is indeterminate (Fig. 3).

"*When the new station is within the great circle,* the strength of its determination increases as it approaches the center of gravity of the great triangle (Figs. 3, 4, and 5).

"There are a number of graphic solutions, but all save three are better suited to the drafting-room with its appliances than to the conditions which exist in the field.

"*Lehmann's method* of solution is the simplest and most direct, and applies under all circumstances. The directions are stated in the form of rules.

"The term "*point sought*" will be understood to mean the true position on the sheet of the projected point of the station

* Department of Commerce and Labor, U. S. Coast and Geodetic Survey. Report of 1905.

occupied. The surveyor is assumed to be facing the signals, and the directions right and left are given accordingly.

"*Rule* 1.—The point sought is always distant from each of the three lines drawn from the three fixed points in proportion to the distances of the corresponding actual points from the

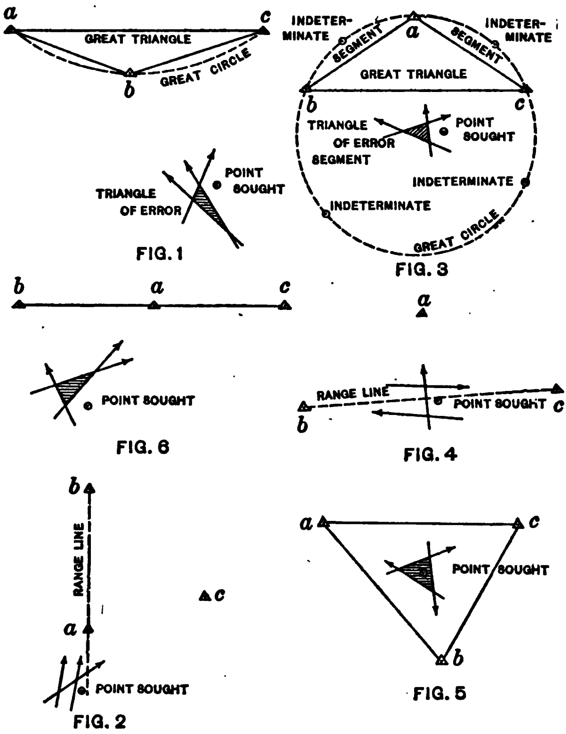


ILLUSTRATION 29.—THE THREE-POINT PROBLEM.

station occupied, and it will always be found on the corresponding side of each of the lines drawn from the fixed points.*

"The simplest case for the application of this rule occurs when the station to be determined is within the triangle formed

* That is, if it is on the right side of one line it will be on the right of the other two.

by the three fixed points; the point sought must then be within the triangle of error to satisfy the conditions (Fig. 5, p. 117).

"Although Rule 1 is sufficient in itself for the solution of the problem, there are two subordinate rules which materially assist the topographer in reaching a decision as to the proper location of the point sought with reference to the lines from the fixed points.

"*Rule* 2.—When the point sought is without the great circle, it is always on the same side of the line from the most distant point as the intersection of the other two lines (Fig. 1, p. 117).

"*Rule* 3.—When the point sought falls within either of the three segments of the great circle formed by the sides of the great triangle, the line drawn from the middle point lies between the point sought and the intersection of the other two lines (Figs. 3, 4, and 6, p. 117).

"**Application of Rules.**—In practice the topographer first decides the relation of the new station with reference to the fixed points, whether it is within the great triangle or in one of the segments or outside the great circle. He then determines the position of the point sought with reference to one line (if within one of the segments or without the great circle by Rule 2 or 3); it then follows from Rule 1 that it must be on the corresponding side of the other two lines. Finally, he estimates the relative distances of the three actual points from him and marks the position of the point sought a proportionate distance from each of the three lines."

Practical Applications.—The Coast Survey method of solving the three-point problem is extensively used by topographers of the U. S. Coast and Geodetic Survey and the U. S. Geological Survey. The length of time necessary to locate the station occupied is much less than one might suppose, and a solution is possible for all positions of the table (except on or near the great circle when the problem is indeterminate). Some of the other graphical methods depend upon construction lines which, for certain positions of the plane-table relative to the known points, will fall off the paper. The Coast Survey solution is certainly preferable to the mechanical method of the previous problem; for in damp weather the tracing cloth will soon become unfit for use, and in windy weather the topographer will experience some difficulty in keeping the cloth flat as it is shifted about the board.

Problem 46. The Two-point Problem with Plane-table (Mechanical Solution)

Object.—To locate the position of an unknown point *P* by resection on two known points *A* and *B*, and to orient the plane-table.

Procedure.—(1) Plot the two known points on the plane-table sheet. (2) Set up the plane-table over a fourth point *C* from which the other three points can be seen and which will give intersection angles from *P* and *C* to *A* and *B* between 30°

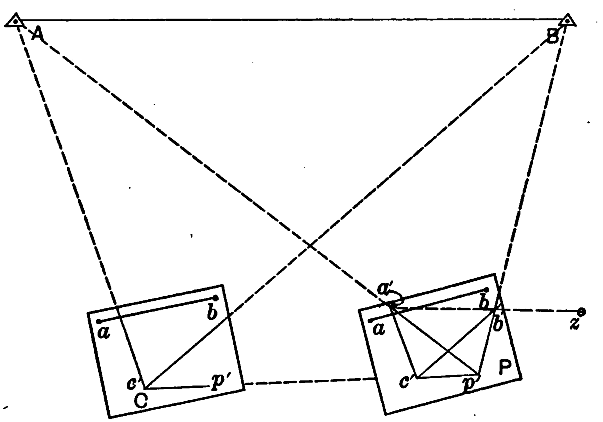


Fig. 30(*a*).

and 150°. Fasten a piece of tracing cloth to the board. Assume a point *c'* on the tracing cloth corresponding to *C* on the ground; and, with the straight-edge of the alidade passing through this point, take sights to *A*, *B*, and *P*, and draw lines of indefinite length, Fig. 30(*a*). (3) Estimate the distance from *C* to *P*, and plot *p'* (the estimated position of *P*) on the line drawn toward *P*. (4) Set up the plane-table at *P*, and orient the board with respect to the line *PC*.* With *p'* as the center take sights to *A* and *B*, and draw lines until they intersect the corresponding lines from *c'*, at the points *a'* and *b'*. (5) Remove the alidade, and shift the tracing cloth until the line *a'b'* falls on the line *ab*, Fig. 30(*b*), page 120. Fasten the cloth in this position. (6) Replace the alidade on the tracing cloth and

* That is, bring the line *p'c'* into the same vertical plane with *PC*.

with the straight-edge along one of the lines *p′a′*, *p′b′*, or *p′c′*, revolve the board until the corresponding actual point *A*, *B*, or *C* is sighted, Fig. 30(*c*). The plane-table is now oriented.

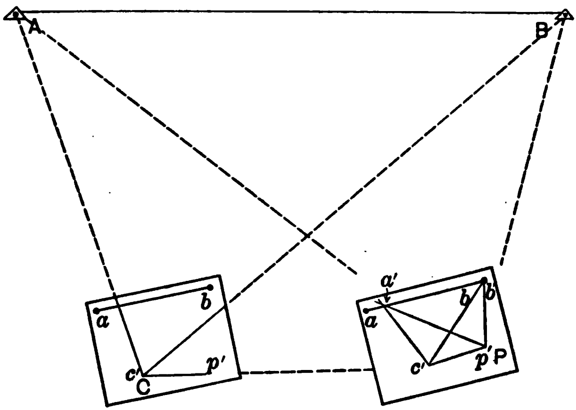


FIG. 30(*b*).

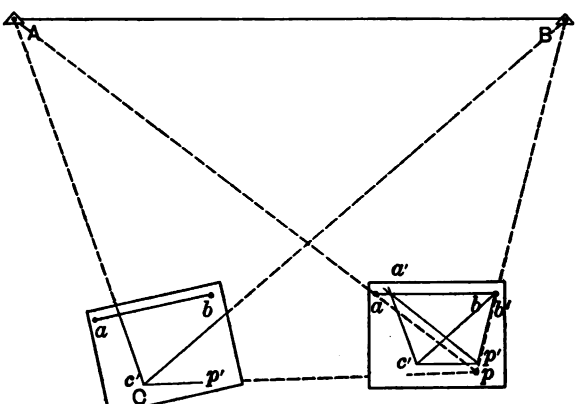


FIG. 30(*c*).

(7) Remove the tracing cloth. Resect through *a* from *A*, and through *b* from *B*, and draw lines on the plane-table sheet. The intersection of these two lines, shown as dotted lines in the figure, will be the position of *p*, the point sought.

Hints and Precautions.—(1) If the fourth point C is chosen in range with the known points A and B, the plane-table can be oriented at the start. This makes the problem comparatively simple, for when P is occupied, the table can be correctly oriented by the line drawn from c toward P. It is then only necessary to resect through a and b. (2) If the true length of the line CP be known, the tracing cloth may be shifted so that a' will coincide with a, and b' with b. The correct position of the point sought will then be at p'. (3) Intersecting lines should form angles between 30° and 150°. The position of the point C should be chosen with this in mind.

Practical Applications.—Owing to the length of the problem it is not practical except where other methods fail. If the known points are accessible and are near the station to be occupied, it will be found much simpler to use the method explained in Problem 43, page 112, or to run a traverse to the unknown point. If, however, the known points are at a distance or are inaccessible, as would be the case with church spires or chimneys, the two-point problem must be solved.

Problem 47. The Two-point Problem with Plane-table (Graphical Solution)

Object.—To locate the position of an unknown point P by resection on two known points A and B, and to orient the plane-table.

Procedure.—(1) Plot the two known points on the plane-table sheet. (2) Set up the plane-table over a fourth point C from which the other three points can be seen and which will give intersection angles from P and C to A and B between 30° and 150°. Orient the board as nearly as possible with the compass. Assume a point c', Fig. 30(a), page 119, on the plane-table sheet corresponding to C on the ground; and, with the straight-edge of the alidade passing through this point, take sights to A, B, and P and draw lines of indefinite length. (3) Estimate the distance from C to P and plot p' (the estimated position of P) on the line drawn toward P. (4) Set up at P, and orient the board with respect to the line PC. With p' as a center, take sights to A and B, and draw lines until they intersect corresponding lines from c' at the points a', b'. The line $a'b'$ will be parallel to a line between the corresponding actual points. (5) With the straight-edge of the alidade along the line $a'b'$, note the position of the line of sight on some prominent object of the landscape or set a point *z* on line at some distance from the table. Move the alidade to the line *ab*, and turn the table until the same point is sighted. The plane-table is now oriented. (6) By resection through *a* and *b*, locate the point sought.

Hints and Precautions.—Observe the hints and precautions of Problem 46 that apply to this problem.

Practical Applications.—This method of solving the two-point problem is in favor with many topographers of the U. S. Coast and Geodetic Survey and U. S. Geological Survey. The fact that no extra equipment is needed makes it preferable to the solution of the previous problem.

Problem 48. Adjustments of the Sextant

(1) To Make the Index Mirror Perpendicular to the Plane of the Sextant.

Test.—Set the vernier to read about 30°. With the arc held away from the body and the eye slightly above the plane of the sextant examine the arc and its reflected image in the index mirror. If the mirror is in adjustment, the arc and its reflection should form a continuous curve.

Correction.—If there is a break in the continuity of the curve, change the inclination of the mirror by the screws provided for that purpose or by inserting slips of paper under its base. If the reflected image appears above the arc, the top of the mirror should be moved backward; if below, forward.

(2) To Make the Horizon Glass Perpendicular to the Plane of the Sextant.

Test.—Sight at some well-defined point a mile or more away (a star or the sun may be used). Move the index arm back and forth until both direct and reflected images are brought as near as possible to coincidence. If the images may be brought to exact coincidence, the desired relation exists.

Correction.—If the glass is not in adjustment, move the screws at the back or foot of the horizon glass until coincidence occurs. This adjustment should be made after the adjustment of the index mirror.

(3) To Make the Horizon Glass Parallel to the Index Mirror for a Zero Reading of the Vernier.

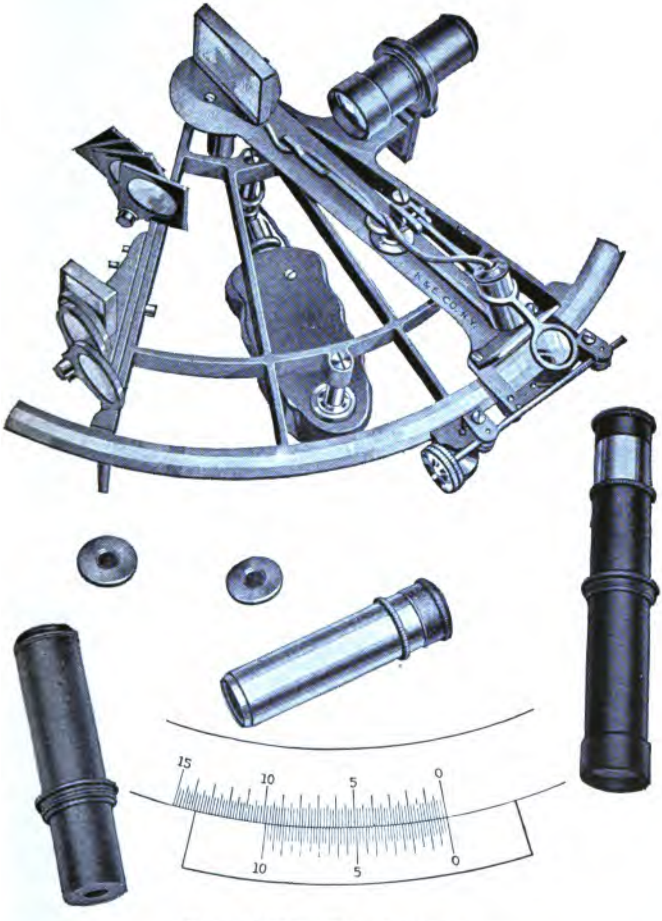


PLATE VIII.—THE SEXTANT.
(From a half-tone furnished by Keuffel & Esser Co.)

(*Facing page 122.*)

Test.—Set the vernier to read zero and sight at a distant object as in Adjustment (2). If coincidence of images occurs the mirrors are parallel.

Correction.—If not, move the horizon glass about an axis perpendicular to the plane of the sextant. Many sextants have no provision for such adjustment. With such instrument the index error is the reading of the vernier at the conclusion of Adjustment (2).

4. To Make the Line of Sight of the Telescope Parallel to the Plane of the Sextant.

Test.—Place the sextant on some solid object about 20 ft. from a wall with the plane of the graduated arc in a horizontal position. Make two wooden blocks of length equal to the height of the axis of the telescope above the plane of the graduated arc. Set the blocks on the graduated arc, and sighting over them, mark a point on the wall. Sight in the same direction with the telescope.

Correction.—If the line of sight (center of square) strikes above or below the point, the desired relation does not exist. Correct by tipping the collar into which the telescope is screwed. If the lines of sight come within an inch of coinciding at 20 ft., no correction need be made. Some sextants have no provision for this adjustment.

PROBLEM 49. ANGLES OF A TRIANGLE WITH SEXTANT

Object.—To measure each of the angles of a triangle with the sextant; both with the handle below the telescope and with the handle above. The sum of the angles should be within 05′ of 180°.

Procedure.—(1) Sight at some clearly defined distant object, and move the vernier arm until the image of the same object reflected from the index mirror nearly coincides with that one directly viewed through the telescope. (2) Clamp the vernier arm in this position, and bring the images into exact coincidence by use of the tangent-screw. Read the vernier. The reading thus obtained is the index error and should be added to or subtracted from each observed angle according as the zero of the vernier lies to the right or to the left of the zero on the arc. (3) Place flag poles at two vertices of the triangle, and standing on the third point, with the telescope directed toward one of the flag poles, move the vernier until the images of the two flag poles nearly coincide. Clamp the arm in this position, and bring the images exactly together with the tangent-screw. Read the vernier and record the angle. (4) Repeat with the instrument inverted. (5) From the mean of these two readings add or subtract the index correction. (6) Repeat the process for the other two angles of the triangle.

Hints and Precautions.—(1) Beginners usually find considerable difficulty in getting both images in the field of the telescope. The reflected image will usually be the dimmer of the two. Estimate the angle and set it off on the arc. Sight at the flag pole which is to furnish the direct image, and move the sextant slightly about the axis of the telescope. If two images fail to come into the field of view, move the vernier arm back and forth, at the same time rocking the sextant slightly about the axis of the telescope until they do appear. If the observer finds that he is unable to distinguish two images in this manner, he may remove the telescope and place his eye in the center of the ring. In this way he can distinguish the images without difficulty, and having brought them approximately together, can bring them into exact coincidence after replacing the telescope. (2) Use the eyepiece intended for terrestrial work. (3) The colored shades are intended for solar observations. (4) Notice that the angles measured are horizontal angles only when the eye of the observer and the two objects are in a horizontal plane. (5) The possible accuracy of angle determination with the sextant depends upon the size of the angle and the length of sight. The angle actually measured has its vertex not at the eye, but at the point where the ray coming directly to the eye and the ray striking the movable mirror would intersect were it not for the intercepting mirror. It is evident that the distance to this intersection will increase as the angle decreases and that it will be large for very small angles. For this reason the sextant is not an instrument of precision for distances less than 1000 ft. and angles less than 15°.

Practical Applications.—The advantage of the sextant over other surveying instruments is that sights may be taken to two objects simultaneously. While other instruments must be kept stationary between backsight and foresight, the sextant may be held in the hand. For this reason, in surveying it is used principally on hydrographic surveys for locating the position of soundings. Two angles are taken from the boat to three known points on shore at the instant of taking the sounding. This may be accomplished by one observer with two sextants or by two observers each having a sextant. The sextant is also used in astronomical observations.

CHAPTER VI

FIELD ASTRONOMY

PROBLEM 50. LATITUDE BY OBSERVATION ON THE SUN AT NOON

Object.—To determine the latitude of the place by an observation on the sun at local apparent noon with the engineers' transit.

Procedure.—(1) Set up the transit some time before the sun is at the highest point in the heavens, with the telescope pointing in the general direction of south, and determine the index error of the vertical circle. (2) With the horizontal cross-hair on the lower limb of the sun, follow it along until a maximum altitude has been reached and the sun begins its apparent descent. Record the vertical angle and watch time. (3) Correct the watch time by comparing it with a clock keeping standard time, and determine the Greenwich mean time (*G.M.T.*) of the observation by adding to the corrected time the longitude (in hours) of the meridian from which standard time is reckoned. (4) In the Nautical Almanac* (page II for month) or other solar ephemeris giving values for Greenwich mean noon, look up the apparent declination δ_0 for mean noon, and the hourly change of declination on the day of the observation, noting that the declination is *positive* when *north* and *negative* when *south*. (5) Calculate the change in declination $\Delta\delta$ between Greenwich mean noon and the Greenwich mean time of observation.

$$\Delta\delta = (\textit{hourly change} \times G.M.T.)$$

(6) Compute the apparent declination for the Greenwich mean time of the observation.

$$\delta = \delta_0 \pm \Delta\delta$$

(7) Compute the true altitude h of the sun's center by correcting the observed vertical angle h' for the index error, re-

* *American Nautical Almanac*, published for each year by Nautical Almanac Office, U. S. Naval Observatory, Washington, D. C.

fraction and parallax, and the sun's semi-diameter. The sun's semi-diameter may be taken as 16′. (For a closer value consult the Nautical Almanac.) The correction for parallax and refraction depends upon the altitude. It may be found in Table I, page 243. Calculate the latitude by the equation

$$\phi = 90^\circ - h + \delta$$

Follow the accompanying form, page 128.

(8) Some of the pocket solar ephemerides give declinations for *Greenwich apparent noon.* If the ephemeris used is of this kind, after step (3) change Greenwich mean time to Greenwich apparent time by equation

$$G.A.T. = G.M.T. \pm E$$

in which E is the equation of time or difference between mean solar time and apparent solar time at the Greenwich apparent time of observation. The value of E is found by interpolating. The values given in the table headed "*Equation of Time*" are for Greenwich apparent noon. The sign of the equation of time as expressed at the head of the column is for changing *apparent* time to *mean* time; since here the reverse operation is desired, the opposite sign must be used. The operation of finding the declination will now be similar to that explained in steps (4), (5), and (6), except that Greenwich apparent time will be used. Since the maximum rate of change of declination is less than 01′ per hour and since the equation of time is always less than 17^m, the changing of mean time to apparent time may often be neglected.

Hints and Precautions.—(1) Particular care should be taken to apply corrections with their proper sign. The parallax and refraction correction will *always be negative;* the semi-diameter correction will be positive when the lower limb of the sun is observed; and the index error will depend upon the instrument used. (2) If the longitude of the place of observation is known, the time of observation need not be determined. Thus, if the hourly change in declination be denoted by c, the apparent declination is

$$\delta = \delta_0 + c\lambda$$

in which λ is the longitude *in hours.* (3) Prismatic eyepieces with colored shades are usually furnished with instruments for use in solar observations. In case none is to be had,

14.

Prob. 50. Latitude of Town Hall, Gorham, Me.

Observation on Sun at Apparent Noon.

Field Work.

Circle	Obj.	Time	V. Circle	Index E.	
L	"A"		+2°58'30"		h_1
R	"A"		+2°55'30"		h_2
				−0°01'30"	
L	☉	$11^h 34^m 01^s$	48°05'00"		h'

Computations.

Watch	Time			11^h	$34^m\ 01^s$
"	Slow				29
G.M.T.	(E.S.T. + 5^h)			4	34 30
Eq. of	Time (from Ephemeris) +				04 40
G. A. T.				4^h	$39^m\ 10^s$
δ_0	(Declin. at G. A. Noon)			3°	09' 59.3"
$\Delta\delta$	(57.7" × 4.65)			−	04' 18.5"
δ				3°	05' 40.8"
Obs.	h' on ☉			48°	05.'0
Index-error				−	01.'5
Ref.	& Parallax (Table I)			−	00.'7
Sun's	Semi-Diameter			+	15.'9
h	(Corrected Altitude)			48°	18.'7
δ				3°	05.'7
ϕ	$= 90° - h + \delta$			44°	47.'0

Wm. Bolton }
H. L. Brown } Observers.
Sept. 15. 1913

Remarks. Fair, Warm, Calm.
"A" is Mark on Barn
400 ft. South. B. & B. Transit No. 142.
See Note. Waltham Watch.
By Watch. 29^s slow E. S. Time.

NOTE:– Index – Error found by
Reversal on Point "A"

$$\text{Index } E = -\frac{(2°58'30") + (2°55'30")}{2}$$

" " $= -0°01'30"$ with Circle Left.

Ephemeris gave Values
for G. A. Noon

Latitude of Town Hall.

SAMPLE NOTES 50.

and the sun is not at a high altitude, very good observations can be made by allowing the sun's image to come to focus on a white card held several inches from the eyepiece. A pointing on the sun should be made as nearly as possible by sighting over the telescope; the eyepiece should then be drawn out and the objective focused until the sun's image and the cross-hairs are clearly seen on the card. If the eyepiece of the telescope is erecting, the image on the card will be inverted; if the eyepiece is inverting, the image will be erect. (4) In using the card with an instrument equipped with stadia hairs, the observer should make sure that the *middle* cross-hair is brought tangent to the sun; this can be ascertained by revolving the telescope a few minutes in altitude until all the cross-hairs have been seen. (5) With a transit having a full vertical circle the index error may be most accurately found by noting the vertical circle readings to some well-defined point with the telescope in both positions (circle right and circle left). This will not only take into account the errors due to vertical axis not being truly vertical and zero of vernier's being displaced, but will also dispose of the error due to line of sight's not being parallel to axis of telescope bubble-tube. The determination of the index error by this method is shown in Sample Notes 50, h_1 and h_2 being the observed vertical angles to the mark. The reason for the sign of the correction should be ascertained. (6) Some engineering instrument companies issue pocket ephemerides for use with the solar attachment. There is on the page for each month a table of refraction corrections with argument expressed in hours before or after noon. It should here be noted that *these refraction corrections are to be applied to apparent declinations* (*when using the solar attachment*), *not to observed altitudes*. (7) In the United States the watches and clocks usually keep standard time. Eastern, central, mountain, and western standard times are respectively five, six, seven, and eight hours slower than Greenwich mean time. Thus when it is 12 o'clock central standard time it is 6 o'clock Greenwich mean time.

Practical Applications.—The above method provides a rough value of the latitude for use in determining the meridian by astronomical observations. Under favorable conditions the error need not exceed 02′, and the mean of a series of observations will render a closer determination.

Problem 51. Azimuth by Direct Solar Observation

Object.—To determine the true azimuth of a line by an observation of the sun with the engineers' transit.

Procedure.—(1) Set up the transit over one end of the line and level very carefully. If the vertical vernier is not adjustable, note the index error of the vertical circle with the telescope level and pointing in the general direction of the sun. Set the *A* vernier to read zero on the horizontal circle, and note the reading of the *B* vernier. Sight along the line, and clamp the lower motion. (2) Loosen the upper motion, and bring the vertical and horizontal cross-hairs tangent to the sun in the upper left-hand quadrant if the observation is in the forenoon, in the lower left-hand quadrant if in the afternoon. Record the time of tangency and horizontal and vertical circle readings, reading all verniers. (3) Loosen the upper motion, plunge the telescope, and bring the vertical and horizontal cross-hairs tangent to the sun in the lower right-hand quadrant if in the forenoon; in the upper right-hand quadrant if in the afternoon. Read and record the horizontal and vertical angles, and note the time of observation as before. (4) Loosen the upper motion, and again sight along the line with the instrument in the plunged position. Record the horizontal circle readings. (5) Compare the watch used with a time-piece keeping correct standard time, and note the correction. (6) Compute the Greenwich mean time of observation by adding five, six, seven, or eight hours (according to the time belt in which the place of observation is located) to the corrected mean of the times of observation. (7) From the Nautical Almanac or Solar Ephemeris, determine the apparent declination δ of the sun for the Greenwich mean time of observation as explained in the previous problem. (8) If the ephemeris gives values of the declination for Greenwich apparent noon, follow Procedure (8) of the previous problem. (9) Correct the mean of the observed altitudes for index error, and refraction and parallax. The latter correction may be found in Table I, page 243, and is applied as in the previous problem. (10) Compute the azimuth of the line (using five-place logarithmic tables) by the formula

$$\operatorname{Cos} A = \frac{\sin \delta}{\cos h \cos \phi} - \tan h \tan \phi$$

in which ϕ is the latitude of the place, h is the corrected mean altitude to the sun's center, and A is the angle east or west of south according as the observation is made in the morning or afternoon. (11) Calculate the azimuth of the line by algebraically subtracting the mean of the observed horizontal angles from the computed azimuth of the sun, angles taken in a clockwise direction from reference line to sun being considered as positive. (12) Make several similar observations. Determine the most probable value of the azimuth and the probable error of a single observation and of the mean (Procedure (5), page 98). See Sample Notes 51, page 132.

Hints and Precautions.—(1) The time between sights should be short, but the observer should not read the vernier carelessly in an endeavor to hurry. An interval of five minutes between sights will not cause an appreciable error in the final result. (2) The reason for taking one pointing with the instrument direct and the other with the instrument inverted is to eliminate instrumental errors. As an additional precaution, eccentricity of circles is eliminated by reading both verniers. If the transit has two vertical verniers, they should both be read and a mean value taken. (3) On account of the size of the sun, a sight can not be taken to its center with any degree of precision; but by taking the mean of the observations as already described, the position of the sun's center is determined with accuracy. If for any reason, however, it is desired to obtain the azimuth of a line by a single pointing, the sun may be brought tangent in any of the quadrants. The altitude correction for the sun's center will then be the amount of the sun's semi-diameter or approximately 16′;* the horizontal angle correction will be equal to 16′ × *secant of the altitude*. (4) Great care should be taken in leveling the instrument before making the observation; first level with the plate bubbles and then, as a final test, see that the telescope bubble remains in the center of the tube when the instrument is revolved about its vertical axis. (5) If no solar eyepiece is to be had, a card held in the rear of the eyepiece may be used as explained in the previous problem. If the eyepiece of the telescope is erecting, the image on the card will be inverted; thus the image of the sun will appear in the upper right-hand quadrant when it is in the lower left-

* For a closer determination of the sun's semi-diameter see Nautical Almanac.

Prob. 51. Azimuth of Line Δ 46 – Δ 63

Direct Solar Observation.

Circle	Object	Time	Vert. Cir.	Horiz. Circle Vert. A	Vert. B
R	Δ 63			0°00′00″	180°00′00″
R	☉	$8^h42^m48^s$	34°46′	331°37′40″	151°38′20″
L	☉	$8^h47^m23^s$	34°55′	152°05′40″	332°05′20″
L	Δ 63			179°59′40″	359°59′20″

Watch	Time	$8^h45^m05.5^s$		– LO	GS –
"	Slow +	33		sin. δ	8.73595
C.S.	Time	$8^h45^m38.5^s$		cos. h	9.91431
G.M.	T.	$2^h45^m38.5^s$		cos. φ	9.85112
Eq.	Time +	04 42.1			8.97052
G.A.	T.	$2^h50^m20.6^s$		tan. h	9.84235
δ_0		3°09′59.3″		tan. φ	9.99672
Δ δ	–	02 43.7			9.83907
δ		3°07′15.6″	– N	UMBER	S –
h′		34°50.5′	$\frac{\sin\delta}{\cos h}$	$\cos\phi$ =	0.09344
Ref.	& Par. –	01.2	– tan. h	tan. φ = –	0.69034
h		34°49.3′	c	os A = –	0.59690
				A =	126°39′
				H =	28°08′
					154°47′

15.

Queen & Co. Transit.	J.N. MacDougal.
Watch 33ˢ Slow.	W.W. Wilson.
C.S. Time.	Sept. 15, 1913, A.M.
	Hot, Windy.

Hor. Angs. on Azimuth Circle.

Note: Ephemeris gave Values for G.A. Noon.

Azimuth of Sun from North.

Angle from Line to Sun.

Azimuth of Line (from North).

SAMPLE NOTES 51.

hand quadrant. If the eyepiece is inverting, the image will appear in its true position. (6) For good results, observations should be taken between the hours from 8 to 10 a.m. and 2 to 4 p.m.; for, if taken when the sun is near the horizon, the refraction correction is large and uncertain, and if taken near the hour of noon, the resulting astronomical triangle will be poorly proportioned. Under favorable conditions of atmosphere and time, the azimuth of a line may be determined within 01′ by a single observation. A series of observations will render a much closer determination.

Practical Applications.—The method of determining the meridian by a single altitude of the sun is much used in surveys where a high degree of accuracy is not necessary. Although more accurate results may be obtained by an observation on Polaris, this necessitates work at or near night, which is not always desirable. The solar attachment makes it possible to turn to the meridian at the time of the observation and eliminates the solution of the spherical triangle; but the direct method possesses the advantage that no additional instrumental equipment is needed besides being the more accurate method. At the present time the direct method is looked upon by surveyors with increasing favor.

PROBLEM 52. AZIMUTH WITH SAEGMULLER SOLAR ATTACHMENT

Object.—To determine the true azimuth of a line by a single observation with the Saegmuller solar attachment.

Procedure.—(1) Look up the sun's declination for the Greenwich mean time of the proposed observation as explained in Problem 50, and correct the result for refraction by Tables III and III(*a*), pages 245 and 247, *adding* the correction if the declination is *positive* and *subtracting* if the declination is *negative.* (2) Set up the transit, with solar telescope attached, over one end of the line the azimuth of which is to be determined, carefully leveling the instrument. Note the index error of the vertical circle. (3) Sight the transit telescope at some well-defined point, and bring the lines of sight of both telescopes into the same vertical plane by a similar pointing with the solar telescope. (4) Set the horizontal vernier to read zero. (5) Incline the transit telescope an amount equal to the *corrected declination* (allowing for index error if such exists), depressing the objective end if the declination is north and elevating it if the declination is south. Level the solar telescope without disturbing the transit. Set off

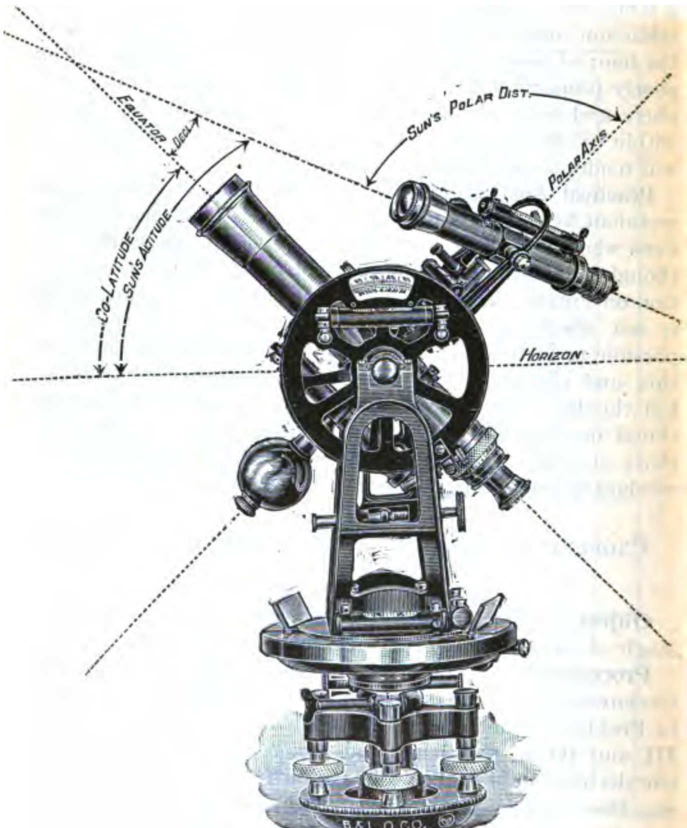


Plate IX.—Solar Attachment Mounted on Transit.
(In position for determining the meridian.)
(From an electrotype loaned by Bausch & Lomb Optical Co.)

a positive angle on the vertical circle equal to the co-latitude of the place (allowing for index error). (6) Point the transit telescope as near south as can be determined with the magnetic needle. With the lower motion unclamped, and with the solar telescope free to revolve about the polar axis, bring the sun into the center of the lined square of the solar telescope near the time for which the declination was computed. Clamp the lower motion and the polar axis. Make an exact setting by moving simultaneously the tangent-screw controlling the lower motion of the transit and the tangent-screw controlling the movement of the solar telescope about the polar axis. If no mistake has been made in laying off the co-latitude or declination, the line of sight of the large telescope will lie in the plane of the meridian and the polar axis will point toward the pole. If this condition is realized, since the declination changes very slowly, it will be possible to keep the sun in the center of the square for some time by simply revolving the solar telescope about the polar axis. *This should be done as a check.* Plate IX shows the solar attachment mounted on transit with the line of sight of the transit pointing south in the plane of the equator and the solar telescope pointing at the sun. (7) Loosen the upper motion, and turn to the line. Read the horizontal angle, and record the azimuth of the line. (8) Repeat the observation for other times until several satisfactory determinations have been made. (9) Compute the most probable value of the azimuth, and probable error of a single observation and of the mean (Problem 34, Procedure 5, page 98). If conditions are similar, compare the results with those of the previous problem.

Hints and Precautions.—(1) Many of the suggestions of the previous problem apply to observations with the solar transit. (2) The solar attachment should be in perfect adjustment, and vertical angles should be set off very carefully, for an error in the declination or co-latitude will be increased in the meridian determination. (3) The refraction corrections to be applied to declinations for latitude 40° with arguments in hours before and after noon may be found in Table III, p. 245. For latitudes other than 40° the refraction may be found by multiplying the value for 40° by the coefficient corresponding to the latitude of the place of observation, Table III(*a*), p. 247. These tables are particularly adapted to observations with the solar transit, and it should be noted that *these corrections are for declinations, not altitudes.* (4) To the beginner, the operation of getting the sun into the center of the lined square of the solar telescope presents some difficulties. This can best be done as follows: Point the solar telescope (as near as can be judged) along a vertical plane through the sun. Hold it in that position with one hand, and with the other hand revolve the transit about its vertical axis. Since the solar telescope is free to revolve about the polar axis, the effect of this movement will be to elevate or depress the line of sight. The sun will soon appear in the field, and a final setting may be made as previously explained. (5) Where observations are likely to extend over some time, a table of declination settings should be made out before going into the field. Declinations, corrected for refraction, should be computed for intervals of time depending upon the rate of change of declination and the least reading of the vertical circle. Thus, if the vertical vernier reads to 20″ and the hourly change in declination is 58″, declinations should be calculated at intervals of say 15^m. (6) The method of procedure with other attachments is essentially the same as with the Saegmuller solar attachment. On the Burt solar attachment, the declination is laid off on a declination arc, and the sun's image is brought to the center of a ruled square on a silver disc.

Practical Applications.—As a quick method of determining the meridian in the field the solar attachment possesses a decided advantage over the ordinary transit. Most engineers, however, agree that its cost and the bother of adjusting it more than offset the advantage gained, unless it is to be used very frequently, as is sometimes the case in mining and city surveying. Under the most favorable circumstances its accuracy is less than that to be obtained by the direct method of observation. For this reason most engineers do not view the use of the solar attachment with favor.

Problem 53. Time by Observation on the Sun at Noon

Object.—To determine the correction of a time-piece by observing the sun as it crosses the meridian. It is assumed that the longitude of the place of observation has already been determined either by astronomical observations or from a map; and that a meridian has been established.

Procedure.—(1) Set up the transit over the north end of the established meridian, carefully level the instrument, and sight along the meridian. Elevate the line of sight to in-

					16.
Prob. 53. Time By Observation					On Sun at Noon, Nov. 13, 1913.
Meridian at Observatory, Urbana.					Young & Sons Transit. J.C. Moore.
					Watch. E. Williams.
Circle	Object	Time			Fair, Cold.
L	△ 20				So. End of Meridian.
L	d	$12^h07^m32^s$			
R	△ 20				" " " "
R	b	$12^h09^m49^s$			
					Note:- Longitude of
Time of Passing of Suns Ctr.				$12^h08^m40.5^s$	Observatory $5^h52^m54.1^s$
Eq. of Time at G.A. Noon			−	15 40.8	From Ephemeris.
Longit. x Hourly Ch. (5.87×0.34^s)			+	02.0	" "
Watch Time of L.M. Noon				$11^h53^m01.7^s$	
△ T_m			+	06^m58^s	Correction to Local Mean Time.
△ T_s			−	08^s	" " Central Std. "

SAMPLE NOTES 53.

tercept the path of the sun. (2) At the instant of tangency between the western limb of the sun and the vertical cross-hair, note the time. (3) Quickly plunge the telescope, reverse in azimuth, and again sight along the meridian. Elevate the telescope, and note the time when the vertical cross-hair is tangent to the eastern limb of the sun. The mean of the two times will be the time of transit of the center of the apparent sun over the meridian. (4) If the longitude of the place of observation is expressed in degrees, change to hours by dividing by 15. Since the sun transits at apparent noon, and longitude is reckoned from Greenwich, the longitude in hours is the Greenwich apparent time of observation. (5) From the Nautical Almanac or Solar Ephemeris determine the equation of time for the Greenwich apparent time of observation. Then if T be the observed time, E the equation of time, and ΔT_m the correction of the time-piece to local mean time,

$$\Delta T_m = 12^h - (T \pm E)$$

The sign of the equation of time will be positive or negative according as mean solar time is slower or faster than apparent solar time. (6) If the correction to give standard time is desired, it may be found by the equation

$$\Delta T_s = \Delta T_m \pm \Delta\lambda$$

in which $\Delta\lambda$ is the difference in longitude in hours between the place of observation and the meridian at which standard time and local mean time agree. If the place of observation is east of this meridian, the sign of $\Delta\lambda$ will be negative; if west, positive.

Hints and Precautions.—(1) The length of time taken by the sun in crossing the meridian will depend somewhat upon its declination and semi-diameter but will not vary much from two and one-third minutes. The observer should, therefore, lose no time while reversing the transit for a second sight along the meridian and the sun's eastern limb. (2) If for any reason it is impracticable to observe but one limb, the time in seconds earlier or later that the sun's center passes the meridian is approximately $\frac{4S}{\cos\delta}$, in which S is the sun's semi-diameter in minutes of arc (approximately 16′) and δ is the declination of the sun. It is best, however, to take the time of passage of both limbs with a reversal of the instrument between sights, for by so doing, instrumental errors are eliminated. (3) If the transit is not equipped with eyepiece for astronomical observations and the sun is not too high, a card held back of the eyepiece may be used as explained in Problem 50, p. 127. (4) If the ephemeris used gives values for Greenwich mean noon, an exact determination of the equation of time will necessitate changing the Greenwich apparent time of observation (Procedure (4)) to Greenwich mean time by means of an approximate equation of time. The true equation of time may then be found by interpolating between values given for the preceding and succeeding mean noons. Since the equation of time is always less than 17^m and since its rate of change is relatively small, for ordinary observations for the purpose of determining time, the change from apparent to mean time need not be made. (5) The problem may be readily reversed; if the time is accurately known, the longitude may be computed.

Problem 54. Azimuth and Longitude by Solar Observation

Object.—To determine the azimuth of a line and longitude of the place of observation by an observation on the sun with the repeating theodolite. It is presumed that the theodolite used is a high-grade instrument, carefully adjusted, and that the time is taken with an accurate time-piece keeping standard time.

Procedure.—(Fieldwork similar to that of Problem 51, p. 130.) (1) Between the hours of 8 and 10 a.m. set up the instrument at one end of the line whose azimuth is to be determined. Level the instrument very carefully. (2) With upper motion clamped, sight along the line, clamp the lower motion, and record the horizontal circle readings (circle right). (3) Loosen the upper motion, reverse the telescope, and again sight along the line. Record the horizontal circle readings as before (circle left). (4) Loosen the upper motion, and turn to the sun, bringing the sun tangent to the cross-hairs in the upper left-hand quadrant (circle left). Record the time at the instant of tangency and the horizontal and vertical circle readings. (5) Loosen the upper motion, and again sight on the sun, repeating step (4) (circle left). (6) Loosen the upper motion, reverse the telescope, and bring the sun into the lower right-

15.

Prob. 54. Azimuth and						Longitude at White Rock, Ky.			
	Observation on Sun.					Lat. 37°03'35"			J.M. Kirk.
		Field Work							F.C. Camp.
		A.M.							July 2, 1901.
Circle	Obj.	Time	Horizontal		Circle	Vertical Circle			Temp. 97°F.
			Ver. A	Ver. B	Mean	Ver. A	Ver. B	Mean	Berger Theod. No. 43.
R	B		352°37'40"	38'00"	352°37'50"				Chronometer 06.8s –
L	B		172 37 20	37'00"	37'10"				Fast.
Mean					352°37'30"				E. S. Time.
L	☉	9h35m15s	291 40 00	40 20	111 40 10	44°17'30"	17'30"	44°17'30"	
L	☉	36 10	291 49 00	49 00	111 49 00	44 29 00	29 00	44 29 00	
R	☉	37 40	112 45 20	45 40	112 45 30	45 11 00	11 30	45 11 15	
R	☉	38 47	112 56 20	56 40	112 56 30	45 24 00	24 30	45 24 15	
Means		9 36 58.0			112°17'50"			44°50'50"	
Mean Hor. Ang. Line to Sun.					119°40'20"	Par. & Ref. –		0'78	
R	☉	9 39 46	113 06 10	06 20	113 06 15	45 36 00	36 00	45 36 00	
R	☉	40 49	113 17 00	17 00	113 17 00	45 48 30	49 00	45 48 45	
L	☉	42 20	112 49 00	49 00	112 49 00	45 41 30	42 00	45 41 45	
L	☉	43 24	112 59 10	59 20	112 59 15	45 54 30	54 30	45 54 30	
Means		9 41 34.8			113°02'50"			45°45'26"	
Mean Hor. Ang. Line to Sun.					120°24'30"	Par. & Ref. –		0.76	
L	B		172 38 00	37 30	352 37 45				
R	B		352°38'50"	39 10	39 00			Note: Hor. Angs. are	
Mean					352°38'20"			Clockwise from Line A-B to ☉	

SAMPLE NOTES 54.

hand quadrant (circle right). Record the time and readings of circles. (7) Repeat step (6) (circle right). The four readings taken to the sun thus far constitute a set which for convenience will be called Set 1. (8) Without again turning to the line, sight on the sun taking the first two observations with circle right and the last two, with circle left. (9) Again turn to the line (circle left) and record the horizontal circle readings. (10) Reverse the telescope and repeat (9) (circle right). (11) Record the temperature. This completes Set 2. (12) Between the hours of 2 and 4 p.m. take two more sets in a similar manner except that the sun may best be observed in the upper right-hand and lower left-hand quadrants. (13) Compute the mean values of horizontal and vertical angles and time, for each set. (14) Correct each mean altitude for parallax and refraction (Table I, page 243) and look up the apparent declination for the Greenwich mean time of each set. (15) Calculate the azimuth of the sun for the mean time of each set by the equation

$$\text{Cot}^2 \tfrac{1}{2} A = \frac{\sin (s - \phi) \sin (s - h)}{\cos s \cos (s - p)}$$

in which A is the angle to the sun east of south in the morning, west of south in the afternoon; ϕ is the latitude of the place of observation; h is the corrected altitude; p is the polar distance of the sun or $(90^\circ - \delta)$; and $s = \frac{1}{2}(\phi + h + p)$. Compute the mean azimuth of the line. (16) Determine the azimuth of the line by combining these results with the mean of the corresponding horizontal angles to the sun as shown in the sample computation. (17) Calculate the hour angle of the sun at the mean time of each set by the equation

$$\tan \tfrac{1}{2} t = \frac{\sin (s - h)}{\cot \tfrac{1}{2} A \cos (s - p)}$$

in which t is the hour angle expressed in degrees of arc. This expressed in hours is the local apparent time. (18) Convert into local mean time by applying the equation of time, the latter being added or subtracted according as mean time is faster or slower than apparent time. (19) Calculate the longitude of the place of observation. The difference between this local mean time of each set and the correct standard time of each set is the difference in longitude expressed in hours between the place of observation and the meridian at which

17.

Prob. 54. (Cont'd.) Azimuth and Computations.

	Set	1	2	3	4
	h	44° 49'.7	45° 44'.5	44° 05'.3	41° 28'.5
	φ	37 03.6	37 03.6	37 03.6	37 03.6
	p	66 55.5	66 55.5	66 56.7	66 56.8
	2 s	148° 48'.8	149° 43'.6	148° 05'.6	145° 28'.9
	s	74 24.4	74 51.8	74 02.8	72 44.4
	s−p	7 28.9	7 56.3	7 06.1	5 47.6
	s−h	29 34.7	29 07.3	29 57.5	31 15.9
	s−φ	37 20.8	37 48.2	36 59.2	35 40.8
Check	2 s	148° 48'.8	149° 43'.6	148° 05'.6	145° 28'.7
colog	cos s	0.57056	0.58316	0.56090	0.52767
colog	cos (s-p)	0.00371	0.00418	0.00334	0.00222
log	sin (s-h)	9.69339	9.68723	9.69842	9.71516
log	sin (s-φ)	9.78293	9.78743	9.77933	9.76586
log	cot² ½ A	0.05059	0.06200	0.04199	0.01091
log	cot ½ A	0.02530	0.03100	0.02100	0.00546
Az. of	⊙ from N	93° 20'.2	94° 05'.2	267° 13'.8	269° 16'.8
Ang. ⊙	to Line	240 19.7	239 35.5	66 27.1	64 23.4
Az. of	A−B	333° 39'.9	333° 40'.7	333° 40'.9	333° 40'.2
Mean	Az	333° 40'.4			
Cal.	Bear.	N 26° 19'.6 W			

Longitude at White Rock, Ky.

For Field Notes See P. 15 & 16.

Computed by J. C. Kirk.

Checked by F. C. Camp.

July 2, 1901.

Set	1	2	3	4
colog cos (s-p)	0.00371	0.00418	0.00334	0.00222
log sin (s-h)	9.69339	9.68723	9.69842	9.71516
colog cot ½ A	9.97470	9.96900	9.97900	9.99454
log tan ½ t	9.67180	9.66041	9.68076	9.71192
t°	50° 19' 00"	49° 10' 12"	51° 13' 57"	54° 30' 32"
t^h	−3h 21m 16.0s	−3h 16m 40.8s	3h 24m 55.8s	3h 38m 02.1s
E	3 40.2	3 40.2	3 43.4	3 43.5
Local M. T.	8 42 24.2	8 46 59.4	3 28 39.2	3 41 45.6
Obs. Time	9 36 58.0	9 41 34.8	4 23 10.8	4 36 22.8
Δ T_m	− 54 33.8	− 54 35.4	− 54 31.6	− 54 37.2
Δ T_s	− 06.8	− 06.8	− 06.9	− 06.9
Δ λ	54m 27.0	54m 28.6	54m 24.7	54m 30.3
Mean	54m 27.6 = 13° 36'.9			
λ	88° 36'.9			

SAMPLE COMPUTATIONS 54.

standard time is also local mean time. If local mean time is ahead of standard time, the place of observation is to the east of the standard meridian, and the longitude is, therefore, less; if behind, to the west, and the longitude is greater. Record notes and computations as shown in Sample Notes 54 and Sample Computation 54, pages 140 and 142.

Hints and Precautions.—(1) Observe the hints and precautions of Problem 51, page 130. (2) Notice that Sample Notes 54 has not included the afternoon observations for which computations are shown.

Practical Applications.—The above method is employed by the U. S. Coast and Geodetic Survey in connection with magnetic observations. For longitude determination a reliable time-piece is of the utmost importance, since error in time will make a corresponding error in longitude. By this method, under favorable conditions the error in azimuth determination need not exceed 10″, and in longitude determination, 30″.

Problem 55. Latitude by Observation on Polaris at Culmination

Object.—To determine the latitude of the place by observing Polaris at upper or lower culmination.

Procedure.—(1) Determine the approximate local mean time of culmination of Polaris by Table IV, page 248. Change to standard time. Look up the star's polar distance $p = 90° - \delta$ in Table VIII, page 261. The polar distance may be determined more accurately by looking up the value of the declination δ in the Nautical Almanac (Table I). (2) A few minutes before culmination set up the transit leveling it very carefully (as a final test the telescope bubble should remain centered for all pointings) and note the index error of the vertical circle or adjust vernier to read zero. (3) Locate the star with the naked eye, and if the latitude is roughly known, set off its value, plus or minus the star's polar distance, on the vertical circle. Sight along the barrel of the telescope to range in the star; sight through the telescope and move the instrument about the vertical axis until the star is near the center of the field. (4) Illuminate the cross-hairs, and follow the star until it no longer appears to move off the horizontal hair but moves along it. Note the time, and record the vertical angle as quickly as possible. (5) Plunge the telescope, and again sight on the star. Record the time and vertical angle as before. (6) Correct the mean of the observed vertical angles for refraction (Table II, page 244) and index error if such exists; and compute the latitude, $\phi = h \pm p$, in which the sign of p is positive or negative according as the star is at lower or upper culmination.

Hints and Precautions.—(1) The mean of the times of observation should agree closely with the computed time of culmination; this furnishes a check upon the star sighted. (2) Observations are taken with the telescope in both positions to eliminate instrumental errors. This would, of course, be impossible with a transit not equipped with a full vertical circle. (3) Knowing the local time of culmination, to find the standard time of culmination, add or subtract the difference in longitude expressed in hours between the place of observation and the standard meridian, *i.e.*, the place in the time belt of the observation where standard and local mean time agree, as the 75th meridian for eastern standard time, or the 90th meridian for central standard time. The difference in longitude will be added to local mean time of culmination if the place of observation is to the west of the standard meridian; and subtracted, if it is to the east. The longitude of the place of observation may be determined with sufficient accuracy for the problem from any good map. (4) Some transits are furnished with reflectors with which to illuminate the cross-hairs at night. The illumination is accomplished by holding a light to one side of the telescope. With the ordinary transit not so equipped the cross-hairs can be illuminated sufficiently by holding the light a few inches in front of the objective and a little to one side of the telescope barrel, thus causing the rays of light to enter the telescope diagonally. If held so that the rays strike the eye of the observer directly, the light will be so strong as to cause the star to disappear. A drop of candle grease in the center of the objective lens is also often used. The grease should be shaved to a very thin layer. (5) It should be noted that, while a double sight eliminates error due to line of sight not being parallel to axis of bubble, it will not eliminate error in the vertical circle reading due to the vertical axis not being truly vertical, and for this reason the instrument should be leveled with great care. (6) The position of Polaris in the sky may be easily determined by means of the two neighboring constellations, Cassiopeia and Ursa Major

(Fig. 31). The seven most brilliant stars of the latter constellation are known as the Great Dipper; the two stars forming that part of the bowl farthest from the handle are called the "*pointers*" because a line through these stars points very nearly at the Celestial Pole. Since Polaris is only a little more than a degree distant from the Pole, and since there are no other bright stars in its vicinity, the correct star can be readily

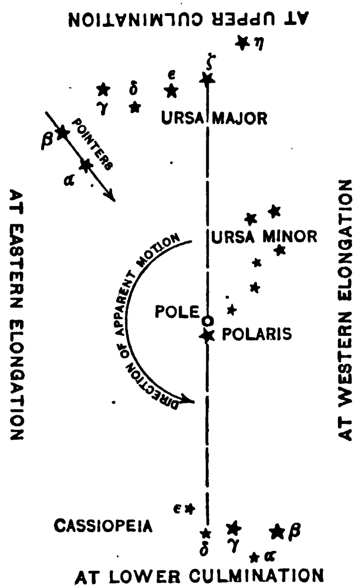


FIG. 31.—POSITIONS OF CONSTELLATIONS NEAR THE NORTH POLE WHEN POLARIS IS AT CULMINATION AND ELONGATION.

picked out with the eye. (7) If tables for determining the time of culmination of Polaris are not at hand, the position of the star relative to the Pole may be roughly estimated by noting the position of the stars δ Cassiopeia and ζ Ursa Major. A line through these stars passes very near Polaris.

When δ Cassiopeia and ζ Ursa Major (Fig. 31) are in the same vertical plane, Polaris is at upper or lower culmination, depending on whether δ Cassiopeia is above or below the Pole. (8) Observations may be taken on Polaris some time before it is dark enough to discern the star with the naked eye, providing the objective is properly focused and the air is clear. Government surveyors who find it necessary to make frequent observations, often mark on the barrel of the telescope the position of the objective slide for this focus. When the air is very clear, as in high altitudes, it is possible in this way to observe the star when the sun is still above the horizon. A method of focusing the objective often used when it is desired to observe near dusk is to get a clear definition of the evening star which can be clearly seen with the naked eye before sunset. The objective should be moved toward or from the cross-hairs until the star appears as a fine point.

Practical Applications.—Same as for Problem 50, p. 126. The method will, however, give more reliable results, and should be used when an error greater than 01′ is undesirable.

Problem 56. Azimuth by Observation on Polaris at Elongation

Object.—To determine the azimuth of a line by observation on Polaris at eastern or western elongation.

Procedure.—(1) Look up the local mean time of elongation in Table IV, p. 248, and change to standard time as explained in Problem 55. (2) A few minutes before the star is at elongation, set up the transit very carefully over one end of the line the azimuth of which is to be determined. (3) With the objective focused so that the star appears as a fine point, and the cross-hairs illuminated, follow the star until it no longer appears to travel horizontally but moves along the vertical hair. Depress the telescope, and carefully set a point at least 300 ft. distant. (4) Reverse the instrument, and again sight on Polaris. Depress the telescope, and set a point beside the first. If the instrument is in good adjustment, these two points may coincide; if they do not coincide, set a point midway between them. (5) Look up the azimuth of Polaris in Table V, p. 250. (6) By daylight determine by repetition (Problem 18, page 62) the angle between the line established by setting points beneath the star and the line of which the azimuth is desired. Calculate the azimuth of the line.

Hints and Precautions.—(1) The hints and precautions of Problem 55 should be noted. Fig. 31, p. 145, shows the relative positions of the neighboring constellations, Ursa Major and Cassiopeia, when Polaris is at elongation. (2) It is not essential that the exact latitude be known. It will be seen from Table

V that an error of 1° of latitude will make a relatively small error in the deduced azimuth. It will therefore, for ordinary observations, be sufficient, if the latitude of the place of observation has not been previously determined, to estimate it from a reliable map; or having no map, to assume that it is the vertical angle to the star at time of elongation. In the latter contingency it will, of course, be necessary to slightly alter the course of procedure outlined. (3) The azimuths given in Table V are deduced from mean values of the declination for each year. If very accurate results are desired, the apparent declination of Polaris for the date of observation should be determined from the Nautical Almanac; careful observations for latitude should be made (Problem 55); and the azimuth of the star should be computed by the formula $\sin A = \frac{\cos \delta}{\cos \phi}$ in which δ is the apparent declination and ϕ is the latitude. If the latitude is accurately known, the resulting azimuth should be in error less than 05″. Six place logarithms should be used. (4) If an error of 01′ in the calculated azimuth of the line is allowable, points need not be set beneath the star. Instead, the horizontal angle between the line of reference and the line of sight to the star may be measured with circle right and circle left as in direct solar observations (Problem 51, p. 130). (5) A very satisfactory target for night work is a strongly illuminated card upon which a line has been drawn of sufficient weight to be clearly seen through the telescope from the observer's station. The source of illumination should be shielded from the observer, for otherwise the target will appear indistinct. (6) The time between sights to the star should be as short as possible, but there is no necessity for extreme haste in setting the ground points, since the azimuth of the star is changing very slowly. At latitude 40° this change amounts to about 01.5″ when the star is 05^m from elongation, and about 06″ when the star is 10^m from elongation. (7) If it is necessary to observe Polaris when not at elongation but *near* that position, the correction to be applied to the azimuth at elongation is expressed approximately by $C = 0.06n^2$ (for observations south of latitude 50°) in which n is the time in minutes before or after elongation and C is in seconds of arc.*

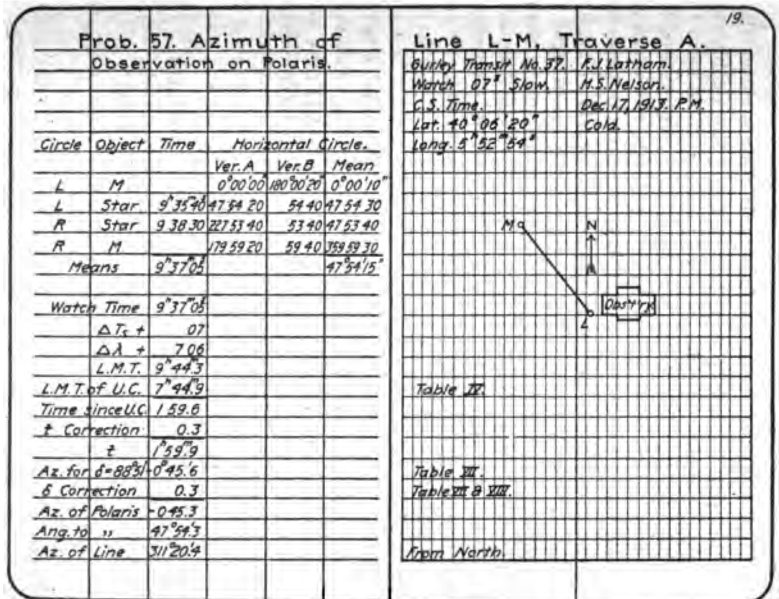
* More exactly the correction is $0.052n^2$ for latitude 30°, $0.058n^2$ for latitude 40°, and $0.069n^2$ for latitude 50°.

Practical Applications.—Because azimuth determination by observation on Polaris does not depend upon a time-piece and because the horizontal movement of the star is comparatively slow near the time of elongation, this method is superior to all others in point of precision. With modifications it has been extensively used on the U. S. Coast and Geodetic Survey, the Great Lakes Survey, the U. S. Geological Survey, and many other surveys of less magnitude. On some surveys this method is used to the exclusion of all others. Its chief disadvantage is that under the most favorable conditions observations can be taken but twice daily, and it often necessitates work at inconvenient hours.

Problem 57. Azimuth by Observation on Polaris at any Time

Object.—To determine the true azimuth of a line by observation on Polaris at any hour when the star can be seen, the latitude and longitude of the place of observation being known. See Sample Notes 57.

Procedure.—(1) Measure the horizontal angle between line and star with circle right and circle left, and note the time of the passing of the star across the vertical cross-hair at each setting as in direct solar observation (Problem 51, page 130). (2) Correct the mean of the two times of observation by comparing the time-piece used with some clock keeping correct standard time. (3) Change this correct standard time to local mean time by adding or subtracting the difference in longitude expressed in hours between the place of observation and the standard meridian (meridian where standard time is local mean time), adding, if the place of observation is east of the meridian; subtracting, if west. (4) From Table IV, page 248, determine the local mean time of the upper culmination of Polaris on the date of observation. (5) Compute the interval in hours before or after upper culmination. If Polaris were moving at the same angular velocity about the pole as the sun, this time interval would be the hour angle before or after upper culmination. In reality, Polaris is gaining on the sun at the rate of about 3.95^m per day or 10^s per hour. Add this gain to the time interval and the result is the hour angle of the star before or after upper culmination. (6) By Table VII, page 253, determine the azimuth of the star for declination 88°51′, the above hour angle, and the



Prob. 57. Azimuth of
Observation on Polaris.

Circle	Object	Time	Horizontal Circle.		
			Ver. A	Ver. B	Mean
L	M		0°00′00″	180°00′20″	0°00′10″
L	Star	9h35m40s	47 54 20	54 40	47 54 30
R	Star	9 38 30	227 53 40	53 40	47 53 40
R	M		179 59 20	59 40	359 59 30
Means		9h37m05s			47°54′15″
Watch Time		9h37m05s			
	ΔTs +	07			
	Δλ +	7 06			
	L.M.T.	9h44m.3			
L.M.T. of U.C.		7h44m.9			
Time since U.C.		1 59.6			
t Correction		0.3			
	t	1h59m.9			
Az. for δ=88°51′		0°45′.6			
δ Correction		0.3			
Az. of Polaris		-0 45.3			
Ang. to "		47°54′.3			
Az. of Line		311°20′.4			

19.

Line L-M, Traverse A.

Buff. Transit No. 37. F. J. Latham.
Watch 07s Slow. M. S. Nelson.
C. S. Time. Dec. 17, 1913. P.M.
Lat. 40°06′20″ Cold.
Long. 5h52m54s

M
N
L
Obs'y

Table IV.

Table VI.
Table VII & VIII.

From North.

SAMPLE NOTES 57.

latitude of the place of observation, interpolating if necessary. By Table VIII, page 261, determine the mean declination for the time of observation, and by last columns of Table VII correct the azimuth for change in declination. (7) Compute the azimuth of the line.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 55 and 56. (2) If results closer than the major part of one minute are desired, the declination and right ascension of Polaris should be ascertained from the Nautical Almanac (Table I) for the upper culmination for the 90th meridian west of Greenwich on the date of observation; and the sidereal time should be determined, by adding the local mean time, the correction of Table III, Nautical Almanac, whose argument is the Greenwich mean time of observation, and the sidereal time of Greenwich mean noon. The latter quantity will be found in the Nautical Almanac, page II for month. For method of computing Greenwich mean time see Hints and Precautions (7), page 129. It can be shown that

$$t = \theta - \alpha$$

in which t is the hour angle after upper culmination, θ is the sidereal time, and α is the right ascension. When t exceeds 12 hours it must be subtracted from 24. Table VII may now be used as before, or the azimuth of the star may be calculated by the equation

$$\tan A = \frac{\sin t}{\cos \phi \sin \delta - \sin \phi \cos t}$$

in which A is the angle east or west of the meridian, t the hour angle expressed in degrees of arc, ϕ the latitude, and δ the declination. (3) It should be noted that Polaris is east of the meridian for 12^h before upper culmination; west, for 12^h after.

Practical Applications.—It will be seen that the above method theoretically is not dependent upon the position of the star. From inspection of the preceding formula or by Table VII it will be noted that when the star is near upper or lower culmination any small variation in the hour angle will produce a relatively large variation in the azimuth of the star, the angular rate of change of the star being about 01′ of arc for $2\frac{1}{2}^m$ of time (for latitude 40°). For this reason the method should not be expected to give accurate results when the star is far from elongation unless a trustworthy time-piece is used.

CHAPTER VII

TOPOGRAPHIC SURVEYING

PROBLEM 58. STADIA INTERVAL FACTOR (FIXED HAIRS)

Object.—To determine the stadia interval factor (K) of transit, level, or plane-table alidade.

Procedure.—(1) Establish an approximately level line 800 ft. long, setting stakes by tape measurement every 100 ft. (2) Measure the focal distance (f) and the distance from the center of the instrument to the plane of the objective (c). Set a stake at a distance ($f + c$) back of the zero end of the line just established. (3) Set up over this last point, and with the telescope approximately level, determine the spaces intercepted by the upper and middle, and lower and middle hairs on a rod held at each of the base-line hubs. (4) Calculate K for each distance for upper, lower, and full intervals, and take the mean of the computed values as the factor for the instrument used. Follow the accompanying form, page 152.

Hints and Precautions.—(1) Before laying out the base-line the instrumentman should make certain that the projections of the lower and upper hairs, when the telescope is level, will fall on the rod for all distances up to the length of the proposed base-line. On fair days it is essential that the line of sight of the lower hair should be some distance above the ground because of the very unequal refraction in the layers of air close to the earth. (2) The focal length f is approximately equal to the distance from the plane of the objective to the plane of the cross-hairs when the objective is focused for a distant object. (3) Placing the instrument at a distance ($f + c$) in the rear of the zero end of the base-line eliminates the constants ($f + c$) and therefore simplifies the computations. With the instrument in this position the stadia interval factor whether for upper, lower, or full interval may be determined by the formula

$$K = \frac{D}{S}$$

30.

Prob. 58. Interval Factor (K)
Determination.

Dist. (ftc)	Lower Interval	Upper Interval	Full Interval	K for Low. Int.	K for Up Int.	K for Full Int.
100	0.49	0.50	0.99	204.1	200.0	101.0
200	0.98	0.99	1.97	204.1	202.0	101.5
300	1.48	1.49	2.98	202.7	201.4	100.7
400	1.97	1.99	3.96	203.0	201.0	101.0
500	2.45	2.48	4.94	204.1	201.6	101.2
600	2.94	2.98	5.93	204.1	201.3	101.2
700	3.41	3.48	6.91	205.3	201.1	101.3
800	3.92	3.99	7.90	204.1	200.5	101.3
Means				203.9	201.4	101.3

H.R. Sargent, ⊼
Gurley Transit No. 98. M.D. Rand, Rod.
(ftc) = 1.25 ft. June 14, 1911.
Warm, Cloudy,
No Wind.

Distances Measured with Tape
Inst. Set at Dist. (ftc) Back
of Zero of Base-line.

Sample Notes 58.

in which D is the distance from zero of base-line to rod and S is the rod interval at the above distance. It will probably be found that values of K will differ somewhat for the various distances. This will be due partly to errors of observation and partly to unequal atmospheric refraction. (4) It will facilitate the calculation of the rod interval if the lower hair is set on a footmark, for then the subtrahend will be a whole number of feet. This may be done without appreciable error if the footmark nearest the apparent position of the hair is chosen. That is, if with the instrument leveled the lower hair cuts the rod at 1.57 ft. throw the telescope out of level until the hair falls on 2.00 ft.

Practical Applications.—The stadia interval factor should be determined before any sort of a stadia survey is attempted, and to secure a good average value a series of tests should be made under a variety of weather conditions and at various hours of the day. On an extensive survey, since radical changes in weather conditions may alter the value of the interval factor, occasional tests should be made. For most instruments the value of this factor is so nearly 100 that for side shots or short traverses it may be considered as such without introducing an appreciable error, but on long traverses its true value should always be used.

Problem 59. Topographic Survey by Transit-Stadia Method. (Elevations Carried Forward by Direct Leveling)

Object.—To collect sufficient data for making an accurate topographic map of an assigned tract. (The data may be used in Problem 90, page 229.) See Sample Notes 59(*a*) and 59(*b*).

Procedure.—(1) Make a rapid reconnaissance of the tract, picking out the most advantageous points for instrument stations. (2) Run a closed azimuth traverse through these points, observing the stadia intervals and vertical angles (where the latter are so great as to appreciably change the stadia distance when reducing to the horizontal). Note the angular error of closure. *Allowable error in minutes* = $\sqrt{\textit{number of sides}}$. (3) Calculate the horizontal distances, latitudes and departures (Problem 73, page 183), and linear error of closure of the traverse. *Allowable error of closure* = $1/500$. (4) Run a level circuit from some point of known elevation for the purpose of determining the elevations of the traverse stations and such other points as will facilitate the taking of topography. *Allowable error in feet* $= 0.05 \times \sqrt{\textit{distance in miles}}$. (5) Occupy each of the traverse stations, and with the instrument correctly oriented, observe the azimuth, stadia distance, and vertical angle to all changes in ground-slope and to other natural and artificial features which are to appear on the finished map and which are within range of the instrument. (6) With the stadia slide-rule,* reduction diagram,* or by Table IX* compute the horizontal distance, difference in elevation, and elevation of each side shot.

Hints and Precautions.—(1) The line of zero azimuth may be either an assumed or true meridian, but in case an assumed meridian is used, for convenience in checking with the magnetic needle, it will be found advantageous to have the assumed and magnetic meridian closely related. In any case the azimuths of the traverse lines should ultimately be referred to the true meridian. (2) The hubs of the traverse lines should be referenced to permanent objects in such manner as to be easily re-located in case of loss, and a careful description of the location of the traverse stations should appear in the notes. Suitable witness stakes marked with the number of the line and station should be driven near the hubs. (3) In running the traverse the magnetic bearings of all lines should be recorded and immediately compared with the calculated bearing (deduced from the azimuth of the line). It is obvious that, if this method is adhered to, there will be no possibility of large angular errors. If the magnetic and calculated bearings are not referred to the same meridian, that is, if they do not agree, there should always be the same difference, and this difference is the check. (4) Vertical angles and stadia intervals should be observed from both ends of each traverse line; if they agree closely, their numerical mean is used in the later computations. Vertical angles are usually determined by sighting at a point as far from the foot of the rod as the center of the horizontal axis is above the station occupied; if for any reason this is impracticable, some other point is sighted upon (usually a whole

* Based on formulas $d = Kr \cos^2 v + (f + c) \cos v$ and $h = \frac{1}{2} Kr \sin 2v + (f + c) \sin v$ in which h and d are vertical and horizontal distances, v the vertical angle, r the rod interval, K the interval factor, and $(f + c)$ the stadia constant. Though the derivation of these formulas should be understood, the time and labor consumed in computing makes their direct use prohibitive.

31.

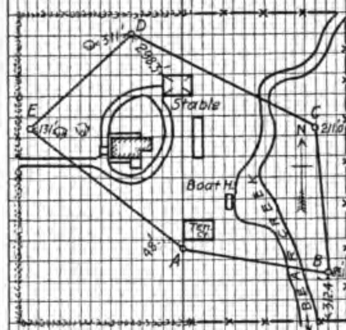
Prob. 59 a. Stadia Traverse for Horizontal Control.

Sta.	Obj.	Az.	Mag. B.	Rod Int.	Ver. Ang.	Hor. Dist.
B	A	279°00′	N 81° W	9.09	+2°41′	908
	C	356°14′	N 4° W	8.94	+0°31′	
C	B	176°14′	S 3°30′ E	8.98	−0°32′	899
	D	296°56′	N 63° W	13.45	−1°17′	
D	C	116°56′	S 63° E	13.50	+1°17′	1350
	E	221°49′	S 42° W	8.49	+4°42′	
E	D	41°49′	N 41° E	8.47	−4°41′	845
	A	127°57′	S 52° E	11.90	−1°27′	
A	E	307°57′	N 52° W	12.00	+1°27′	1198
	B	98°58′	S 81° E	9.05	−2°40′	

Black Estate.

Wisconsin Transit
(f+c) = 1.25 ft. K = 100.2

G. Burke, ⊼
J. Norris, Notes.
F.J. & K.D., Rods.
Apr. 3. 1913. Fair, Cold.

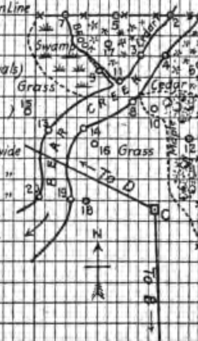


SAMPLE NOTES 59(*a*).

Prob. 59 b. Topographic Details, Black Estate. 32.

Inst. at C. El. 423.9, H.I. = 4.4

Obj.	Az.	Rod Int.	Ver. Ang.	Hor. Dist.	Diff. El.	Elev.	
							Wisconsin Transit G. Burke,
							(f + c) = 1.25, K = 100.2 M. D. Rand, Notes.
							F. J. & K. D., Rods.
B	176° 14′						Apr. 4, 1913.
1	10° 21′	7.23	− 3° 11′	723	− 40.2	383.7	Water's Edge - Corner Cloudy, Cold.
2	3° 14′	7.02	− 3° 17′	702	− 40.2	383.7	" " - On Line
3	352° 45′	5.64	− 4° 11′	563	− 40.9	383.0	" "
4	7° 18′	5.76	− 4° 04′	575	− 40.9	383.0	" "
5	349° 10′	(7.14 x	4) + 3.3	714	− 31.9	392.0	Line (Intervals)
6	16° 55′	5.50	− 2° 50′	551	− 27.3	396.6	"
7	315° 20′	(7.86 x	5) − 2.5	786	− 36.8	387.1	" (") 15
8	349° 15′	4.13	− 5° 46′	410	− 41.4	382.5	Water's Edge
9	339° 30′	5.40	− 4° 22′	539	− 41.1	382.8	Bank Brook 6′ wide
10	0° 05′	3.71	− 4° 12′	371	− 27.2	396.7	" " 15′ "
11	344° 40′	4.85	− 4° 54′	484	− 41.4	382.5	" " 15′ "
12	25° 00′	2.86		288	+ 1.2	425.1	Direct Levels.
13	307° 45′	4.88	− 4° 56′	487	− 42.0	381.9	Water's Edge.
14	319° 10′	4.02	− 5° 56′	400	− 41.6	382.3	" "
15	309° 45′	5.80	− 3° 00′	581	− 30.7	393.2	
16	318° 25′	3.27	− 4° 36′	327	− 26.3	397.6	
B	176° 15′	ck.					
17	340° 00′	6.34	− 3° 08′	635	− 34.7	389.2	
18	278° 35′	2.51	− 5° 43′	250	− 25.0	398.9	
19	276° 20′	3.07	− 7° 56′	303	− 42.3	381.6	Water's Edge.
20	277° 40′	4.24	− 5° 40′	422	− 41.9	382.0	" "



Sample Notes 59(*b*).

number of feet above or below the point for true vertical angle) and a note is made to that effect. For ordinary stadia work, inclined distances with vertical angles of less than 3° may be considered as horizontal without appreciable error. (5) Rod intervals can be most easily determined by sighting the lower hair on a footmark. (6) When the distance between stations is too great to allow a single reading to be taken the stadia distance may be found by adding readings between the lower and middle hair, and between the middle and upper hair; or by adding the readings taken from a point approximately midway between the two stations. The objection to the last method is that it allows no check. (7) The stadia constant $(f + c)$ should not be omitted from the calculations; with the ordinary transit and in stadia work of the usual precision this may be assumed to be 1 ft. (8) When a station is to be occupied for some time (as when side shots are being taken), it is well to record the azimuth to some prominent object as soon as the transit has been oriented. Any motion of the lower plate can then be detected by again sighting upon the object before leaving the station. (9) The azimuths of topographic details such as roads, streams, and changes of slope need be read only to the nearest 05′, without the use of the vernier; the rod intercept should be read to the nearest 0.01 ft. unless the scale of the prospective map be small; vertical angles should be observed to single minutes. (10) Many shots can be taken by reading the rod with the telescope leveled as in direct leveling. This should be done when possible on account of the labor saved in later computations. (11) The location of points where elevations would be useless can often be advantageously determined by azimuths from two or more traverse stations. (12) The middle hair should not be mistaken for one of the stadia hairs. (13) Because of the lack of checks on side shots, the instrumentman and noteman should form the habit of judging distances and angles with the eye. This precaution together with the method of sketching the approximate position of all shots on the right-hand page of the note-book will often prevent absurd mistakes. (14) Care should be taken to make clear, complete notes; the sketches should be made approximately to scale, and except for contour lines, should show all features that are to appear upon the finished map. The accompanying pages of notes are of a convenient form for traverse and side shots.

(15) Many topographers believe that time can be saved when vertical angles are not large by using the "*interval*" or "*stepping*" method for determining the difference in elevation, instead of reading the vertical angle and computing this difference with the stadia slide-rule. To illustrate this method suppose it is desired to determine the elevation of the ground at *a* (Fig. 32). The transit is sighted on the rod held at *a*, and the rod interval is noted as 4.55 ft. The telescope is then lowered in the same vertical plane until level, and the position of the *horizontal* line of sight on some prominent

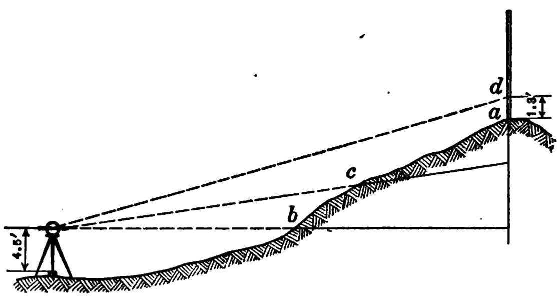


FIG. 32.—THE STEPPING METHOD.

mark of the landscape is noted at *b*. The telescope is raised until the lower hair rests on *b*, and the position of the upper hair on the landscape is noted at *c*. The lower hair is brought up to *c*, and the upper hair rests at *d* which is 1.3 ft. on the rod. The difference in elevation between the known point at the instrument and *a*, assuming that no error is introduced by reading the rod with line of sight inclined, is $4.5 - 1.3 + 2 \times 4.55 = 12.3$ ft. It is evident that for vertical angles of sufficient magnitude to cause an appreciable difference in the rod interval or to necessitate horizontal corrections this method would not be applicable, and it seems doubtful if, even at long distances, it possesses any advantage over the vertical angle method for angles greater than two or three intervals. (16) Where a high degree of precision is desired, as will generally be the case with large scale and small contour interval, the engineers' level is often used in conjunction with the transit. With the level a number of points are found on each contour; and with the transit, the horizontal position of these points is determined by angle and stadia distance. (17) It is sometimes advantageous, particularly if there is a large number of details, to plot the map in the field as the work progresses, and thus not only portray the topography with greater veracity but avoid taking voluminous notes as well. The usual method is to divide the area into a number of parts, plotting the control points of each subdivision on a sketch-sheet large enough for a complete map of that area and of a convenient size for field sketching. With this sheet mounted on a small drawing board, the draftsman or sketcher plots the map as fast as data is obtained. When all the field-sheets have been completed, they are assembled, any irregularities adjusted, and a map of the whole area made by tracing the assembled sheets.

Practical Applications.—The transit-stadia method of topographic surveying is in general use on railroad preliminary, hydraulic, municipal, highway, and many other surveys where the contour interval is small and moderate precision is required. The method of procedure may be somewhat altered from that above, but the principles are the same. For a more accurate horizontal control of surveys covering a large area, triangulation systems are established and traverses are run with transit and tape; for a less accurate vertical control, or on surveys where government or other bench-marks can be tied onto from time to time, direct leveling may be dispensed with and elevations may be carried forward by stadia and vertical angles. The transit-stadia method is advantageous over the plane-table in that less time is required in the field, a higher precision can be obtained, and progress is not hindered by showers or very cold weather; but more time is consumed in office work. It is advantageous over location of contours by direct leveling in that less time is consumed in the fieldwork and a smaller party is required; but the precision is lower, and since the position of contours must be interpolated, the resulting map is more generalized. In heavily wooded country the method is not practicable, but on cleared or partially cleared land it is, for the accuracy obtainable, one of the most economical methods in use.

Problem 60. Survey of Tract for Deed Description and Topographic Map

Object.—To obtain sufficient data for a proper legal description and an accurate topographic map by a combination of tape and stadia measurements. (The data may be used in Problem 90, page 229. For method of computing area within traverse see Problem 73, page 183.)

Procedure.—(1) Run an azimuth traverse referred to the true meridian about the field, measuring the sides with a steel tape, and setting hubs at the corners. Determine the angular error of closure, as in Problem 59. *Allowable angular error of closure in minutes* = $\frac{1}{2} \times \sqrt{\textit{number of sides}}$. (2) Calculate the latitudes and departures, and linear error of closure; balance the survey, and compute the area of the tract by the double meridian distance method. *Allowable linear error of closure* = $\frac{1}{5000}$. (3) If it is impossible to obtain sufficient topography from the corner stations, establish other stadia stations at advantageous points, and tie these stations to the main traverse by azimuths and stadia distances. (4) Conduct the remainder of the survey as in Problem 59.

Hints and Precautions.—The same as for Problem 59.

Problem 61. Preliminary Survey for Railroad by Transit-stadia Method

Object.—To obtain data for plotting a topographic map of a proposed railroad route between two governing grade-points (the elevations of which are known) approximately 2 miles apart. (The data may be used in Problem 90, page 229.)

Procedure.—(1) Run a stadia azimuth traverse between the two points establishing stadia stations at advantageous points near where it appears that the line will eventually be placed. (2) Determine the distance between adjacent stations by reading the stadia distance on both forward and back sights. (3) Observe the vertical angles between instrument stations on both forward and back azimuths. (4) While running the traverse, take side shots 300 to 800 ft. on either side of the traverse line according to methods of Problem 59, p. 153. (5) Determine azimuth at the beginning and end of traverse by astronomical observations (Problem 51 or 56, pages 130 and 146). (6) Note the quality of the land, as to whether it is clay, rock, sand, or gravel. Note the condition of the land as to whether it is cleared land, pasture land, or woods, and if woods the kind of trees and density of growth.

Hints and Precautions.—(1) In determining the differences of elevation, and horizontal distances between traverse points, use the mean of the two vertical angles and mean of the two stadia readings taken along the line joining these points. (2) The width of the tract over which topography must extend will depend to a large extent upon the character of the country. If in a valley or on a ridge and the ground slopes are steep, it will be necessary to map only a narrow strip, but if the country is nearly level or rolling, side shots must be taken over a considerable width. (3) Before taking side shots about the station occupied, set the next stadia station in advance. (4) For other precautions see Problem 59, page 153.

Practical Applications.—Methods made use of in this problem are applicable to preliminary surveys for railroads, canals, and highways where the route is through open or partially cleared country. The accuracy obtainable is well within the allowable error of such surveys. The contour interval is usually 5 ft., but may be greater in rough country.

PROBLEM 62. RAILROAD PRELIMINARY SURVEY. (TOPOGRAPHY WITH HAND-LEVEL)

Object.—To obtain data for a topographic map along the route of a proposed railroad by locating the contours directly by hand-level and tape measurements.

The staff on a survey of this kind is made up of three parties, led by a chief who directs the course of line. The transit party, made up of the transitman, chainmen, flagman, and axemen, establish the horizontal control for the survey. Sample Notes 20(*a*), page 68, are of the form commonly used. The level party, made up of the leveler and rodman, establish vertical control for the survey by the methods of Problem 28, page 85. The topographer's party, made up of topographer, rodman, and tapeman, locate the details for a topographic map of a narrow strip of country on both sides of the transit line. It is with the latter operation that this problem is chiefly concerned, though the work of the transit and level parties is incidentally mentioned.

Procedure.—(1) Run a deflection angle or azimuth traverse along the apparent general route of the proposed railroad. Set stakes every 100 ft. (by tape measurements) along the line. Mark the station number so that it will read down the stake; when the stake is driven the number should face toward the zero end of the line. Establish permanent hubs at all instrument stations, and mark the station and plus of the hub on a witness stake. Also reference the hubs so that they may be replaced in case of loss. (2) By direct leveling determine the elevations of all stations and such other points as are needed to plot an accurate profile of the line. Establish permanent bench-marks every 3000 ft. (3) With hand-level, metallic tape, and topographers' rod, determine the position of 5-ft. contours at each station, and at such plusses as are necessary over a distance of 300 to 800 ft. on either side of the line; and the chainage of points between stations at which 5-ft. contours cross the traverse. (4) Measure distances from the traverse line to other topographic features such as land lines, fences, streams, roads, lakes, and buildings; and determine the chainage of intersections of other lines with the traverse line. (5) Note the quality of the land, as to whether it is clay, rock, sand, or gravel. Note the condition of the land, as to whether it is cleared land, pasture land, or woods, and if woods, the kind of trees and density of growth. (6) Tabulate the numerical data on the left-hand page of the notebook, keeping the notes up the page. On the right-hand page make a neat sketch showing the approximate position of contours, roads, streams, and all other details. Follow the accompanying form.

Hints and Precautions.—(1) To avoid large angular errors in the traverse, the magnetic bearing of each line should be taken, and immediately compared with the calculated bearing; to avoid errors of 100 ft. in chaining, the rear chainman should call out the station number at which he is holding the tape, and the head chainman or stakeman after marking and setting the forward stake should see that these numbers increase consecutively. As a further check the rear chainman as he leaves each station may enter its number in a small notebook. (2) The bench-marks should be placed on roots of trees, stumps, or large rocks if possible and should be marked

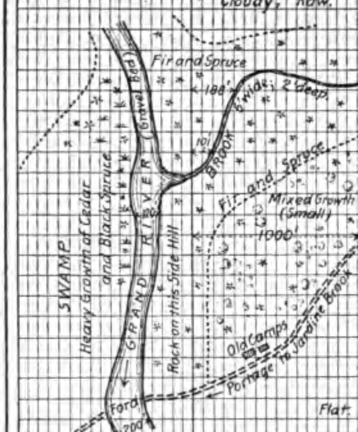
Prob. 62. Topography
I.N.R. Preliminary.

Sta.	Lt.	Ctr. El.	Rt.
651	R/230 40/215 45/91 50/38	455.0	55/67 55/140 60/320
			50/272 55/240 60/275 65/300
+56		450	55/37 65/129 50/149 Bk./188
+06		445	
			65/218 70/249 75/288 80/340
650	River = 36/124 40/111	444.1	45/95 45/108 50/149 55/170 60/193
+92		442.1	
+56		445	
+23		450	
649	R/159 40/149 45/108 50/57	455.0	60/33 65/68 70/104 75/148
+55		460	
648	R/170 40/158 45/105 50/46 55/27 60/10	463.6	65/7 70/25 75/92 80/175 85/296
+90		465	
+33		470	
	River = 36/250 40/224		
647	45/114 50/108 55/92 60/77 65/65 70/17	471.8	75/46 80/137 85/248 90/357
+75		470	
	River = 36/196 40/182		
646	45/130 50/61 55/46 60/32 65/17	466.9	70/28 75/102 80/197 85/290

33.

W. Wyman, Topog.

Locke Level.	P. Martin,	Rod.
50-ft Tape, Rod.	L. Parker,	Tape.
Pocket Compass.	May 10,	1912.
	Cloudy,	Raw.



SAMPLE NOTES 62.

in such a manner as to be easily recognized. The station and plus of all bench-marks, their offset from the traverse line, and the name of objects on which they are placed should appear in the level notes. (3) At each station contours are located with the hand-level as follows: The topographer, having already obtained the elevations of the traverse stations from the level notes, stands over the station from which distances to contours are to be measured, and directs the rodman on a line perpendicular to the traverse until a point, indicated by the rod reading, is found which has an elevation of a multiple of the contour interval. The distance from this point to the station occupied is measured with the tape and recorded by the topographer. He then takes up a new position at the point where the rod was held, and the process is repeated. This is continued until topography has been taken over a strip sufficiently wide for the purposes of the survey. The distances to other important features are taken in the meantime. The directions may be determined with the pocket compass or estimated with the eye. Points a remote distance from the line may be located by pacing. (4) The ordinary leveling rod may be used, but a special topographers' rod makes more rapid progress possible. This has an adjustable base so that the zero point may be fixed at a distance from the foot equal to the height of the observer's eye above the ground. The rod is graduated in half-feet over a length of 10 ft. and the numbers increase from the zero point toward the ends. In reading the rod, tenths of feet are estimated. To illustrate its use, suppose the observer to be at a station the elevation of which is 481.7, and that he wishes to locate the 485-ft. contour, which is 3.3 ft. higher than the station. He will then direct the rodman on the high side of the line until the line of sight of the hand-level as it is held to the eye cuts the rod at 3.7 ft. below the zero point. (5) If the Locke hand-level is used it will be found after a little practice that a much clearer sight can be obtained with both eyes open. (6) In flat country the engineers' level is often used in place of the hand-level because of the great horizontal distance between contours and the fact that the survey usually extends over a much wider belt.

Practical Applications.—This method was formerly much used on railroad preliminary and other surveys of a like nature, but at the present time it is used chiefly in wooded country where stadia work would be impractical. It is advantageous over the transit-stadia method in that the contours are located on the ground, but with an experienced topographer the increased accuracy is slight and is more than offset by the extra cost.

Problem 63. Topographic Survey of Park for Landscape Architect's Study. (Hand-level and Sketch-sheets)

Object.—To make accurate large scale field-sketches of the topography of a given area showing such details as the position, kind, and size of trees, walks, drives, buildings, and fences; the location of streams, hydrants, manholes, etc.; and the representation of the relief by contours located directly with the hand-level, the individual sketches being so constructed that together they will form a complete map of the area surveyed.

Procedure.—(1) With transit and tape establish a base-line along the middle of the area to be mapped, setting stations every 500 ft. (2) Through each station of the base-line run laterals (with transit and tape) at right angles to the base, setting stakes every 100 ft. to the boundary of the area. (3) By direct leveling determine the ground elevations at stakes on base-line and laterals. (4) Plot the base-line and laterals to the scale of the proposed map, showing the ground elevations at all stakes. Cut this sheet into sketch-sheets that may be fastened to a drawing board of convenient size for field use. (5) The topographer locates the position of contours crossing the laterals and crossing lines parallel to the base-line between corresponding stations on the laterals, using the hand-level as explained in the previous problem. (6) The sketcher plots contours on the sketch-sheet as fast as their positions are determined, taking note of the shape of the ground between contour crossings and sketching the contours accordingly. (7) The topographer locates the other details by offsets and distances along line. (8) The sketcher plots these details using conventional signs (Figs. 44–47, pages 218–222) when it is possible, but showing everything that should appear on the finished map. (9) When all sketch-sheets have been completed, assemble the drawing and paste to a backing. Make a tracing of the whole map.

Hints and Precautions.—(1) Observe the hints and precautions of Problem 62 which apply to this problem. (2) The topographer in locating contours should call out the number of the contour and its distance from the station away from which he is moving. The elevation of this station should be given by the sketcher to the topographer on his arrival. (3) The topographer should check his work by comparing distances between stations with corresponding distances of the transit survey, and elevations should be compared in the same manner. The allowable discrepancy between these corresponding measurements will depend on the contour interval, the scale of the map, and the character of the country, but should not be of a magnitude sufficient to appreciably alter the position of contour or other object on the sketch-sheet. Small discrepancies may be assumed to have gradually accumulated and the topography should be adjusted accordingly. A large discrepancy indicates a mistake, and the work should be retraced until the error is discovered. (4) Usually it will be sufficiently accurate to locate many of the minor details by pacing or by estimating distances with the eye.

Practical Applications.—The preceding method is particularly adapted to large-scale topographic surveys where the details are many and varied in character and the country is rolling. It is simple, chances of error are reduced to a minimum by checks, and the entire ground must be covered, hence it may be used to good advantage by surveyors having little or no experience in topographic work; for field sketching the absence of computations make it preferable to the transit-stadia method; the map is made in the field, therefore field notes, which must necessarily be very copious for a survey of this kind, are unnecessary; and since the map is made in the field, much can be sketched with the eye that would otherwise have to be located by field measurements. Essentially the same methods have been used in numerous small surveys* throughout the country. It seems probable, however, that in the hands of an experienced topographer the plane-table would become a more efficient instrument for constructing the detail of the map, being sufficiently accurate providing horizontal and vertical control points are established with transit and tape and level, and requiring a much smaller number of these control points than the method of this problem.

* For description of such a survey see "Topographic Surveying Methods Used on the Vanderbilt Estate, Biltmore, N. C.," *Jour. Assoc. Eng. Soc.*, April, 1897.

PROBLEM 64. SURVEY FOR CONTOUR MAP BY "SQUARE" METHOD (TRANSIT AND LEVEL)

Object.—To obtain sufficient data for an accurate topographic map of large scale and small contour interval. The area to be mapped is small, very regular, and possesses few details. (The data may be used in Problems 89 and 90, pages 227 and 229.)

Procedure.—(1) Divide the tract to be mapped into 100-ft. squares, setting stakes at the corners. (2) Locate the position of buildings, roads, fences, etc., by offsets, ties, etc. (Problem 23, page 74), and show these measurements on the sketch. (3) Run levels over the area taking rod-readings at the stakes, high and low points, and changes in slope. Keep the notes as shown in Sample Notes 64, page 168.

Practical Applications.—The above method may be profitably used in flat country where the contour interval is 1 or 2 ft. and the scale of the map is from 20 ft. to 100 ft. to the inch. The resulting map may be the basis of a study of problems in grading or land drainage. It will be noticed that, unlike the preceding problem, contours are not located directly.

PROBLEM 65. TOPOGRAPHIC SURVEY WITH PLANE-TABLE (LARGE SCALE)

Object.—To make a large-scale topographic map of an assigned area using the plane-table. The map is to be accurate and all other details as well as contours are to be shown, as in Problem 63, p. 165.

Procedure.—(1) Make a reconnaissance for the purpose of determining advantageous positions for plane-table stations. (2) Run a closed azimuth traverse between these points with transit and stadia as in Problem 59, p. 153. (*Allowable error of closure,* $\frac{1}{500}$.) Run differential levels over the course establishing bench-marks at or near traverse points. (*Allowable error of closure,* 0.1 *ft.* $\times \sqrt{\textit{distance in miles}}$.) (3) Plot the traverse on the plane-table sheet; show the elevations of bench-marks. (4) At any station orient the plane-table by sighting at an adjacent station as described in Problem 40, Procedure (3), page 108. (5) Determine the height of instrument above the station occupied or other bench-mark. (6) Observe the vertical angle and stadia reading to a change

34.

Prob. 64. Survey of Proposed Rice Farm, C.K. Perry.

For Contour Map.

Scale 1" = 50', Contour Int. = 1 ft.

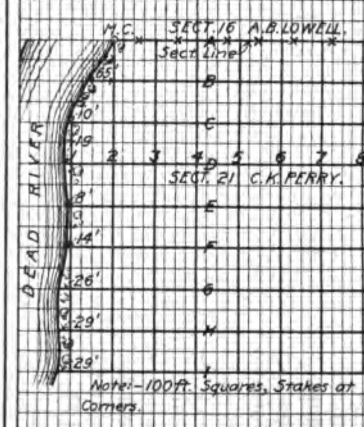
Sta.	B.S.	H.I.	F.S.	El.	
B.M.₁	5.31	105.31		100.00	(Assumed)
A-8			7.3	98.0	
B-8			6.0	99.3	
C-8			4.1	101.2	
D-8			2.2	103.1	
E-8			1.4	103.9	
E-7			2.9	102.4	
D-7			4.2	101.1	
C-7			5.0	100.3	
(B+50)-7			5.6	99.7	
B-7			4.8	100.5	
A-7			3.2	102.1	
A-(7+60)			6.2	99.1	
A-6			2.5	102.8	
(A+50) 6			0.4	104.9	
B-6			1.9	103.4	
C-6			3.6	101.7	
D-6			4.2	101.1	
E-6			3.5	101.8	
E-(5+40)			6.7	98.6	
E-5			6.1	99.2	

Buff Transit No. 290. B.C. Pride, ⊼

C.L. Cotton, Rod.

Aug. 9, 1911. Fair, Hot.

Spike in Fence Post at A-(8+90).



SAMPLE NOTES 64.

of slope, land line, fence, road, building, or other necessary point; draw a line along the straight-edge of the alidade that will include the point to be plotted. (7) Deduce the elevation (if on the ground) and horizontal distance, using the stadia slide-rule or reduction diagram. Plot the point and mark its elevation in an unmistakable manner. (8) Locate other points in like manner. Sketch in all details except contours as soon as they are located using conventional signs (Figs. 44–47, pages 218–222). Sketch in contours by interpolating between the elevations of plotted points (Problem 89, page 227), when a sufficient number of ground points have been located to show the trend of the contours. (9) When all the topography that can conveniently be taken from one station has been plotted, proceed to the next point in the traverse and continue the work in the same manner. If it is found that transit traverse stations have been so located that portions of the area can not be mapped, run short secondary traverses with the plane-table (Problem 41, page 110), and carry elevations forward from transit traverse stations by vertical angles and stadia. From the secondary stations locate the omitted topography in the usual manner. (11) When the map is completed make a neat title showing name and location of area mapped, purpose of map, scale, contour interval, date, and members of surveying party.

Hints and Precautions.—(1) If the scale of the map is very large, the point on the paper representing the station occupied should be approximately over the corresponding point on the ground. The table should be roughly oriented before any attempt is made to fulfill the above condition. Many plane-tables have a plumbing arm for suspending a plumb-bob beneath the point on the plane-table sheet. If no such device is at hand, the relative positions of the two points may be determined with sufficient precision by dropping a pebble from beneath the estimated position of the point on the paper. (2) Care should be taken to keep the plane-table as nearly level as possible, and it should be low enough to allow sketching without resting elbows or body on the board. (3) It is especially desirable that the board should be level where the vernier is fixed, as is the case with some alidades, for when not level the index error will vary for every direction of the telescope; but however much care is taken, it will be found, unless the table is unusually rigid, that as the alidade is moved about on the sheet, the board will spring somewhat. For this reason, no vertical angle should be taken without ascertaining the index error if the vernier is fixed or without making sure that such error has been eliminated if the vernier is movable. (4) If the vernier has an attached bubble, it should be centered before reading each angle; if the vernier is movable, the zeros should be brought together with the telescope level before taking each sight; if the vernier is not movable, the index error should be determined and the final vernier reading should be corrected accordingly. (5) Use direct leveling when possible; it simplifies computations and is more accurate. (6) If it is possible, measure vertical angles by sighting at a point as far from the foot of the rod as the instrument is above the point from which the height of the instrument is determined. (7) Before leaving a plane-table station check the orientation by sighting to an adjacent station or other mark, the position of which has been correctly plotted. Any movement between initial and final sights will be indicated by the distant point, its plotted position, and the plotted position of the station occupied not falling on a straight line. (8) Do not take long sights; the topographer can not accurately judge topographic details far from the station occupied. (9) If the scale is larger than 50 ft. = 1 in., contours should be exactly located by direct leveling unless the ground is very even, and distances to important details should be measured with the tape. When the contour interval is as small as 1 or 2 ft. and the ground is uneven the engineers' level may well be used in conjunction with the plane-table, one rodman serving both instruments at the same time. The rodman is directed to a particular contour by the leveler; the plane-table operator observes distance and direction and plots the point with its elevation. This method makes it possible to cover more ground from a single plane-table station, particularly if slopes are steep. (10) If the scale is as small as 100 ft. = 1 in., the contour interval is as great as 5 ft., and the area to be mapped is small, horizontal control may be carried forward, as the work of taking topography progresses, by plane-table traverse using stadia (Problem 41, p. 110); and vertical control, by trigonometric leveling with the plane-table. (11) Note the hints and precautions of Problems 39, 40, and 41 which apply to this problem.

Practical Applications.—The plane-table has not been used to any great extent on large-scale work. This may be due to the scarcity of surveyors who understand its use and possibilities, or to the quite prevalent idea that it is fit only for rough surveys and large contour intervals. It would seem that it might properly be used with as good results as could be obtained by any method and at less cost, regardless of the scale, providing proper vertical and horizontal control were established.

Problem 66. Topographic Survey with Plane-table (Medium Scale)

Object.—To make a medium-scale topographic map of an assigned area using the plane-table. It is assumed that horizontal control has been established by a system of triangulation, or by transit traverses, or by both; that vertical control has been established by direct leveling; and that signals, which may be seen for long distances, have been erected at these control stations.

Procedure.—(1) Plot the position of triangulation or traverse points and bench-marks on the plane-table sheet. (2) Choose a favorable site for plane-table station, and locate the position of the station and orient the board by one of the methods of resection (Problems 43–47, pages 112–122). Determine the elevation of the station by trigonometric leveling from a bench-mark. (3) Take stadia readings and vertical angles to governing points, as along ridge and valley lines and at sharp changes in slope. (4) Reduce and plot the observations as in the previous problem. Note the general form of the slopes and sketch in the contours accordingly, interpolating between points of known elevation (Problem 89, page 227). (5) Take stadia readings or draw intersection lines (Problem 40, p. 108) to other important objects, such as buildings, road intersections, land lines, etc., but sketch less important details without measurement. (6) When all topography has been sketched about the station, move the plane-table to another favorable point and proceed as before. (7) If it is not practicable at any desirable plane-table station to locate the station by resection upon control points, run a traverse (Problem 41, p. 110) to include the proposed station, between control points or to stations that may be located by resection, carrying elevations forward by trigonometric leveling. Check the traverse and distribute the error, assuming that the control points or stations located by resection are correct. (8) When the map is completed, make a neat title showing name of area mapped, purpose of survey, scale, date, and members of party.

Hints and Precautions.—(1) Often certain details, such as buildings or woods, would if placed on the plane-table sheet so crowd certain areas as to seriously interfere with contour sketching. Such details may well be plotted on a piece of tracing cloth temporarily fastened over the plane-table sheet. The tracing cloth may then be removed and not used again until these particular kinds of details are encountered. (2) When vertical angles are small many topographers prefer to determine elevations by the "stepping" or "interval" method as explained in Problem 59, Hints and Precautions (15), page 158. (3) It goes without saying that many of the horizontal control points will be favorable plane-table stations and may be used as such, orientation being obtained by sighting to any signal which can be identified. (4) Often the number of primary control points will not furnish sufficient control for the area to be mapped. It is then customary to locate secondary control stations by graphical triangulation with the plane-table, using the method of intersection or methods of resection. At the same time elevations are determined by vertical angles and scaled distances. Signals are, of course, erected at these secondary points. Thus it is seen that any point located by intersection or resection may become secondary control, providing its strength of determination will so warrant. (5) The sketching of contours, from what can be seen by the topographer and a few elevations of governing points, so that the map will be a truthful representation of the country is the most difficult part of the work, and the only method of learning this art seems to be through experience. The topographer should form the habit of judging distances and slopes and he should check these estimations against more precise measurements whenever possible. It may be taken as a rule that all ground should be viewed from at least two points, and preferably more, before the contour or other object is finally located by estimation. The beginner will find the hand-level very useful for tracing out the trend of contours between determined points, and the clinometer very useful for judging slopes. The rodman should be consulted when there is doubt; he may prepare rough sketches and determine slopes of the ground about points remote from the plane-table. (6) When expansion of the system of horizontal control is achieved through graphical triangulation, secondary vertical control is usually obtained by determining differences of elevation from vertical angles and scaled distances, through the relation that

$$h = D \tan \alpha$$

in which h is the difference in elevation between center of instrument and point sighted, D is the corresponding horizontal distance scaled from the paper, and α is the vertical angle. Since vertical angles will in general be small, it will be seen that a considerable error in the scaled length D will introduce only an insignificant error in the computed value of h. The topographer is furnished with a list of elevations of easily distinguishable points on signals (for example, the elevations of tops of masts) as well as the elevations of benchmarks on the ground. When the plane-table is set up at a station which has been located on the plane-table sheet but the elevation of which is unknown, the topographer measures the vertical angles to two or more of the points on signals contained in his list, scales the corresponding distances, and computes the differences in elevation by the preceding equation. These, when added to or subtracted from the elevations of the corresponding points, will furnish as many independent determinations of the elevation of the instrument as there are vertical angles taken. The work of observing and computing is checked if the discrepancies are reasonably small, and the mean of the elevations thus determined is the accepted elevation of the instrument. (7) If the sights are long, due allowance should be made for the effect of curvature of the earth and refraction of the atmosphere. The correction for curvature and refraction is approximately expressed by the equation

$$h' = 0.57K^2$$

in which K is the distance in miles and h' is the correction in feet to be added to or subtracted from the difference in elevation according as the vertical angle is positive or negative. This correction will be unnecessary for a horizontal distance of less than one mile.

Practical Applications.—The methods of this problem are practically the same as those used by the U. S. Coast and Geodetic Survey on a scale of 1 in 10,000 to 1 in 20,000 and a contour interval of 10 ft. to 50 ft. They are particularly adapted to rough or rolling country such as is often found along the seacoast, and must necessarily be varied to suit topographic conditions. Such methods have been successfully used for a much larger scale and smaller contour interval for constructing topographic maps of cities, drainage areas, and water storage basins.

On small-scale surveys, such as those conducted by the U. S. Geological Survey in which scales of $1/48,000$ or smaller are used, primary control points are relatively few. In rough country, points of secondary control are located by graphic triangulation; in level or wooded country, by plane-table traverses along the roads closing on primary control points, elevations being determined either by trigonometric leveling or very rapid spirit leveling. Distances are often obtained by pacing or by the revolutions of a buggy wheel. Most of the sketching is done from roads already traversed, the positions of only a very few critical points being determined by the topographer. Frequently the topographer has no rodman, and therefore has to depend more upon triangulation for the location of critical details away from the line. In rough country where a large contour interval is used, the elevations of points along the topographical route are determined with the aneroid barometer, it being set at proper elevation at one point of secondary control or bench-mark and checked at the next. On small-scale work, since the number of known points for a given area is very much smaller, the skill of the topographer must be relatively great if a correct interpretation of the relief is to be obtained.

Problem 67. Base-line Preparation

Object.—To prepare a short base-line for precise measurements with the steel or invar tape. It is assumed that the base-line site has already been chosen and that permanent marks have been established at the end points.

Procedure.—(1) Set 3″ × 5″ stakes along the base-line at intervals of the length of tape with which measurements are to be taken. These may be called "*station*" stakes. Use the transit for lining, making sure that the line cuts the middle of the top of each stake, and measure distances with sufficient precision to insure the ends of the base-line tape always falling on stakes when the base is measured continuously from one end, a full tape length at a time. (2) If the soil is yielding, brace each station stake with a smaller stake driven into the ground at an angle of 45° with the horizontal, with top nailed to the larger stake. The braces should be so placed as to best resist movement of the top of the station stake in the direction of the line. (3) Over each end of the base-line build a table by driving four 2″ × 2″ stakes well into the ground (forming a rectangle) and nailing a piece of board to their tops. Brace the table by nailing small diagonal strips to adjacent legs. (4) Tack ½″ × 5″ strips of zinc or copper on the top of each of the tables, and with the transit, *carefully* project each end point of the base-line to the metal strip above. Tack similar strips to the intermediate stakes. (5) At regular intervals between base-line stations set small stakes for supporting the tape on line, making sure that they project from the ground a sufficient distance to be cut by a uniform grade line joining the tops of adjacent station stakes. (6) "Shoot in" grade lines (Problem 31, page 91) setting up the transit at every other station along the base. Drive 10d. nails into the small intermediate stakes at grade, bending them so that they will readily support the tape. (7) Run levels over the line, determining the elevations of tops of station stakes and end tables.

Hints and Precautions.—(1) Because of the great variation of temperature in layers of air very close to the earth's surface, the height of the station stakes should be such that at no place will the grade line come closer to the ground than 1 ft. providing measurements are to be taken in the sunlight; for the sake of convenience in measuring the base-line the station stakes should project from the ground no more than 3 ft. (2) The nails in the intermediate stakes should be driven to form a guide over which the tape may be dragged without slipping off, but should not be sharply inclined else the tape may become pinched between nail and stake. If the intermediate stakes are driven with tops to grade, two nails ½ in. apart, driven upright, will serve as a guide for the tape. (3) The end points of the base-line may be projected to the metal strips on end tables as follows: Set up the transit a short distance, say 25 ft., from the table, and sight to the end point on the ground. Elevate the telescope, and mark two points a few inches apart on the metal strip above. Scratch a straight line between these two points. Now move the transit to another position such that when a sight is taken to the end point the line of sight will form nearly a right angle with the line scratched on the metal strip. Sight at the end point, elevate the telescope, and mark the point of intersection of the line of sight and the scratch. This point will be over the point on the ground. (4) If the base-line is along a railroad tangent, the preparation may consist of placing zinc strips on boards nailed to the ties at a uniform distance from the rail, and of running levels over the line. In this case the tape will be supported by the ground for its full length.

Problem 68. Base-line Measurement

Object.—To measure a given base-line with the steel or invar tape, by such methods that the probable error of the corrected length shall not exceed $1/250{,}000$. It is assumed that the base-line has already been laid out as explained in the previous problem. (The data of this problem may be used in Problem 91, page 230.) See Sample Notes 68, page 178.

Parties for base-line measurement are usually made up of four (sometimes six) persons; two tapemen who hold the tape and exert the required pull with spring balances attached to each end; a rear observer who reads a thermometer, records all observations, and signals when the rear of the tape is at the mark; and a forward observer who scratches a zinc plate opposite the forward end of the tape at the above signal from the rear observer, and reads a second thermometer.

Procedure.—(1) Stretch the tape from the end of the base-line from which preliminary measurements were begun, making sure that it is supported at all intermediate stakes. Suspend two thermometers, one near each end of the tape, so that the bulbs will be at a height approximately that of the grade line above the ground. (2) Exert the required pull through a spring balance at each end, and when there is exact coincidence between the initial point and the rear end of the tape, the rear observer signals to the forward observer. (3) The forward observer carefully scratches the position of the forward end of the tape on the plate at Station 1, using a needle or other sharp-pointed instrument. (4) Release the tension until the tape shows a considerable sag between supports for its full length. Again apply the required pull and make a second scratch on the plate at Station 1. (5) Read the thermometers, and record the temperature. (6) Proceed to the next station, and repeat the operation, holding the rear end of the tape at each of the scratches made at Station 1. (7) At the end of the course, since there will generally be a fractional part of a tape length, mark on the plate at the division that falls nearest the end point, and measure the remaining distance with a finely divided scale. (8) In the preceding manner measure the base-line at least four times.

Hints and Precautions.—(1) If the balance is heavy, proper allowance should be made for having it horizontal instead of in its usual vertical position, unless it is already adjusted for such condition. Assuming that the balance reads zero when freely suspended with the hook down, the reading with balance horizontal corresponding to a required pull may be determined by the equation

$$T = T' - \tfrac{1}{2}(W - E)$$

in which T' is the required pull, W the correct weight of the balance, and E the reading when the balance is freely suspended from the hook. (2) When the required tension has been applied the tape should be set in vibration so that the amount of tension will become uniform throughout its length. The scratch should not be made until all vibration has ceased. (3) The apparent shortening of the tape due to sag varies directly as the square of the distance between supports and inversely as the square of the pull, and friction between supports and tape will have a relatively greater effect upon the sag when the tape is under low tension. Practice seems to indicate that a pull of 12 lb. may be used with good results with supports 25 ft. apart or less, but for intervals in excess of this distance the pull should be correspondingly increased. Thus with the tape supported every 50 ft. a tension of 20 lb. may well be applied. (4) If an invar tape is used, good results may be expected regardless of weather conditions providing the air is calm. If a steel tape is used, good results can not be attained when the tape is exposed to the glaring rays of the sun; for this reason measurements should be taken on foggy or cloudy days or after sundown. (5) The tapeman should take particular care

Prob. 68. Base-Line Measurment.

Sta.	Thermometers		Dist. in Feet.		Mean
	Δ 40 to Δ 41				
	Trials 1 & 2.				
	Rear	Forward	No. 1.	No. 2.	Temp.
Δ 40-1	79.0°F	80.4°F	300.000	300.000	
1-2	80.1	79.2	"	"	
2-3	78.0	77.2	"	"	77.9°F
3-4	78.1	76.2	"	"	
4-Δ41	76.0	75.0	225.2863	225.2808	
			1425.2863	1425.2808	
	Trials 3 & 4				
Δ 40-1	70.4	72.0	300.000	300.000	
1-2	71.0	70.5	"	"	
2-3	70.0	70.7	"	"	70.6°F
3-4	70.0	70.7	"	"	
4-Δ41	70.0	70.8	225.3203	225.3243	
			1425.3203	1425.3243	
	Trials 5 & 6				
Δ 40-1	71.4	71.6	300.000	300.000	
1-2	72.5	73.3	"	"	
2-3	72.5	72.5	"	"	72.2°F
3-4	72.8	72.9	"	"	
4-Δ41	71.5	71.5	225.3000	225.3008	
			1425.3000	1425.3008	

35.

July 28, 1913, 7 P.M.

300 ft. Std. Steel Tape Fair, Calm.
Balances, Tape- H. Raynes.
Stretcher. E. Raymond.
W. LeRiche.
P. Gregory.

July 29, 1913, 4 A.M.
Fair, Calm.

Note:
Supports 25 ft. Apart
12 lb. pull.

SAMPLE NOTES 68.

to hold the spring balance horizontal when applying tension. If it is not counterbalanced he should support it with his hand so that the slide to which the hook is attached will move freely and without friction. It should be held at a height that will barely allow the tape to rest on the adjacent support. (8) There are numerous tape-stretching devices for use with the spring balance, all of them depending upon the principle of the lever. If none of these is to be had, an oak stick 4 ft. long, pointed at one end, may be used in a similar capacity. The balance may be tied to the stick so that it can be readily adjusted to any desired height, and leverage may be obtained by sticking the point in the ground. (9) Care should be taken not to lean against the station stakes or disturb them in any way. (10) An invar tape is very easily damaged. It should, therefore, be handled with great care, and should never be wound except on its special reel. (11) The reductions are made according to the methods of Problem 91, page 230.

Practical Applications.—In any triangulation system it is necessary that the length of at least one side of one triangle in the system be measured before the other sides can be computed. Because of their length and position it is usually impracticable to measure any of the sides of the main triangles of the system. A relatively short line in a favorable location for measurement and for connection with the main triangulation scheme is therefore chosen; and it is measured with much greater precision than is expected to be attained in the triangulation, because the number of operations necessary to expand it through the base-net into one of the long sides causes much of its precision to be dissipated. The above methods have been used on many large surveys and it has been found that the tape when used with proper precautions will furnish the desired accuracy at a cost less than that of any other method.

On *precise traverses* in which the allowable error is less than $1/_{5,000}$, as in city surveying, it is necessary to take temperature and pull into consideration. On such surveys no supporting stakes are set, often tape lengths are marked on the pavement, and much of the chaining is done on the slope. The tapes used on such work are usually short in comparison with those used for base-line measurement.

CHAPTER VIII

OFFICE PROBLEMS

Problem 69. Area of Field Surveyed with Tape

Object.—To determine the area of field surveyed with the tape by subdividing it into triangles. The data of Problem 4, page 31, may be used. The area is to be calculated by each of the three methods outlined therein.

Procedure.—(1) Decide upon a convenient and systematic form for each method, using logarithms where possible, and transcribe the necessary data from field to office book. Check the transcription. (2) Calculate the area of the triangles and the total area in square feet and acres, using four-place logarithms and following each method through before beginning another. Check the results with a slide-rule. (3) Note the time devoted to each method and record this time at one side of the corresponding computation. (4) When the problem is computed compare the three methods and criticise the computations with regard to neatness, clearness, and completeness. (5) Underline final results with red ink.

Problem 70. Area Between Meander Line and Curved Boundary—Offsets at Irregular Intervals

Object.—To calculate the area between meander line and irregular or curved boundary from notes of Problem 5, page 34, or similar data, rectangular offsets having been taken at irregular intervals.

Rule.—Multiply the distance along the course, of each intermediate offset from the first, by the difference between the two adjacent offsets, always subtracting the following from the preceding. Also multiply the distance of the last offset from the first by the sum of the last two offsets. The sum of these products divided by two is the required area.

Procedure.—(1) Choose a form similar to that of the accompanying sample computations. Transcribe the data and check the copy. (2) Calculate the area according to the above rule. Reduce to acres. (3) Check results with the slide-rule. (4) If the data of Problem 5 are used, calculate the areas of triangles as in the preceding problem and determine the total area of the field.

Hints and Precautions.—(1) Show all computations and make sure that they are clear and complete. (2) Do not neglect to prefix to each quantity its proper sign; if the difference

18.

Area Between Meander Line and Bog Brook
on
Brigham Farm.

Field Notes
Drawer 11,
Book T6,
Pages 37-39.

Computations
Aug. 16, 1911.
Comp't'd by J. E. S.
Checked by R. Q. G.

Line E–F.				
Dist. from E ft.	Length of Offset, ft.	Difference	Products. –	Products. +
0 = E	0.0			
20	14.3	– 23.1	462.0	
65	23.1	+ 4.7		305.5
87	9.6	+ 18.5		1609.5
100	4.6	– 8.1	810.0	
131	17.7	– 13.9	1820.9	
148	18.5	– 19.4	2871.2	
160	37.1	– 6.5	1040.0	
200	25.0	+ 12.9		2580.0
225	24.2	+ 2.0		450.0
250	23.0	+ 5.8		1450.0
300	18.4	+ 5.0		1500.0
350	18.0	– 20.4	7140.0	
385	38.8	+ 9.3		3580.5
400	8.7	+ 38.8		15520.0
413.7 = F	0.0	+ 8.7		3599.2
			14,144.1	30,594.7
				14,144.1
			2	16,450.6
				8,225 sq. ft.
			or	0.1888 Ac.

Note: For Area Computations Within Traverse See Page 17 of This Book.

SAMPLE COMPUTATIONS 70.

between adjacent offsets is negative, the resulting double area will be negative. Thus the total double area may come out a negative quantity.

Practical Applications.—This method of determining areas may frequently be used in land surveying where the boundary is a stream, lake, or road. A meander line is run as near the irregular boundary as is practicable, and rectangular offsets are taken from this line at points where the boundary changes direction. The small areas between offset lines are considered as trapezoids, which, although not quite true, gives results well within the allowable error.

Problem 71. Area Between Curved Boundary and Line—Offsets at Regular Intervals (Trapezoidal Rule)

Object.—To compute the area between a curved boundary and traverse line when rectangular offsets from the line to the boundary have been taken at regular intervals, assuming that the areas between offset lines are trapezoids. (The data of Problem 5, page 34, may be used.)

Formula.—*Total Area*

$$= \frac{d}{2}[(h_1 + h_n) + 2(h_2 + h_3 + \ldots h_{(n-1)})]$$

in which d is the common distance between offsets, and h_1, h_2, $\ldots h_n$ are the offsets.

Procedure.—(1) Record the data and calculate the area by the above formula. If different intervals are used along different parts of the boundary, treat each of such parts as a problem by itself. (2) Check all computations. (3) If the data of Problem 5 are used, calculate the areas of the triangles and the total area of the field.

Practical Applications.—The practical applications are the same as those of the preceding problem. It should be noted that with a boundary convex away from the traverse line this method will give results too small; convex toward the line, results too large.

Problem 72. Area by Simpson's One-third Rule

Object.—To compute the area between irregular or curved boundary and traverse line when rectangular offsets have been taken at regular intervals, by using Simpson's one-third rule. (The data of Problem 5, page 34, may be used.)

Formula.—*Area* $= \frac{d}{3}[h_1 + h_n + 2(h_3 + h_5 + \ldots h_{(n-2)}) + 4(h_2 + h_4 + \ldots h_{(n-1)})]$

in which d is the common interval, h_1, h_2, etc., are the offsets, and n is an odd number.

Procedure.—From the data of the previous problem compute the area by the above formula. (2) Check all computations. (3) Compare the result with that of the previous problem.

Hints and Precautions.—(1) SIMPSON'S one-third rule can only be applied to an odd number of offsets taken at regular intervals. The base-line will therefore be divided into an even number of parts. Should the date under consideration be for an odd number of parts, the area of the first or last part may be computed as a trapezoid and the remaining area by the above rule.

Practical Applications.—This method is based on the assumption that the boundary is a parabola. It is held by some that it will give results more nearly correct than those obtained by the trapezoidal rule, but unless the boundary is of very definite character, and therefore located by precise measurements, it seems doubtful if there is any added advantage in its use.

PROBLEM 73. AREA BY DOUBLE MERIDIAN DISTANCE METHOD

Object.—To calculate the area within a closed compass or transit traverse by the double meridian distance method. (The data of Problem 9, 20, or 21, pages 43, 65, and 69 may be used.)

Rules.—Latitude = Distance × cos bearing angle.
Departure = Distance × sin bearing angle.

$\left.\begin{matrix}\text{Latitudes}\\ \text{Departures}\end{matrix}\right\}$ are positive or negative according as they are $\left\{\begin{matrix}\text{north}\\ \text{east}\end{matrix}\right\}$ or $\left\{\begin{matrix}\text{south}\\ \text{west}\end{matrix}\right\}$.

In any closed traverse the algebraic sum of the $\left\{\begin{matrix}\text{latitudes}\\ \text{departures}\end{matrix}\right\}$ must equal zero.

Compass Rule for Balancing.—The correction to be applied the $\left\{\begin{matrix}\text{latitude}\\ \text{departure}\end{matrix}\right\}$ of any course is to the total error in $\left\{\begin{matrix}\text{latitude}\\ \text{departure}\end{matrix}\right\}$ as the length of the course is to the perimeter of the field.

Transit Rule for Balancing.—The correction to be applied to the $\left\{\begin{matrix}\text{latitude}\\ \text{departure}\end{matrix}\right\}$ of any course is to the total error in $\left\{\begin{matrix}\text{latitude}\\ \text{departure}\end{matrix}\right\}$ as the $\left\{\begin{matrix}\text{latitude}\\ \text{departure}\end{matrix}\right\}$ of that course is to the arithmetical sum of all the $\left\{\begin{matrix}\text{latitudes}\\ \text{departures}\end{matrix}\right\}$.

Rules for Double Meridian Distances.—(1) The *D.M.D.* of the first course equals the departure of that course. (2) The *D.M.D.* of any other course equals the *D.M.D.* of the preceding course plus the departure of the preceding course plus the departure of the course itself. (3) The *D.M.D.* of the last course is numerically equal to the departure of that course, but with opposite sign.

The double area of any trapezoid equals the product of its *D.M.D.* and its corrected latitude.

Procedure.—(1) Transcribe necessary data from the field-book into a form similar to that shown in Sample Computations 73, page 186. Check the copy. (2) Calculate the latitude and departure of each course, using logarithms as shown in sample computations. Check results with the slide-rule. (3) Determine the total error in latitude and in departure, and compute the error of closure. (4) Determine the latitude and departure corrections by one of the preceding rules for balancing, using the slide-rule. (5) Apply these corrections, and check by taking the algebraic sum of the corrected latitudes and the algebraic sum of the corrected departures. Each of these sums should equal zero. (6) From the corrected departures compute the *D.M.D.'s*, using the preceding rules and starting from the most westerly point in the survey. If the last *D.M.D.* is not numerically equal to the last corrected departure, it will indicate that a mistake in addition has been made. (7) Compute double areas by the preceding rule, using logarithms and paying special attention to signs. Check computations. (8) Sum up the double areas, divide by two, and transform into acres. (9) Note the time taken for computation, and compare this method with that of Problem 69, page 180.

Hints and Precautions.—(1) Use logarithmic tables with number of places consistent with the precision of the field measurements (Art. 20, page 15). If the bearings have been determined with the surveyors' compass, four-place logarithms will be sufficient; if angles have been taken to the nearest minute (in error less than 30″) with the transit, five-place tables should be used. (2) Checks should be applied after each of the steps in the computations. An absolute check on the work can, of course, only be had by re-computation, using methods that will give as many significant figures in the final result as did the original computations. However, the slide-rule will furnish an approximate check, which is very desirable. (3) If, after having calculated the latitudes and departures and after having checked them against large errors, the error of closure is found to be larger than that allowable, the computer may frequently locate the mistake, whether it be in computations or fieldwork, through the relation of total error in latitudes and total error in departures. Thus, if the mistake is in the length of one line and there are no other large errors, the ratio of the total error in departures to the total error in latitudes will approximately express the tangent of the bearing angle of that line, or if a mistake has been made in the latitude of a line, the departures may nearly close. The computer should, therefore, conduct a critical examination of results, and should then re-compute those values which seem most likely to contain the mistake. If the mistake is not brought to light when all latitudes and departures have been re-checked then, and only then, may he be warranted in concluding that the mistake occurred in the field. (4) The Compass Rule or Transit Rule will be used for balancing latitudes and departures according as the error is assumed to be as much in angles as in distances or as the error is assumed to be mostly due to erroneous lengths. (5) When the error of closure is small, the latitudes and departures may usually be balanced by inspection, without computing the corrections by either of the preceding rules. When the computer knows the conditions surrounding the fieldwork, he may often distribute the error according to his own judgment rather than by any fixed rule. (6) Often neither calculated nor magnetic bearings of lines are shown in the transit notes. If deflection or interior angles were taken, it will be convenient to assume one of the lines in the traverse as the meridian and calculate the bearings of other lines accordingly. If magnetic bearings are recorded in the field notes, they should not be confused with calculated bearings and used as the basis of computations, for their precision will not warrant such use. (7) Corrections for erroneous

Prob. 73. Area of Balsam Park, Island Pond, Vermont
By D.M.D. Method.

47.

Field Notes
Book No. 3
Page 47.

Computations
Aug. 17, 1913.
Computed and
Checked by J. Q. M.

Line	Cal. Bear.	Dist. 66 ft. Ch.	Latitudes N	Latitudes S	Departures E	Departures W	Corrected Lats.	Corrected Deps.	D.M.Ds.	Double Areas +	Double Areas −
A−B	S 80°29½' W	34.464		5.694		33.991	− 5.693	−33.990	61.812		351.89
B−C	S 33 04 W	25.493		21.364		13.911	−21.361	−13.911	13.911		297.15
C−D	S 33 46¾ E	33.934		28.205	18.867		−28.201	+18.867	18.867		532.06
D−E	N 87 58¼ E	28.625	1.013		28.607		+ 1.013	+28.608	66.342	67.21	
E−A	N 0 27 E	54.235	54.234		0.426		+54.242	+ 0.426	95.376	5173.51	
		176.751	55.247	55.263	47.900	47.902	ΣL=0	ΣD=0		5240.72	1181.10
				55.247		47.900				1181.10	
				.016		.002				2 | 4059.62	
										2029.81 Sq. Ch.	
										or 202.981 Ac.	

$E. \text{ of } C. = \frac{.016}{176.751} = \frac{1}{11,000}$ $E = \sqrt{.016^2 + .002^2} = 0.016$ Chains.

Line	A−B	B−C	C−D	D−E	E−A
Lat.	5.694	21.364	28.205	1.013	54.234
Log. Lat.	0.75542	1.32968	1.45032	0.00573	1.73427
Log. Cos.	9.21805	9.92326	9.91969	8.54899	9.99999
Log. Dist.	1.53737	1.40642	1.53063	1.45674	1.73428
Log. Sin.	9.99399	9.73689	9.74509	9.99973	7.89535
Log. Dep.	1.53136	1.14331	2.27572	1.45647	9.62964
Dep.	33.991	13.911	18.867	28.607	0.426
Log. Cor. Lat.	0.75534	1.32962	1.45026	0.00561	1.73434
Log. D.M.D.	1.79107	1.14336	1.27570	1.82179	1.97944
Log. D.A.	2.54641	2.47298	2.72596	1.82740	3.71378
D'ble Area	351.89	297.15	532.06	67.21	5173.51

Note:
Survey Balanced
by Transit Rule.

SAMPLE COMPUTATIONS 73.

length of chain or tape should not be overlooked. Constant errors of this sort will have no effect on the error of closure. (8) By starting with the most westerly point in the survey all the *D.M.D.'s* become positive; it is not necessary for the solution of the problem that this point be chosen, but it is customary.

Practical Applications.—The double meridian distance method of calculating the area *within a closed traverse* is universally used in preference to subdividing into triangles. It is generally agreed that it takes less time, is more systematic, and offers more easy checks; through the use of latitudes and departures, the error of closure is readily determined.

A method essentially the same as the preceding is that of double parallel distances used by some surveyors. In principle the two methods are identical, the only difference being that in double parallel distances (*D.P.D.'s*) the bases of trapezoids are along a line perpendicular to the meridian, while in double meridian distances they lie on the meridian itself. Thus, the rules for finding *D.M.D.'s* may be changed to rules for finding *D.P.D.'s* by substituting the word "latitude" for "departure"; and the rule for finding double areas will then be as follows: *The double area of any trapezoid equals the product of its D.P.D. and its corrected departure.*

PROBLEM 74. OMITTED MEASUREMENTS—BEARING AND LENGTH OF ONE SIDE UNKNOWN

Object.—To determine the length and bearing of one side of a closed traverse, these measurements not having been taken in the field. It is assumed that the error in the known lines of the traverse is so small as to be inconsiderate.

Procedure.—(1) Calculate the latitudes and departures of the known lines as in the previous problem, and find their totals. (2) On the preceding assumption and since the algebraic sum of latitudes and of departures for any closed traverse is zero, it follows that the latitude and departure of the unknown line are numerically equal to the sums of corresponding quantities for the known lines, but with opposite sign. Therefore, determine the bearing and length of the unknown line by the equations:

$$\text{Tan bearing angle} = \frac{\text{Departure of line}}{\text{Latitude of line}}$$

and

Length of line =

$$\sqrt{Lat^2 + Dep^2} = \frac{Lat.\ of\ line}{cos\ Bear.\ ang.} = \frac{Dep.\ of\ line}{sin\ Bear.\ ang.}$$

Hints and Precautions.—(1) Observe the hints and precautions of the previous problem that are here applicable. (2) It should be noted that the omission of measurements makes checking fieldwork through computations impossible, and throws all error into the unknown length and bearing. (3) It is well to plot the traverse with scale and protractor, thus graphically determining the unknown length and bearing.

Problem 75. Omitted Measurements—Bearing of One Side and Length of Another Side Unknown

Object.—To determine the length of one side and the bearing of another side of a closed traverse, these measurements having been omitted in the field. It is assumed that the errors in the traverse measurements are so small as to be inconsiderate.

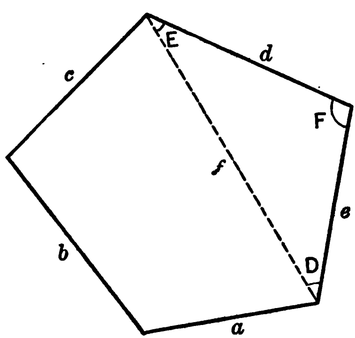


Fig. 33.

Procedure.—(1) Compute the latitudes and departures of known sides as in Problem 73, p. 183. (2) Suppose that *a*, *b*, and *c* (Fig. 33) are the known lines, *d* the line of unknown bearing, and *e* the line of unknown length. Then compute the bearing and length of the side *f* as in the previous problem, thus forming a closed traverse *abcf*. (3) The bearings of *e* and *f* being known, compute the angle *D*. (4) In the triangle *def*, two

Prob. 75. Omitted Measurements and Area. 53.
Black Addition.

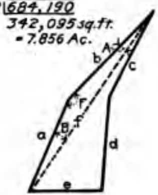
Field Notes
Book No. 11
Page 4.

Computations.
Dec. 4, 1905.
Comp'td by F.K.R.
Checked by R.J.P.

Line.	Cal. Bear.	Dist. Ft.	Latitudes N	Latitudes S	Departures E	Departures W	D. M. Ds.	Double Areas +	Double Areas −
a	N 20° 00 E	*622.0* (omitted)	*584.50*		*212.73*		*212.73*	*124,350*	
b	*N 45 00 E* (omitted)	800.0	*565.69*		*565.69*		*991.15*	*560,700*	
c	S 30 00 W	500.0		433.01		250.00	*1306.84*		*565,700*
d	S 5 04 E	720.0		717.18	63.58		*1120.42*		*803,540*
e	W	592.0		0.00		592.00	*592.00*		
			Lat. f =	+1150.19	63.58	842.00		*685,050*	*1,369,240*
						63.58			*685,050*
					Dep. f =	+778.42			2)*684,190*
									342,095 sq. ft.
									= 7.856 Ac.

Line.	c	d	e	*a*	*b*		f
Lat.	433.01	717.18	0.00	*584.50*	*565.69*	Log Dep.	2.89121
Log Lat.	2.63650	2.85563		*2.76678*	*2.75258*	Log Lat.	3.06077
Log cos	9.93753	9.99830		*9.97299*	*9.84949*	Log Tan	9.83044
Log Dist.	2.69897	2.85733		*2.79379*	*2.90309*	Bear.	N 34° 05.3' E
Log Sin	9.69897	8.94603		*9.53405*	*9.84949*	Log cos	9.91812
Log Dep.	2.39794	1.80336		*2.32784*	*2.75258*	Log Dist.	3.14265
Dep.	250.00	63.58	592.00	*212.74*	*565.69*	Dist.	

Angle	B	F	A	Line	a	b
Bear. f	N 34° 05.3' E			Log f	3.14265	
Bear. a	N 20 00 E			Log sin A	9.27712	
Ang B	14° 05.3'			Log sin F	9.62593	
Log sin B		9.38637		Log a	2.79384	
Log f		3.14265		Dist.	622.0	
Log b		2.90309		Bear. a		N 20° 00' E
Log sin F		9.62593		Ang (180-f)		+ 25 00
Ang.		155° 00'	10° 54.7'	Bear		N 45° 00' E



Note:
Necessary Computations for Omitted Measurements Show in <u>*Vertical*</u> *Figures.*
Further Computations Necessary to Determine Area Shown in <u>*Slope*</u> Figures.

41.

SAMPLE COMPUTATIONS 75.

sides *d* and *f*, and the angle *D* are known; solve for the remaining unknown quantities, and from the bearing of *f* and angle *E*, compute the bearing of *d*. (5) Calculate the latitudes and departures of the sides *d* and *e*. As a check, total the latitudes and departures of the whole traverse; their sums should equal zero. Record in the form shown in Sample Computations 75, page 189.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 73 and 74 that are applicable to this problem. (2) It will be a considerable help to the computer if he first solves the problem graphically. Special attention

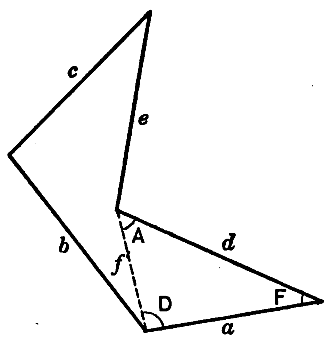


FIG. 34.

should be paid to signs, and *bearings should not be confused with angles.* (3) If the lines of omitted direction or length are not adjacent, they may be treated as though they were; for it is evident that a closed traverse will always close regardless of the order of the lines in that traverse. For example, suppose *a* and *d* (Fig. 33, page 188) are lines with omitted measurements in bearing or distance. Then in Fig. 34 are plotted the known lines parallel to their former positions beginning with *b* and going around the traverse in a clockwise direction. Then *bcef′* is a closed traverse. The bearing and length of *f′* may be found as were the corresponding quantities for *f* in Fig. 33, and the triangle *af′d* may be as readily solved for the unknowns as was the corresponding triangle when the lines containing omitted measurements were adjacent. From Fig. 34 it will be seen that if *f′* becomes zero *a* and *d* will be the same length but running in opposite directions. (4) *By method* *similar to the preceding, a solution may generally be performed when the lengths of two lines are omitted, or when the bearings of two lines are omitted.* In the latter case there are two solutions, hence the approximate direction of the lines must be known. The solution, whether for two sides or two bearings unknown, becomes weak as the closing side f (Fig. 33) or f' (Fig. 34) approaches zero. If the closing side becomes zero, as it may where the sides containing omitted measurements are not adjacent, the problem becomes indeterminate by this method.

PROBLEM 76. SUBDIVISION OF LAND BY A LINE STARTING FROM A KNOWN POINT ON THE BOUNDARY

Object.—To cut off a required area from a closed traverse by a straight line starting from a known point on the traverse. (The data of Problem 9, 20, or 21, pages 43, 65 and 69 may be used.) See Sample Computations 76, page 192.

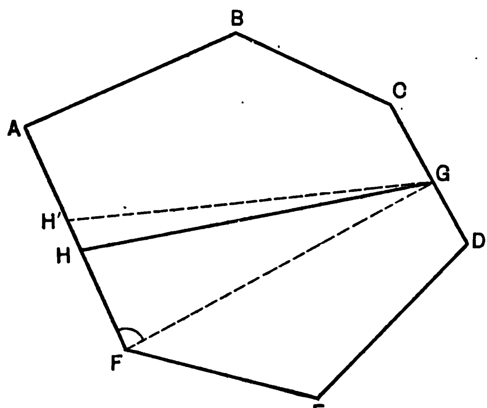


FIG. 35.

Procedure.—(1) Calculate the area within the traverse by method of double meridian distances (Problem 73, page 183). Plot the figure approximately to scale. Let this be represented by Fig. 35, and let G represent the known point from which the subdividing line must start. Draw a line GF to that corner of the traverse which, from inspection

Prob. 76. Computations for Balsam Park Division
to Divide in two Equal Parts
by a Line through "A".

55.

Field Notes
Book No. 3, Page 47.
Area Computations
Page 7 this Book.

Comp'td by G.O.F. Aug. 17, 1913.
Checked " N.G.C. " " ".

Total Area (see P.7).

Line	Cal. Bear	Dist. 66 ft. Ch.	Latitudes N	Latitudes S	Departures E	Departures W	D.M.Ds.	Double Areas +	Double Areas −
A-B	S80°29½W	34.46		5.69		33.99	61.81		351.89
B-C	S33 04 W	25.49		21.36		13.91	13.91		297.15
C-D	S33 46¾E	33.93		28.20	18.87		18.87		532.06
D-E	N87 58¾E	28.63	1.01		28.61		66.34	67.21	
E-A	N 0 27 E	54.24	54.23		0.43		95.38	5173.51	
								5240.72	1181.10
							Total	Area 202	.98 Ac.
					Area	A-B-C-D.			
A-B				5.69		33.99	61.81		351.89
B-C				21.36		13.91	13.91		297.15
C-D				28.20	18.87		18.87		532.06
D-A	N27 43 E	62.41	55.25		29.03		66.77	3688.78	1181.10
						Area	A-B-C-D	125.38 Ac.	
					Area	A-B-C-F.			
A-B				5.69		33.99	61.81		351.89
B-C				21.36		13.91	13.91		297.15
C-F	S33°46¾E	25.22		20.96	14.03		14.03		294.08
F-A	N35 12 E	58.75	48.01		33.87		61 93	2973.10	
						Area	A-B-C-F	101.50 Ac.	943.12 Check.

Line	D-A	Log 2 ADF	2.67928	Log D-A-D-F cos θ	1.76534	Line	C-F	F-A
Dist.	62.412	Log D-A	1.79527	Log D-F sin θ	0.88401	Departure	14.028	33.873
Log Dist.	1.79527	Log sin θ	9.94391	Log tan β	9.11867	Log Dep.	1.14687	1.52978
Log cos Br.	9.94704	Log D-F	0.94010	β	7°29'	Log sin	9.74509	9.76075
Log Lat.	1.74231	D-F	8.712	Bear. D-A	N27°43'E	Log Dist.	1.40178	1.76903
Log D.M.D.	1.82457	Log D-F sin θ	0.88401	Bear. F-A	N35°12'E	Log cos	9.91969	9.91230
Log D.Ar.	3.56688	Log cos θ	9.67860	Log F-D	0.94010	Log Lat.	1.32147	1.68133
D.Area	3688.76	Log D-F cos θ	0.61870	Log sin θ	9.94391	Latitude	20.961	48.015
Log Dep	1.46291	D-F cos θ	4.156	Log sin β	9.11498	Log D.M.D.	1.14700	1.79189
Log tan Br.	9.72060	D-A	62.412	Log A-F	1.76903	Log D.Ar.	2.46847	3.47322
Bear.	N27°43'E	D-A−D-F cos θ	58.256	A-F	58.753	D.Area	294.08	2973.10

Formulas.

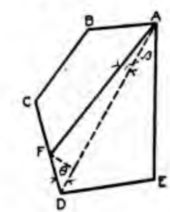
$$F\text{-}D = \frac{2\,A\text{-}D\text{-}F}{AD \sin\theta}$$

$$\tan\beta = \frac{FD \sin\theta}{DA - DF\cos\theta}$$

$$FA = \frac{FD \sin\theta}{\sin\beta}$$

Area A-B-C-D = 125.38 Ac.
Half Park = 101.49
Area A-D-F = 23.89 Ac.

C-D = 33.93
F-D = 8.712
C-F = 25.22



SAMPLE COMPUTATIONS 76.

of the figure, will come nearest being on the required line of division. (3) Calculate the latitude and departure of the line *CG*, and by methods of Problem 74, page 187, compute the latitude, departure, length, and bearing of *FG*. (4) By double meridian distances compute the area of *FABCG*, the amount cut off by the line *FG*. Find the difference between this area and that required. In the figure it is assumed that *FABCG* has an area greater than the desired amount, *GH* being the correct position of the dividing line. Then the triangle *GFH* represents this excess area, and, since angle *F* may be calculated from known bearings, compute

$$HF = \frac{2 \times area\ GFH}{FG \sin F},$$

the angle at *G*, and the bearing and length of the line *GH*. (5) As a check, compute the latitudes and departures of *GH* and *HA* and the area of *ABCGH*. This should equal the required area if the computations contain no mistake.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 73 and 74. (2) Often a line drawn from the known point in the boundary to any corner will nowhere near cut off the required area. Under such conditions or when the traverse has a large number of sides, it is advisable to plot the traverse with protractor and scale and to establish a trial line of subdivision such as *GH'* in Fig. 35, page 191. The planimeter (Problem 78, page 195) may be used to good advantage for finding the approximate area cut off by this trial line, and the line may be shifted until the area cut off agrees closely with that required. The scaled distance *AH'* may be used in the computations. It will be seen that the method of solution will now be identical with that used when the trial line was drawn to a corner.

Problem 77. Subdivision of Land by a Line Running in a Given Direction

Object.—To cut off a required area from a closed traverse by a straight line running in a given direction. (The data of Problem 9, 20, or 21, pages 43, 65 and 69 may be used.)

Procedure.—(1) Calculate the area within the closed traverse by double meridian distances (Problem 73, page 183).

(2) Plot the traverse to scale. Let this be represented by Fig. 36. Through the corner that seems likely to be nearest the line cutting of the required area draw a trial line *DG* in the given direction. (3) In the figure *ABCDG*, the latitudes and departures of *AB*, *BC*, and *CD*, and the bearings of *DG* and *GA* are known. By the methods of Problem 75, page 188, compute the unknown lengths, latitudes, and departures of *DG*

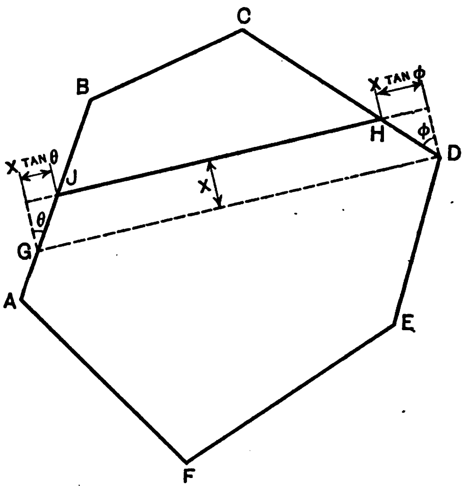


FIG. 36.

and *GA*. (4) By double meridian distances calculate the area cut off by the trial line. The difference between this area and that required is represented by the trapezoid *DGJH*, in which the side *DG* is known and the angles at *D* and *G* may be calculated from the known bearings of adjacent sides. (5) Calculate the angles θ and ϕ. Then

$$\textit{Area of trapezoid} = GD.x + \frac{x^2}{2}(\tan\theta + \tan\phi),$$

in which $\tan\theta$ or $\tan\phi$ is positive or negative according as θ or ϕ lies within or without the trapezoid. Solve this quadratic equation for x, or by trial find a value that will satisfy the equation. (6) Find values of *GJ*, *HD*, and *HJ*, and compute the area of *BCHJ* by double meridian distance method. If this and the required area are the same, the computations for subdivision will be correct.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 73 and 75. (2) It should be noticed that the sign of the squared term in the above equation will depend upon the direction of the lines *GJ* and *DH* with respect to the base of the trapezoid. For example, had these lines formed obtuse angles with the base, the two end triangles would have been added. The above definition of signs for tan θ and tan ϕ makes the equation a general formula. (3) The computations should be recorded in systematic form as in the preceding problem.

Problem 78. Area with Planimeter

Object.—To determine area with the polar planimeter. It is assumed that the figure, the area of which is to be found, is plotted to scale. The data of Problem 32, page 92, may be used as in Problem 83, page 205.

Procedure.—(1) Set the tracing arm so that a complete revolution of the measuring wheel will bear some simple relation to the given scale and unit of measurement. For example, it may be convenient because of the plotted size of the figure to have a complete revolution represent 10 sq. in.; then set the index to the mark on the tracing arm stamped "10 sq. in." (2) Test the accuracy of the setting by constructing a figure of known dimensions, say a 2″ × 5″ rectangle, and with the pole or fixed point in position *outside* the figure, set the tracing point over one corner. Read and record to four places the score on the measuring wheel and disc. (3) Carefully and slowly move the tracing point completely around the boundary *in a clockwise direction* to the point of starting. Again read and record the score of the measuring wheel and disc. (4) It is evident that for the example stated above, the difference between the initial and final readings should equal one complete revolution. If this difference is too small, slightly reduce the length of the tracing arm by means of the slow motion screw, and *vice versa*. (5) In like manner repeat the tests until the desired relation is obtained. (6) Having adjusted the planimeter, traverse the perimeter of the figure the area of which is to be determined, using the same method of procedure as with the figure of known area. Check the operation. (7) Transform the difference between readings into area.

Hints and Precautions.—(1) The reason for moving the tracing arm about the figure in a clockwise direction is that by so doing readings of wheel will increase, or the area will always be positive, when the pole is outside the area. (2) If the pole is placed inside the area, and this is sometimes advantageous with large figures, the difference between initial and final readings will be positive or negative according as the area of the figure is greater or less than that of the *zero circle* (the circle about which the tracing point might be moved without causing the measuring wheel to roll), the direction of motion of tracing point still being clockwise. When the pole is inside the figure, the area of the zero circle must always be algebraically added to the area indicated by the score of the measuring wheel; and the result will always be a positive area, direction of motion being as indicated. (3) The area of the zero circle can be found by measuring any given area with the pole inside and outside of the figure. The area indicated by the difference between the two scores will be the area of the zero circle. (4) If it is of such shape that its area can not be determined with one setting of the planimeter, the figure should be divided into several parts. The area of each of these subdivisions may then be determined as though the figures were separate. (5) The planimeter should be manipulated very carefully, and no result should be accepted as correct until checked. The paper should be stretched on a level surface so that it is free from wrinkles, the contact edge of the measuring wheel should be bright and free from dirt, and the mechanism of the planimeter should be so adjusted that it will move with the utmost freedom, yet without lost motion. The position of the pole should be so chosen that the measuring wheel will stay on the paper as the tracing point is moved about the figure. (6) It is not necessary that a simple relation exist between revolutions of wheel and unit area; the tracing arm may be set at any convenient length and the revolutions of the wheel determined for a known area. Then by proportion, *the known area, is to the revolutions for known area, as any area, is to revolutions for that area.* The first part of the proportion is a constant as long as the length of the tracing arm remains unchanged. (7) On profiles the vertical and horizontal scales are often dissimilar; this will make no difference in finding the area. For example, with a horizontal scale of 100 ft. = 1 in.

and a vertical scale of 10 ft. = 1 in. the value of a square inch on the paper is 1000 sq. ft.

Practical Applications.—The polar planimeter is a specially useful instrument for finding the areas of irregular cross-sections, profiles, and land between curved or irregular boundaries and transit lines, its precision generally being sufficient for such quantities. It is extremely useful as a means of approximately checking areas determined by more precise methods.

Problem 79. Volume of Borrow-pit

Object.—To calculate the volume of earth removed from a borrow-pit. It is assumed that the computer has two sets of level notes, one set giving elevations of ground points before the earth was removed and the other set giving elevations at corresponding points after excavation has been completed.

Procedure.—(1) From the level notes calculate the depth of layer of earth removed at the various points. In the computation book make a sketch showing the plan of the borrow-pit and the amount cut at the various points. Let such sketch be represented by Fig. 37, page 198. (2) Then compute the volumes of the various rectangular, triangular, or trapezoidal truncated prisms by the following rules:

Volume Truncated Triangular Prism (as *abc*) =

$$\frac{A}{3}(h_1 + h_2 + h_3),$$

in which A is the horizontal cross-sectional area and h_1, h_2, and h_3 the corner heights.

Volume Truncated Rectangular Prism (as *defg*) =

$$\frac{A}{4}(h_1 + h_2 + h_3 + h_4).$$

The volume of a truncated trapezoidal prism is the sum of the volumes of the two truncated triangular prisms into which it may be divided (as *jklm*). Figures with more than four sides may be divided into rectangles, trapezoids, and triangles (as *nopqrs*). (3) If several adjacent rectangular prisms have the same cross-sectional area, compute them as one solid as follows: Multiply each corner height by the number of prisms of same cross-section in which it occurs; sum up these results, and divide by four. Multiply this quantity by the cross-sectional area of a single prism. (4) Sum up the individual areas, and transform into cubic yards.

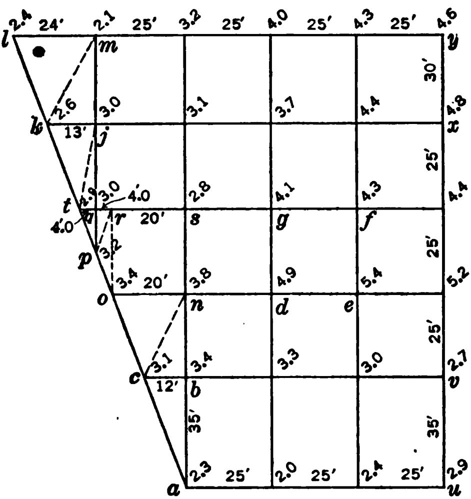


Fig. 37.—Borrow-pit.

Figure	Cor. hts.	Area		Volume	
uvba	82.7	25 x 35	x ¼	7153	
vxjqsb	160.9	25 x 25	x ¼	25141	
xymj	59.9	25 x 30	x ¼	11231	
nors	13.0	25 x 20	x ¼	1625	
klm	7.1	12 x 30	x ⅓	852	
jkm	7.7	6.5 x 30	x ⅓	501	
kjt	8.5	6.5 x 25	x ⅓	460	
jqt	8.9	2 x 25	x ⅓	148	
tqp	9.1	2 x 12	x ⅓	73	
pqr	9.2	2 x 12	x ⅓	74	
opr	9.6	2 x 25	x ⅓	160	
con	10.3	10 x 25	x ⅓	858	
bcn	10.3	6 x 25	x ⅓	515	
abc	8.8	6 x 35	x ⅓	616	
				49407	cu. ft.
			or	1830	cu. yds.

Hints and Precautions.—(1) Data and results may be tabulated as shown in the computations below Fig. 37. (2) Frequently the second set of level notes does not give elevations of points having the same horizontal position as those of the first set. When this is the case the volume can best be found by plotting the two profiles (one, of the ground before excavation, the other, of the ground after) of each line on cross-section paper. The area between original and final profiles is the cross-sectional area of the earth removed, and the volume between adjacent stations is a prismoid. The methods of calculating such areas and volumes is taken up in detail in the next problem.

PROBLEM 80. PLOTTING CROSS-SECTIONS; AND QUANTITIES OF EARTHWORK

Object.—To plot cross-sections from field notes and to calculate quantities of earthwork. It is assumed that the cross-section notes are for railroad (Problem 30, page 88), highway, or similar work, and give cut or fill rather than elevations.

Formulas.—For a three-level section (levels at slope-stakes and center).

$$Area = \frac{h_c(d_r + d_l) + W/2\ (h_r + h_l)}{2}$$

in which h_c, h_l, and h_r are center, left, and right cuts or fills; d_l and d_r are the distances left and right from center to slope-stakes; and W is the width of the roadbed.

For volume between cross-sections the *End-area Formula* is

$$V = \frac{l}{2}(A_1 + A_2)$$

in which A_1 and A_2 are the areas of end bases or cross-sections, l is the distance between cross-sections, and V is the volume in cubic feet.

The *Prismoidal Formula* for volume between cross-sections is

$$V' = \frac{l}{6}(A_1 + 4A_m + A_2)$$

with the same nomenclature as above, except A_m which is the middle area or area half-way between the end sections and which is found *by averaging corresponding dimensions of the end sections,* A_1 and A_2, *not by averaging areas* A_1 and A_2.

Procedure.—(1) Beginning near the top and left-hand end of the sheet or roll of cross-section paper, choose convenient heavy horizontal and vertical lines as grade and center lines. With these as co-ordinates plot the cross-section notes of the first station, counting the number of spaces out from the center and up from the grade line corresponding to these distances in the notes. Usually the scale used on such work is 1 in. = 10 ft. or one space equals 1 ft., but it may be larger or smaller. Mark the plotted points with dimensions identical with those of corresponding points in the notes. Draw straight lines showing roadbed and side slopes of cut or fill and the original ground, thus enclosing the section. Outside and just below the cross-section, and near the center line mark the station number. (2) At a convenient distance below and on the same center line, plot the next section in similar manner. (3) When the bottom of the sheet is reached, plot the next section a little farther to the right and at the top of the sheet; and in this way continue until all plotting is done. (4) Calculate the area of each three-level section by the preceding rule (if it is irregular, by dividing into triangles), and show its value within the section (as 123 sq. ft.). (5) Compute volumes by one of the preceding rules, and show the volume of each prismoid between its end sections (as 97 cu. yd.). (6) Find the total yardage in each cut and fill, and mark these totals conspicuously. (7) Make an appropriate title.

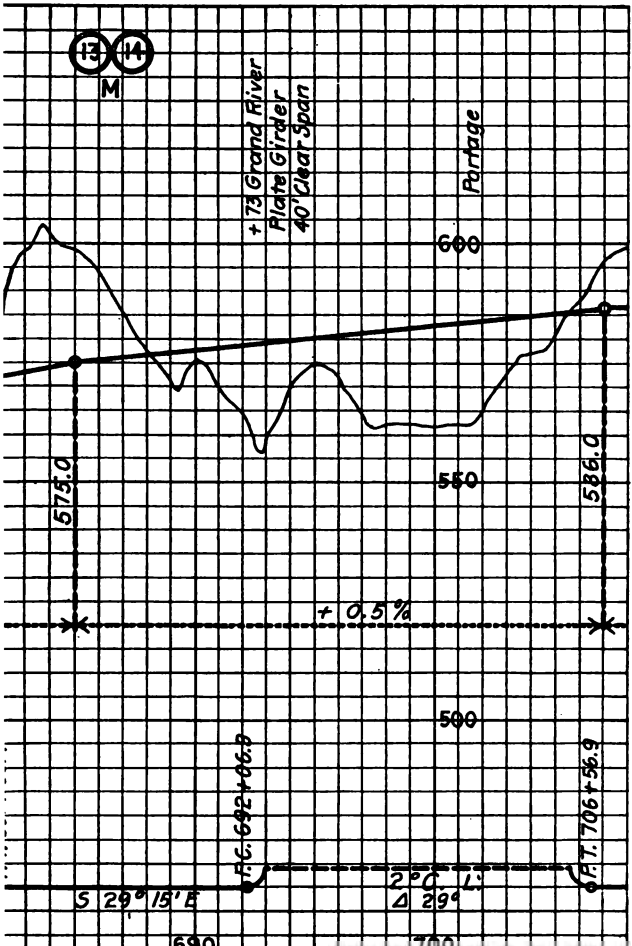
Hints and Precautions.—(1) All plotting and computations should be checked, preferably by someone other than the original computer. In plotting, two men can work together to better advantage than one alone, one calling off field notes while the other plots. When checking the plotted sections it is well to change positions and to reverse the operation by calling out measurements as they appear on the plotted cross-section. (2) Cross-section sheets may be regarded as working drawings, and as such should not receive undue elaboration. Titles should be free-hand, and should show the name of the work, the location of the sections by station numbers, the number of the field note-book and pages containing the original notes, the date of plotting, and the names of computer and checker. It is customary to ink the outlines of the sections in black ink. Areas and volumes should be shown in ink only after having been checked. (3) When cross-sections are plotted continuously throughout a roll of cross-section paper, which is a common method on extended surveys, a title should be placed on each end of the sheet, on the back and near the lower edge of the paper, so that when the roll is filed it may be identified without unrolling. (4) The areas of other and more irregular forms of cross-sections such, for example, as cross-sections of borrow-pit (Problem 79, page 197) may be conveniently determined by plotting on cross-section paper. The method of procedure will be similar to the above, except that elevations must often be plotted instead of cut or fill. For each section it will be convenient to let a heavy horizontal line represent the multiple of 10 ft. nearest the elevations shown in the notes of that section. Elevations can then be plotted by counting spaces from this line. (5) The planimeter can often be used to good advantage in determining the areas of irregular cross-sections (Problem 78, page 195), but the areas of regular three-level sections can be computed by the given formula quicker than with the planimeter. If a planimeter is not to be had, irregular sections must be divided into rectangles, triangles, or trapezoids, and the area of each of the smaller figures must be computed. (6) The *Prismoidal Formula* for volume is generally considered as giving results more nearly correct than the *End-area Formula;* however the latter is more generally used because it is simpler and the theoretical error introduced is small compared with the probable error due to changes of slope between sections which are unnoticed or disregarded in the field. As an example of the magnitude of the error in volume introduced through apparently insignificant variations of the surface, suppose that between two cross-sections 50 ft. apart a sag takes place gradually until at a point midway between the sections it amounts to a depth of 0.5 ft. over a width of 20 ft., thus forming two pyramids of error each with a base of 10 sq. ft. and an altitude of 25 ft. The volume of these pyramids will be $\frac{1}{3} \times 20 \times 0.5 \times 50 = 167$ cu. ft. = 6 cu. yd. The preceding example also shows the uselessness of computing volumes of earthwork closer than to the nearest cubic yard, unless the sections are very close together and the fieldwork is performed with more than the usual care.

Practical Applications.—The methods of computing the volume of earthwork outlined in this problem are the most common in use. Cross-section paper greatly facilitates the plotting, and area computation of irregular sections is thereby simplified. It is not necessary that three-level sections, or other forms of sections not considered in this book but common on railroad and highway work, be plotted in as far as the computations of their total areas are concerned, but these plotted cross-sections are frequently made the basis of preliminary estimates of material excavated, surface lines at the end of each month being plotted on the original section. At any station the area between original surface and the surface at the end of any month as plotted on the cross-section paper shows the cross-sectional area of material moved. If the material is not to grade at any given station at the time of the monthly estimate, that is, if the excavation or embankment is only partially completed, the cross-sectional area of the material will form an irregular figure which cannot be readily computed unless plotted. Frequently, for the purpose of showing the progress of the work, the areas between lines showing monthly progress are tinted with water colors, a different color for each month.

Problem 81. Plotting Profile

Object.—To plot a profile from level notes, and to fix the grade line for railroad, highway, pipe-line, or similar work. (The data of Problem 28 or 29, pages 85 and 87, may be used.)

Procedure.—(1) Choose a horizontal and vertical scale in keeping with the purpose of the profile. If it is for a railroad or highway, and later field measurements will give data for calculating volumes of earthwork, a scale of 1 in. = 400 ft. (horizontal) and 1 in. = 20 ft. (vertical) may be conveniently used with Plate "A" profile paper; if the profile is to be made the basis of earthwork computations, as it may be when it is for a sewer, a scale as large as 1 in. = 40 ft. (horizontal) and 1 in. = 4 ft. (vertical) may be required. (2) Examine the field notes to determine the range between points of maximum and minimum elevation. Number each of the heaviest horizontal lines with its elevation. These numbers, for convenience in plotting the profile, should be multiples of 50 ft. when the vertical scale is small, 10 ft. when the vertical scale is large, and they should be such that a maximum amount of the profile may be plotted without having a break in its continuity. For example, in Fig. 38 the profile may range between elevations of 450 and 650 ft. without leaving the



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sheet. Near the foot of heavy vertical lines record the 100-ft. station numbers, these numbers increasing from left to right, and being multiples of ten for small scales. (3) From the profile level notes plot the profile. The numbers on heavy horizontal and vertical lines will serve as a guide. First locate the vertical line of the station (if it is a plus station the nearest vertical line), then run up this line until the horizontal line corresponding nearest to the elevation is reached, and finally by estimation plot the point. (Using Fig. 38 for example, suppose a point of chainage 642 + 25 and elevation 543.4 is to be plotted. Look along on the profile paper to station 640, then at the second line beyond run up to eight spaces below the heavy line representing elevation 650, and finally plot the point by estimation one-fourth the distance to the next vertical line, and 0.4 the distance to the next horizontal line above.) Through the plotted points draw a free-hand curve. Show the names of streams and roads crossed, directly above their crossings on the profile. Check the profile, and ink it with *black ink*. (4) Fix the grade line. Its determination will depend on the purpose of the survey. If it is for a railroad or similar work, the cuts and fills should nearly balance and the rate of grade should not exceed a stipulated amount; if it is for a sewer, it must allow for drainage and must be below the frost line. At each change of grade draw a small circle, and indicate the elevation (and plus if it does not fall at a full station) of each of these points on a vertical line directly under the small circle. Draw a continuous line through the foot of these verticals, and on it record the rates of grade, as shown in Fig. 38. Ink, *in red*, the grade line and all lines and numbers referring to it. (5) Near the bottom of the paper indicate in *black* ink the horizontal alignment, using a scheme similar to that shown in the figure. (6) Make a title showing the name and location of the work, the horizontal and vertical scale, the date, and the name of the draftsman.

Hints and Precautions.—(1) The profile should be checked by reading elevations and stations from the profile, not the level notes. Two men can work together to good advantage —one reading the notes, while the other plots the profile; when checking the profile, they should exchange places. (2) It is a common mistake to read the elevations of turning-points and bench-marks as ground elevations. This may be avoided by enclosing each of these elevations in the field note-book with a line. (3) When most of the elevations are for full 100-ft. stations, it is common practice for the "reader" not to call off station numbers except at plusses, but the notes and profile are usually tallied every ten stations. (4) A more uniform width of line can be obtained if the profile is inked with a ruling pen rather than with a lettering pen. The draftsman should not endeavor to make it a series of straight lines, for this will not be the condition of the actual profile; on the other hand, he should not round off the summits and depressions an undue amount, for such points will naturally be greatly exaggerated in sharpness on account of the relation between horizontal and vertical scales. (5) Printed profile paper may be obtained in three standard styles: Plate "A" having 4 × 20 spaces to the inch, Plate "B" having 4 × 30 spaces to the inch, and Plate "C" having 5 × 25 spaces to the inch. The use of Plate "A" paper is the most common; it may also be used for cross-section paper. (6) Sometimes the profile of a line is shown on the same sheet with the plan, the horizontal position of points on the profile being determined by projections from corresponding points on the map. Since the plan of a line is not usually straight and will not, therefore, be parallel to the horizontal lines of the profile, the plotted distance between 100-ft. stations on the profile will be a varying quantity depending upon the relative directions of plan and profile. This method of plotting makes comparison between plan and profile very simple, but it can not be used if the change in horizontal direction of line is great within the limits of the sheet. (7) Profiles are often plotted from contour maps. For example, an excellent study of the possible routes of a proposed railway through country that has been mapped by the U. S. Geological Survey can often be made by projecting these proposed routes on the Geological Survey maps. A profile of each route may then be plotted with scaled distances to contour crossings. (8) Profiles of a series of parallel lines on large-scale contour maps, plotted in the above manner, are frequently made the basis of grading estimates. Such profiles are but little different from cross-sections (Problem 80). (9) When profiles are in rolls, besides the title on the face of the drawing, there should be one at each end on the back of the paper, so that unrolling to determine the contents will be unnecessary.

Practical Applications.—Since the location of many lines depends very largely upon the relief of the country through which the lines pass, the profile is as important as the plan. It is made the basis of estimates of cost preliminary to the work of actual construction, and only through its use can grades be laid out to good advantage. The progress of construction can be more readily shown by profile than in any other way; the *progress profile* is usually tinted with water-colors to indicate the work completed, a different color being used for each month.

PROBLEM 82. PROFILE AND QUANTITIES OF EXCAVATION FOR PIPE-LINE

Object.—To plot a profile of pipe-line, and to compute the quantity of excavation by two methods. (The data of Problem 29, page 87, are to be used.)

Procedure.—(1) Plot the profile and grade line from notes as explained in the preceding problem, using scales of 1 in. = 40 ft. (horizontal) and 1 in. = 4 ft. (vertical). (2) With the planimeter determine the area between profile and grade line as explained in Problem 78, page 195. (3) Compute the volume of excavation by multiplying this area by the estimated average width of trench. (4) From the cut as shown in the notes compute the cross-sectional area at each station. Compute the volume by method of average end areas (Problem 80, page 199). (5) Compare the results of the two methods.

Hints and Precautions.—(1) For pipe up to 18 in. in diam. the width of the bottom of the trench may be taken as 2 ft. 6 in. The width of trench for pipes of larger diameter may be assumed as 9 in. greater than the diameter of the pipe. The width of the top of the trench will be somewhat greater, depending upon the nature of the material excavated and the depth of excavation. (2) If Problems 78, 80, and 81 have not already been performed, study them carefully.

PROBLEM 83. PROFILE AND CROSS-SECTIONS OF STREET

Object.—To plot the profile and cross-sections of a proposed street from notes of Problem 32, page 92, and to compute the quantities of earthwork.

Procedure.—(1) Plot the profile of center line, and determine the grade line according to methods of Problem 81, page

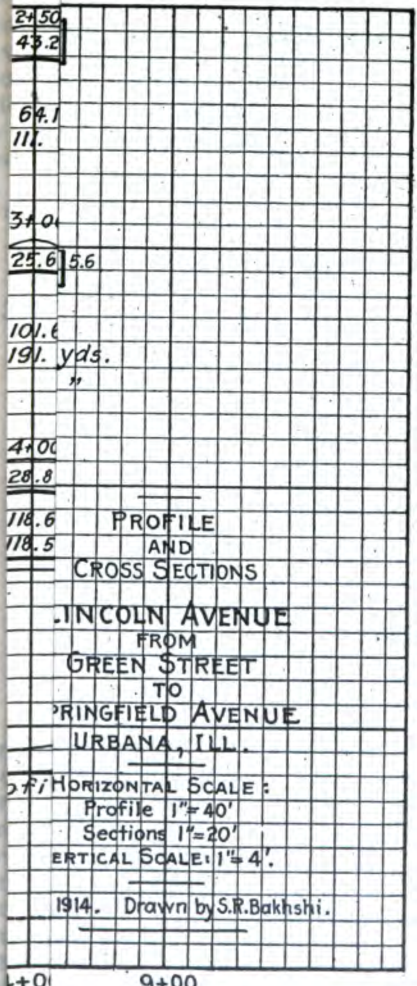
202; use Plate "A" profile paper and scales of 1 in. = 40 ft. (horizontal) and 1 in. = 4 ft. (vertical). (2) Plot the cross-sections on the same sheet, using scales of 1 in. = 20 ft. (horizontal) and 1 in. = 4 ft. (vertical). Use methods of Problem 80, page 199. Plot the sub-grade of sidewalks and street. (3) Find the areas of cross-sections with the planimeter (Problem 78, page 195) and compute the volume by average end-areas (Problem 80, page 199). (4) Make a neat and appropriate title.

Hints and Precautions.—(1) If Problems 78, 80, and 81 have not already been performed, study them carefully. (2) Arrange the profile and cross-sections as shown in Fig. 39.

Problem 84. Plotting by Tangents

Object.—To plot a given traverse by method of tangents. (The data of Problem 9, 20, or 21, pages 43, 65 and 69, may be used.)

Procedure.—(1) Tabulate angles and distances of the given traverse in the computation book. If angles are given as azimuths, change them to deflection angles. (2) Look up and record in tabulated form the natural tangent of each deflection angle less than 45°; the cotangent of each angle greater than 45°. Check the tabulated data. (3) Plot the traverse roughly to small scale using the protractor for angles; note its general form. (4) Carefully plot the first line on drawing paper to the required scale, estimating its position by means of the small-scale sketch. The line should lie so that the drawing will, when finished, be symmetrical with the sheet. Suppose this first line is AB, Fig. 40. (5) If the deflection angle is *less* than 45° prolong AB a distance of 10 in. to b, and carefully erect a perpendicular of indefinite length in the direction of the angle turned. Scale bb' equals 10 *in.* × *tan of deflection angle*. Draw line Bb'. The angle α equals the given deflection angle if no error has been made in plotting. Scale the distance BC. (6) If the deflection angle is *greater* than 45°, as for example β, lay off Cc 10 in. long and perpendicular to BC. At the extremity of this perpendicular erect a parallel to the line BC of indefinite length and scale off distance cc' equals 10 *in.* × *cot deflection angle*. The angle laid off from the perpendicular Cc is $(90° - \beta)$, and if no mistake has been made in plotting, its complement β is the same as the given deflection



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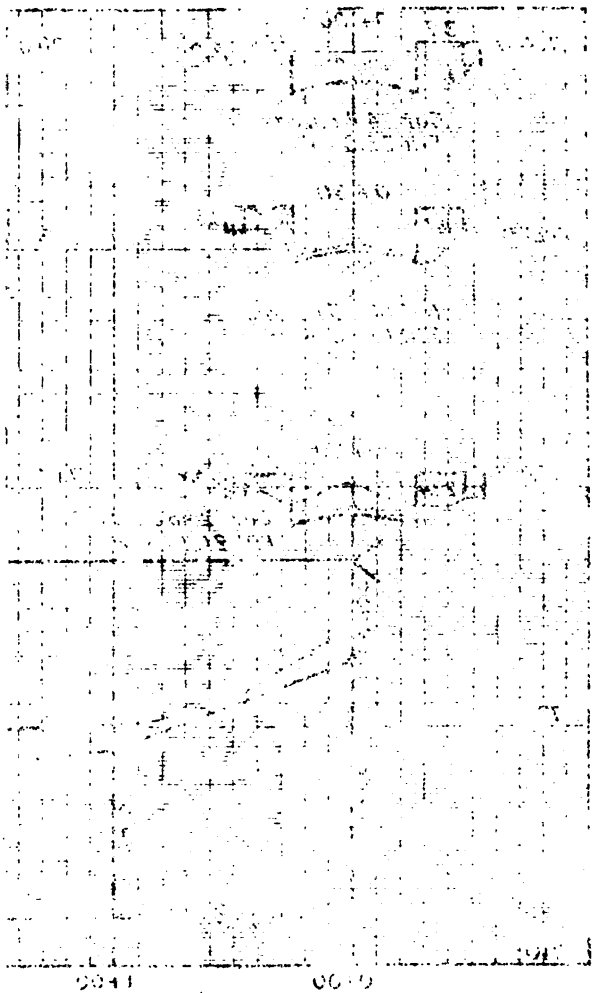


angle. Plot the distance CD. (7) By the above methods plot the entire traverse, being very careful to verify all measurements as soon as they have been plotted. (8) If it is a closed traverse and does not close on the sheet, it may be due to errors in the fieldwork (unless it has been balanced by latitudes and departures) or in the plotting. Carefully go over the traverse, scaling all distances and testing all perpendiculars. (9) If errors are not discovered by such investigation, cut a strip of paper somewhat longer than the combined length of the traverse lines. Let a, b, c, d, etc., be points to be

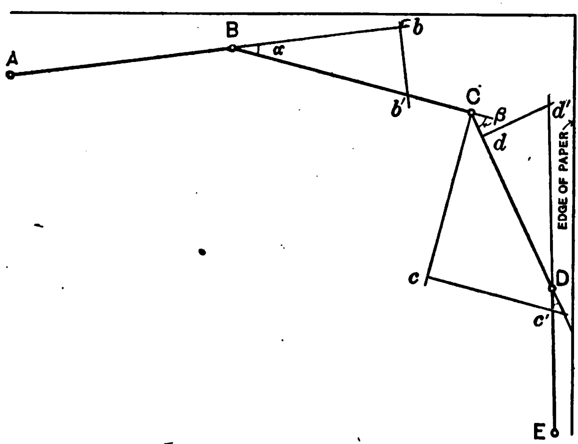


FIG. 40.—PLOTTING BY TANGENT OFFSETS.

marked on this strip corresponding to traverse points A, B, C, D, etc. Near one end mark point a on its edge. Place this mark at A in the traverse and with the edge of the strip along AB, mark point b on its edge opposite B on the traverse. Move the strip to BC with b opposite the corresponding point in the traverse, and mark point c. Continue in this way around the entire traverse. Scale the total length marked off on the strip. This scaled distance should agree very closely with the sum of the lengths in the given data. (10) If no error is discovered, apply the same scheme to tangent offsets. (11) If the traverse still fails to close, the error may be assumed to be in the fieldwork and it may be distributed among the different courses so that the traverse will close. (12) If it is not a closed traverse perform tests (8), (9), and (10). In addition, if possible transfer lines by triangles, parallel ruler, or straight-edge to intersect other lines several courses in advance; scale the tangents of angles of intersection between these lines, and compare these scaled distances with the natural tangents of corresponding calculated angles. (13) Make a neat and appropriate title.

Hints and Precautions.—(1) Particular care should be taken in scaling the lengths of the lines of the traverse and the tangent distances. Points should be pricked with a needle. The scale should be between draftsman and line, and the eye should be above each point as it is plotted. Lines should be drawn with a sharp pencil held so that its point will lie close to the edge of triangle or straight-edge. (2) The perpendicular at any point should not be erected by placing the base of triangle along the base-line with the corner of the triangle at the point, but should be constructed as follows: Fit the hypotenuse of triangle to the base-line, and place a straight-edge in contact with the base of the triangle. Turn the triangle through 90°, and slide it along the straight-edge until its hypotenuse passes through the point. A line drawn along the hypotenuse will be perpendicular to the given base-line providing the triangle has edges that are 90° apart. A second triangle is often used instead of the straight-edge. (3) It is well to test each perpendicular by measuring the hypotenuse of a right triangle of which the perpendicular is one side and the base-line another; if distances of 10 in. are scaled along these sides, then the length of the hypotenuse should equal the square-root of 200 or 14.14 in. It is well to remember this number. (4) Parallels should be laid off with two triangles or with triangle and straight-edge by methods similar to those explained above for erecting perpendiculars. (5) There is no reason why a base of 10 in. should be chosen except for simplicity in computing the tangent offsets; but, although bases of shorter length ordinarily should not be used, because of the increased accuracy of plotted angles it is often advantageous and sometimes necessary on long traverses to use bases of greater length, say 20 in. The distance between graduations on the usual form of natural scale (ten spaces to the inch) make close estimation of tangent distances impossible or at least very uncertain. For this reason it is common practice to divide the tangent distances by two and to plot with the 50 scale. (7) Often when the traverse approaches the edge of the sheet, the usual method of laying angles will not be practicable on account of the construction lines falling off the paper. The method shown in Fig. 40, page 207, for laying off the angle at *D* may generally be resorted to, the base-line *Dd* being measured back along the line *CD* and the tangent offset *dd'* being laid off as usual. The line *d'D* is then prolonged beyond *D*, and *E* is located in the customary manner.

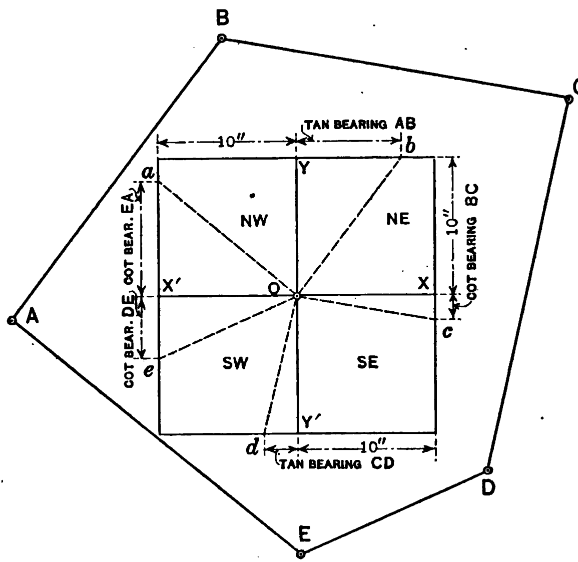


FIG. 41.—PLOTTING BEARINGS BY TANGENTS.

(8) Tangents increase rapidly for angles between 45° and 90°. It is for this reason that the cotangents or, what is the same thing, the tangents of the complements of these angles are plotted. (9) When the directions of lines are given by *bearings or azimuths*, some draftsmen prefer to use the method illustrated by Fig. 41. Four 10-in. squares with sides perpendicular or parallel to the meridian are constructed as shown, corresponding to the four quadrants in which bearings or azimuths may fall. The tangents of bearing angles of less than 45° or azimuths corresponding to such bearing angles are

scaled east or west from the line YY' along the outer sides of squares forming corresponding quadrants; in a similar manner the cotangents of bearing angles greater than 45° or azimuths corresponding to such angles are scaled north or south of the line XX'. Lines joining these points with the intersection of the X and Y axes will have the same bearings as corresponding lines of the traverse.

For example, the line AB has a bearing N 37° E. Its tangent offset is therefore scaled from YY' along the north side of the NE quadrant, thus locating b. The line ob must, therefore, be parallel to AB. The line BC has a bearing of S 71° E which is greater than 45°. The point c is, therefore, located by scaling the cotangent distance south from the line XX' along the east side of the SE quadrant and oc is parallel to BC.

In this way the angles of the whole traverse are laid off. The line ob is then transferred to its proper position with parallel ruler or triangles, and the distance AB is plotted to scale; oc is transferred until it passes through B, and the distance BC plotted to scale, etc.

Practical Applications.—Plotting by tangents is one of the most common methods of plotting traverses. In point of accuracy it is much superior to the protractor and is generally considered as slightly better than the chord method (Problem 85, page 211) but it is inferior to the method of total latitudes and departures (Problem 86, page 213). In point of simplicity of computations it is to be preferred to any other graphical method. Without doubt it is for the latter reason that the method is so commonly used. One of the principal disadvantages of the method is that the accuracy of the position of each line depends upon the accuracy of the position of the preceding line. For this reason it should not be used in plotting long traverses, whether they be closed or continuous. Plotting by tangents of bearing angles or azimuths partly overcomes this difficulty, since the direction of each line is independent of the directions of all other lines. However, there may be a considerable error introduced in transferring the lines from square to traverse, and erroneous lengths will have the same effect as in plotting by tangent offsets of deflection angles.

A very useful labor saving device for plotting by this method is known as the *tangent protractor*. It is nothing more than a scale subdivided into tangent distances for a 10-in. base corresponding to angles of multiples of 10′ between 0° and 45°. The values of angles, not tangent distances, are numbered on the protractor, 45° being 10 in. from the zero end, 30° being 5.77 in. from the zero end, etc. Its use is identical with that of the ordinary scale except that points are plotted opposite given angles rather than opposite pre-determined tangent distances; the use of trigonometric tables is therefore unnecessary. To facilitate the plotting of angles between 45° and 90° by cotangent distances, a second set of angular values are placed under the first, the former decreasing from 90° to 45° as the latter increases from 0° to 45°. The tangent protractor can not be used conveniently for a base other than 10 in. in length.

PROBLEM 85. PLOTTING BY CHORDS

Object.—To plot a given traverse by method of chords. (The data of Problem 9, 20, or 21, pages 43, 65, and 69 may be used.)

This method is much like that of tangents explained in the preceding problem, except that instead of erecting a perpendicular at the end of a 10-in. base-line, an arc of 10-in. radius is struck. The chord distance for the given angle is then scaled from the point of intersection between arc and base-line to a point on the arc. It is evident that the length of this chord is $2R \sin \frac{1}{2}\alpha$, in which R is the radius of 10 in. and α is the given angle.

Procedure.—(1) Tabulate the given data in computation book in form indicated below. If angles are given as bearings or azimuths, change to deflection angles. (2) Compute the half angles and record their natural sines, and compute the

Station	Distance	Def. Ang.	½ Ang.	Sin ½ Ang.	Chord Length

chord lengths (2 × 10 in. × sin ½ *ang.*). Check the computations. (3) Plot the traverse roughly to small scale, using the protractor for angles; note its general form. Carefully plot the first line of the traverse on drawing paper to the required scale, estimating its position by means of the small-scale sketch. The line should lie so that, when the drawing is finished, it will be symmetrical with the sheet. Let this first line be AB, Fig. 42, page 212. (4) Prolong AB an indefinite length. About B as a center swing an arc of 10-in. radius through b in the direction of the angle. With the scale plot the point b', the distance bb' being made equal to the corresponding tabulated chord length. Draw a line through Bb', and plot the distance BC. (5) With the ordinary scale it will be inconvenient to plot distances in excess of 12 in. This chord length, for a radius of 10 in., corresponds to an

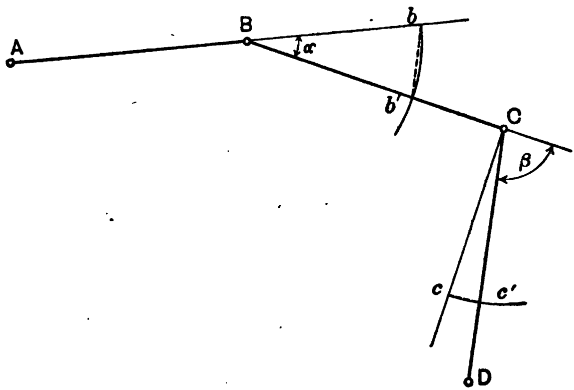


FIG. 42.—PLOTTING BY CHORDS.

angle of 73° 46′. For an angle in excess of this amount, compute the chord length corresponding to the complement of the angle. Let β be such angle. At C erect a perpendicular and swing an arc of 10-in. radius as before. From c scale the computed chord distance cc'. If no mistake has been made in plotting, Cc' is at the given angle with BC prolonged. Scale the distance CD. (6) By the above methods plot the entire traverse. (7) Check the traverse by methods similar to those of the preceding problem. (8) Make a neat and appropriate title.

Hints and Precautions.—(1) The whole of Problem 84 should be understood before attempting the chord method, for on account of the similarity of the two methods, much of the detail is omitted from the above description. (2) It is essential that the plotting and checking be done with as great care as with the tangent method. In swinging an arc with radius of 10 in. it is best to use beam compasses, for the ordinary dividers when extended to such length, even though they have an extension arm, are too flexible to make the drawing of a true circular arc a certainty. A very good substitute for beam compasses is a thin strip of wood pierced by two very small holes, so that when a needle is inserted in one hole and the sharp point of a pencil in the other, the two points will be exactly 10 in. apart. (3) If a table of chords is available * the chord lengths may be obtained directly, and no computations are necessary. (4) When directions of lines are given by bearings or azimuths instead of deflection angles, some draftsmen prefer to use a method of plotting by chords corresponding to that explained for plotting by tangents (Problem 84, Hints and Precautions (9)), and illustrated by Fig. 41, page 209, except that instead of using a 20-in. square divided into four 10-in. squares, a 20-in. circle is divided into four quadrants. This method of plotting azimuths or bearings by chords has the same advantages and disadvantages as the corresponding method for plotting by tangents.

Practical Applications.—The chord method of plotting is not as extensively used as the tangent method, and is generally regarded as being less accurate. There seems to be no reason why its precision should be less, provided the arcs are properly constructed with beam compasses. On account of computations, it is not as practicable as the method of tangents, unless the draftsman has access to a table of chords.

PROBLEM 86. PLOTTING BY TOTAL LATITUDES AND DEPARTURES

Object.—To plot a given traverse by the method of total latitudes and departures or co-ordinates. The data of Problem 9, 20, or 21, pages 43, 65, and 69 may be used.

Rule.—The $\left\{\begin{array}{l}\text{total latitude}\\ \text{total departure}\end{array}\right\}$ of any point equals the algebraic sum of $\left\{\begin{array}{l}\text{latitudes}\\ \text{departures}\end{array}\right\}$ of lines lying between that point and the $\left\{\begin{array}{l}\text{reference parallel}\\ \text{reference meridian}\end{array}\right\}$ passing through the origin. $\left.\begin{array}{l}\text{Total latitudes}\\ \text{Total departures}\end{array}\right\}$ are positive or negative according as the corresponding points lie

$$\left\{\begin{array}{l}\text{north or south of the reference parallel.}\\ \text{east or west of the reference meridian.}\end{array}\right\}$$

* See TRAUTWINE'S *Engineer's Pocket-book* (page 143, 1909 edition).

Procedure.—(1) Transpose the given data to computation book in form shown in the accompanying sample computations, page 216. If directions of lines are given in azimuths or deflection angles, calculate bearings referred either to the true or an assumed meridian. (2) Compute the latitudes and departures of the traverse lines, and if it is a closed traverse, balance the survey (Problem 73, page 183). Assume one of the traverse points as the origin of co-ordinates (*A* in computations), and calculate total latitudes and departures by the preceding rule. (4) Check all computations. (5) From the total latitudes and departures determine the size of the enclosing rectangle, the four sides of which pass through the extreme eastern, western, northern, and southern points of the traverse. (6) Roughly plot the enclosing rectangle and traverse to small scale, and note their relative positions. (7) On drawing paper plot the enclosing rectangle to the required scale, estimating its position on the sheet by means of the small-scale sketch. The traverse, not the rectangle, should be symmetrical with the sheet; therefore, the sides of the rectangle may or may not be parallel to the edges of the paper. (8) Test the accuracy of the plot by scaling the length of the diagonals. The scaled and calculated distances should agree very closely. Let *HJKL*, Fig. 43, be the enclosing rectangle of the traverse of which the computations are shown. (9) Plot the reference meridian (if it is not one of the sides of the enclosing rectangle) by scaling *JA* and *HM* equal corresponding computed total departure. Check by scaling *ML* and *AK*. In a similar manner plot and check the reference parallel. (10) If the drawing is of large size and there are a considerable number of points in the traverse, to facilitate plotting, construct other meridians and parallels so that the area will be divided into squares the sides of which are less than the length of the scale used (less than 12 in. with the ordinary scale) and represent some whole number of hundreds or thousands of feet at the given scale. For example, in the figure, meridians are drawn at intervals of 1000 ft. west of the reference meridian; parallels, at intervals of 1000 ft. north of the reference parallel. If the scale of the drawing is 100 ft. = 1 in., the actual length of the sides of the resulting squares is 10 in. Number each of these lines with its distance from the reference meridian or parallel. (11) Locate each traverse point by plotting its total latitude and departure.

Mark it with a fine dot enclosed in a small circle; draw traverse lines up to this circle, but not inside. Check by scaling the length of the preceding traverse line. For example, with origin at *A*, *F* has total N latitude of 1250 ft. and total E departure of 394 ft. (See Sample Computations 86, page 216.) Then, above parallel 1000 N, scale *ab* and *cd* equal 250 ft.,

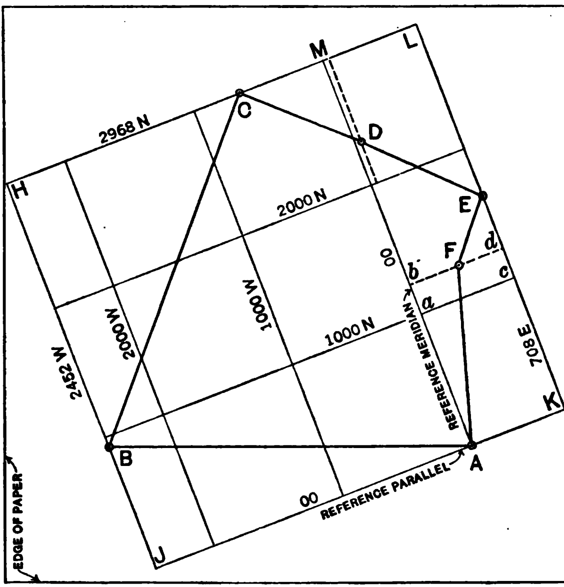


FIG. 43.—PLOTTING BY TOTAL LATITUDES AND DEPARTURES.

draw line *bd*, and scale *bF* equals 394 ft. Scale *EF*; it should equal the distance given in the notes, if no mistake has been made in plotting or computing. (12) Finish the drawing in pencil without erasing construction lines. Make a neat and appropriate title.

Hints and Precautions.—(1) Meridians and parallels should be accurately constructed. Parallel lines should not be drawn with the T-square by the common method of sliding its head along the edge of the drawing board, for such lines are seldom truly parallel. Having, by trial, constructed a true enclosing rectangle, the direction and position of all other nes should be based entirely upon linear measurements. 'he T-square may be used as a straight-edge, but that is all. 'he greatest pains should be taken in plotting the enclosing ectangle, since upon its precision depends the accuracy of the

48.

Prob. 86. Carry Pond Traverse. Total Lats. and Deps.

Field Notes Book No. 4 P. 27
Log. Computations on P. 47 this Book.

Comp't'd by J.E.L.
C'k'd by C.M.F.
Nov. 4, 1912:

Sta.	Cal. Bear.	Dist. Ft.	Latitudes N	Latitudes S	Departures E	Departures W	Adjusted Lats.	Adjusted Deps.	Total Lats. N	Total Lats. S	Total Deps. E	Total Deps. W
A									0	0	0	0
	N69°07'W	2622	934			2450	+ 936	−2454				
B									936			2452
	N41°50'E	2722	2030		1816		+2032	+1812				
C									2968			642
	S46°20'E	946		652	686		− 652	+ 682				
D									2316		40	
	S44°52'E	950		672	670		− 670	+ 668				
E									1646		708	
	S38°30'W	506		396		314	− 396	− 314				
F									1250		394	
	S17°30'W	1312		1252		392	−1250	− 394				
A									0	0	0	0
		9058	2964	2972	3172	3156	±2968	±3162				
				2964	3156							
				8	16							

$E.\ of\ C. = \frac{18}{9058} = \frac{1}{503}$

Sample Computations 86.

'hole drawing. (2) If latitudes and departures are comuted solely for the purpose of map construction, they should ot be determined with greater precision than can be used in lotting. For a scale of 1 in. = 100 ft. total latitudes and epartures should be correct to the nearest foot; for a scale of

1 in. = 500 ft., to the nearest 5 ft. (3) It sometimes happens that the length of a line will check when its direction is slightly in error. This will occur when the angle between line and meridian or line and parallel is small; for this reason points upon which such lines depend should be checked by re-measuring the total latitude and departure as well as by scaling the length of the line. (4) Some draftsmen prefer to consider the southwest corner of the enclosing rectangle as the origin of co-ordinates, since by so doing all the total latitudes and departures will be positive. (5) Long traverses that do not form closed figures may often be more conveniently plotted without constructing the enclosing rectangle; in fact often as much of it would fall off as on the map. The general method is to construct reference meridian and parallel as accurately as possible, and with these as a basis to construct other meridians and parallels, forming squares of a convenient size only where such squares are necessary. (6) For small drawings, construction lines other than those of the enclosing rectangle will be unnecessary.

Practical Applications.—The method of plotting by total latitudes and departures is generally recognized as the best method for plotting the average traverse. Its single disadvantage as compared with the method of tangents or chords is in the increase in the amount of computations necessary. This is not, however, always true, since in the case of closed traverses it is often necessary to compute the area; latitudes and departures are therefore necessary, and the increased labor of calculating co-ordinates is very slight. The principal advantages of the method are that the size and shape of the drawing can be accurately determined beforehand, that the accuracy of location of any point does not depend upon the accuracy of previous lines in the traverse, that the method of checking is simple and unlike the method of plotting, and that in closed traverses, the survey is balanced before plotting is begun. It is the only practical method for large drawings with many traverse lines, whether such drawings be of closed or continuous traverses.

Problem 87. Conventional Signs

Object.—To construct a plate of the more common of the standard conventional signs shown in Figs. 44–47, pages 218–222.

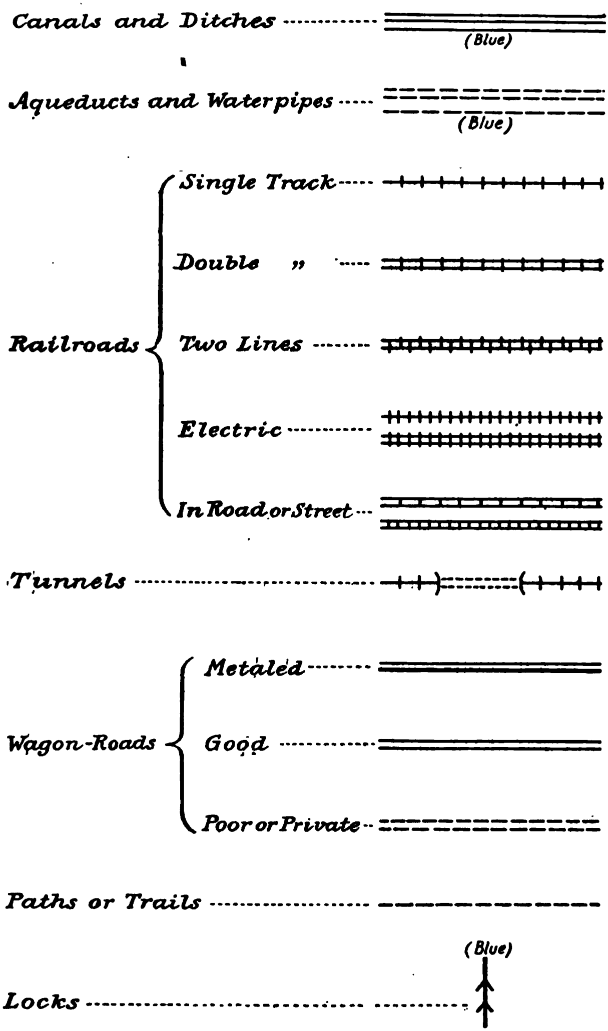


FIG. 44(*a*).—CONVENTIONAL SIGNS; PUBLIC AND PRIVATE WORKS.

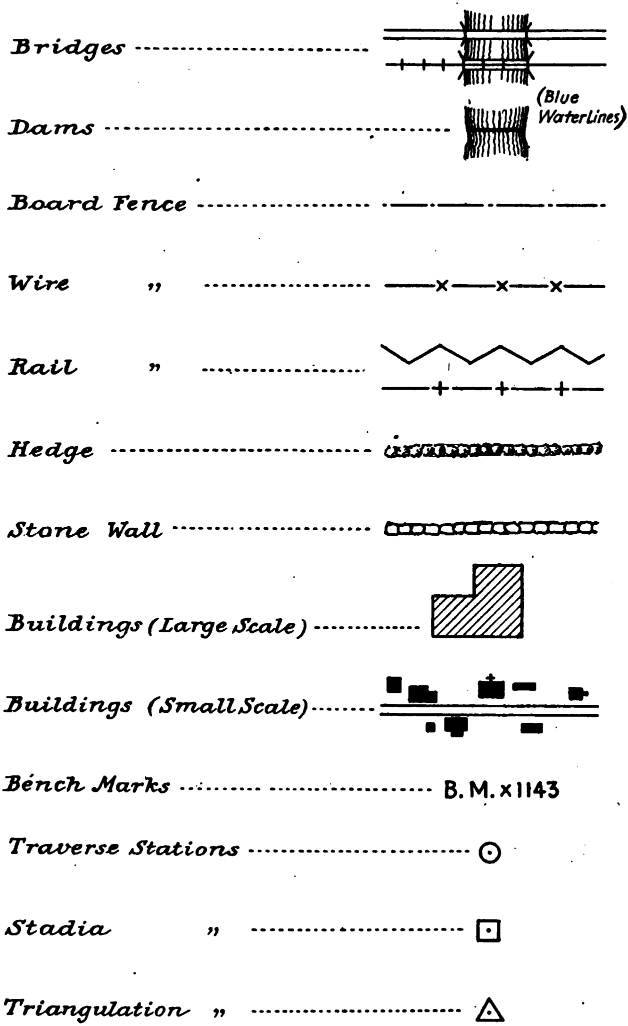


FIG. 44(*b*).—CONVENTIONAL SIGNS; PUBLIC AND PRIVATE WORKS.

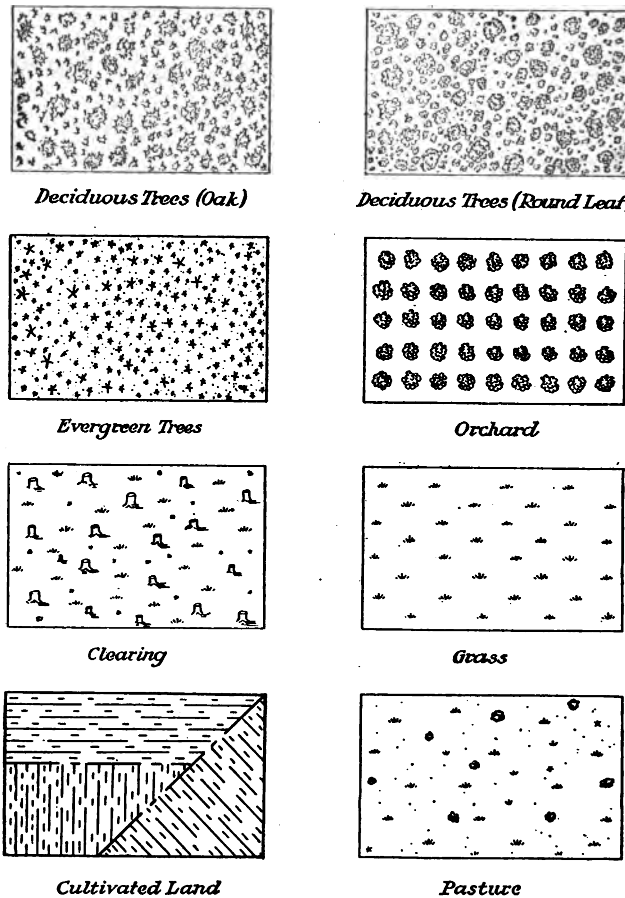


FIG. 45.—CONVENTIONAL SIGNS; CULTURE.

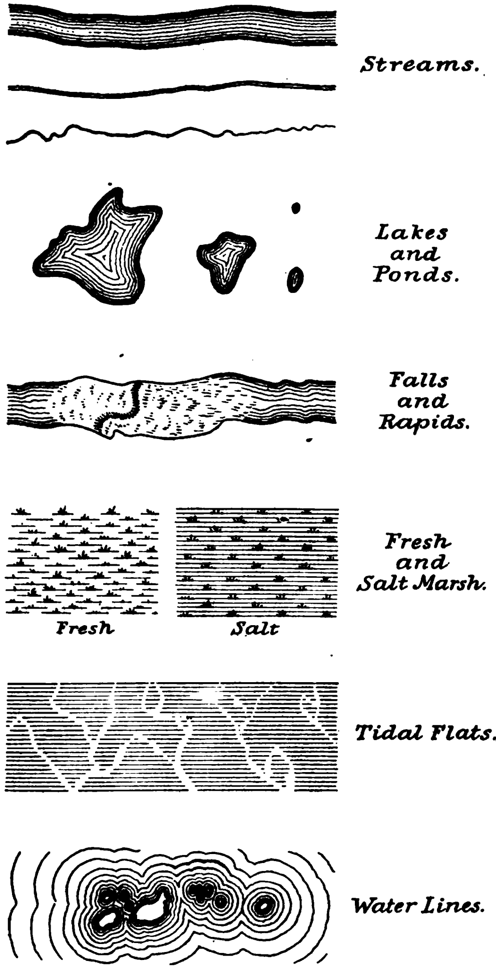


FIG. 46.—CONVENTIONAL SIGNS; HYDROGRAPHY.

Procedure.—(1) Divide a sheet of good quality drawing paper by lines forming rectangles or strips, and in pencil sketch such signs as those for fences, property lines, streets, paths, railroads, buildings, bridges, tunnels, etc., representing the works of man; deciduous and evergreen trees, meadow land, cultivated land, etc., representing culture of land; streams, lakes, fresh and salt marsh, tidal flats, etc., representing hydrography; and contours, depression contours, hachures, etc., representing hypsography or relief. (2) Opposite each

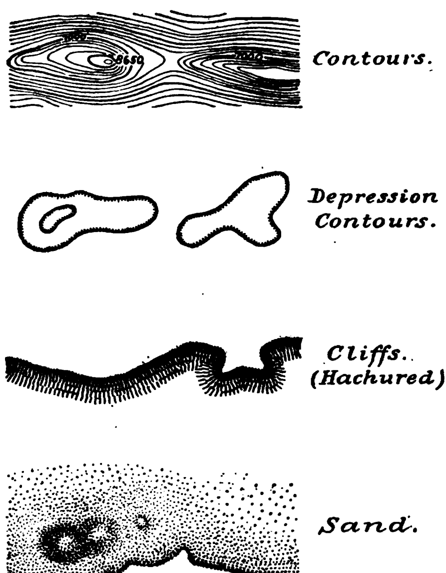


FIG. 47.—CONVENTIONAL SIGNS; HYPSOGRAPHY.

sign show its name in Italics (Fig. 49, page 224). (3) Ink, *in black*, the symbols representing works of man and culture of land; ink, *in blue*, the symbols representing hydrography; and ink, *in brown*, the symbols representing relief. (4) Make a neat and appropriate title in Roman letters (Fig. 49, page 224), and surround the plate with a simple border line.

Hints and Precautions.—(1) It is a common fault for inexperienced draftsmen to construct the symbols for trees, cultivated land, marsh, and meadow land much too large or too heavy. (2) Water-lines and other hydrographic features may be made with prussian blue water-color with a touch of burnt sienna; relief, with burnt sienna water-color. The general tendency is to mix the colors much too dark. (3) The construction of water-lines is perhaps the most difficult of any work the draftsman will be called upon to do. Water-lining can not be done hastily even by the expert draftsman; if the beginner is to obtain satisfactory results he will, therefore, have to proceed very slowly and carefully. Beginning

SLOPE FORM.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz 1234567890

Extended: PRESUMSCOT RIVER.

Compressed: AGRICULTURAL BUILDING. Sta. 43+71.2, El. 721.05.

VERTICAL FORM.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz 1234567890

Extended: ROCHESTER STREET.

Compressed: BANCROFT ESTATE. Note: Gravel in this Side Hill. Etc.

FIG. 48.—REINHARDT'S STYLE LETTERS.

at the shore line, the space between water-lines should *increase* while the width of the lines *diminishes*. Starting with the first water-line, the draftsman, with short, quick strokes of the pencil, should exactly parallel the shore line for its full length, and so on, line by line, except that as the distance from shore increases, the minor irregularities apparent in the shore line should become less and less noticeable. The pencil lines should be very light. The necessary weight is given to the lines when they are gone over with the pen, but when inked they should not be made unduly heavy. (4) The general tendency is to make contours too heavy. Every fifth or tenth contour, counting from the datum plane, is made

ROMAN

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

ITALIC OR STUMP WRITING

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

GOTHIC

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

INCLINED GOTHIC

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

HAIRLINE ANTIQUE

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

FIG. 49.—STYLES OF LETTERING.

heavier than the rest, and at intervals it is broken to allow showing its elevation. The breaks in the accentuated contours should not be made haphazardly. When there are several such contours the elevations should appear one above the other. Elevations on the same contour should appear at intervals sufficiently frequent to make reading the map a simple matter. Elevations should be recorded before the contour is drawn, otherwise the draftsman is likely to ink over the intended break; and the numbers should be of the same color as the contour. (5) The inking of water-lines or contours may be done with a contour pen or with a ruling pen; either one will give a more uniform width of line than the usual lettering pen. Some draftsmen prefer to use a fine lettering pen that has become well worn so that the nibs are slightly spread. Such a pen will produce a line of uniform width with but very little or no pressure on the part of the operator. (6) Tufts of grass, the conventional sign for meadow land, should not be placed too thickly; the blades of the individual tufts should be made to radiate from a common center some distance below. (7) A single line representing a stream should gradually increase in width from a line barely discernible at the source.

Practical Applications.—Ordinary maps usually show but few of the conventional signs, those for boundaries, fences, streams, roads, and buildings being the most common. Detail maps not showing the relief, sometimes called *flat topographic* maps, and contour or complete topographic maps will often show as many symbols as there are features in the area surveyed.

Problem 88. Map Construction

Object.—To plot a map from field notes of Problem 23, page 74, or similar data.

Procedure.—(1) Plot the traverse to given scale by method of tangents (Problem 84, p. 206), chords (Problem 85, p. 211), or total latitudes and departures (Problem 86, p. 213). (2) Plot details by methods the reverse of those used in the field. (3) By conventional signs represent features shown in Figs. 44–47, pages 218 to 222; represent objects not included in the figures by symbols, and on the map construct a legend or key to such symbols. Use a style of lettering in keeping with the purpose of the map (Art. 27, page 22). (4) Show the direction of the true or magnetic meridian by a simple but neatly constructed meridian needle (Art. 26, page 21). (5) Make a neat and appropriate title of Gothic capitals (Art. 28, page 23).

Hints and Precautions.—(1) The protractor should be used for laying off angles to details. Where a number of angles have been taken from a single traverse point, if the protractor is not constructed so that angles and distances can be plotted at the same time, it is best to plot all the angles before scaling any of the distances. There are so many varieties of protractors that no attempt will be made to give explicit directions as

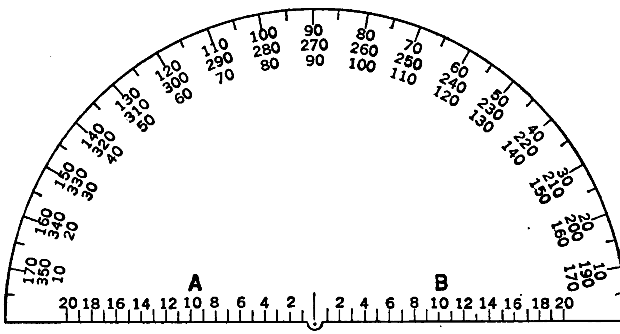


FIG. 50.—PROTRACTOR FOR PLOTTING DETAILS.

to their use. However, the student should first make certain of the location of the center of the protractor, no matter what kind he is using. (2) For plotting side shots a very good protractor can be made by pasting a half-sheet of polar coordinate paper to heavy drawing paper in the form shown in Fig. 50. It is well to have a small piece of transparent celluloid with a hole for a needle point, pasted at the center. The scale along the diameter should be plotted with fine lines and with spaces corresponding in length to those of the ordinary scale. The angle numbers are so arranged that either azimuths or deflection angles may be laid off by moving the protractor about the station as a center until the graduation on the protractor representing the given angle coincides with the reference line; the distance is then found on the diametrical scale. With such protractor, when angles are given in azimuths, a meridian line must be constructed through each transit station. To lay off a line of given azimuth (from North) move the protractor about the station as a center until the graduation corresponding to the given azimuth coincides with the south end of the meridian line. The distance will then be found on scale *A* or *B* according as the azimuth is less or greater than 180°. (3) Points located by linear measurement from two traverse stations may be plotted by intersecting arcs of the given radii drawn with dividers. (4) When objects, such as roads and streams, have been located by perpendicular offsets from transit line, these offsets may usually be plotted by estimating the perpendiculars with the eye. (5) The location of each point or object should be checked. The scaled lengths of sides of buildings should compare with field measurements, and if the buildings are rectangular, the angles formed by sides should be tested with the protractor. It often happens that mistakes in the fieldwork give an object more than one location or that a portion of the field notes are confusing. In such case, it is best to proceed with the plotting of other portions of the survey, for these portions may help in clearing up the doubtful point. (6) Note the hints and precautions of Problem 87, page 217, concerning the construction of conventional signs.

Problem 89. Interpolation of Contours

Object.—To interpolate contours on a sheet upon which have been plotted streams, lakes, ridge and valley lines, and representative ground points with their elevations. (The data of Problem 64, page 167, may be used.)

Facts Concerning Contours.—(1) All points in any one contour line have the same elevation. (2) Every contour closes upon itself either within or without the limits of the map; in the latter case it is drawn to the edge of the sheet. (3) Two contours can not cross each other except at an overhanging cliff or cave, in which case they must cross each other *twice*, and the lower contour must be shown as a dotted line. (4) The area within a closed contour is either a summit or depression. In the latter case, *when there is no water in the depression*, the contours are marked as shown in Fig. 47, page 222. (5) The distance between contours varies inversely as the slope. (6) Contours are evenly spaced where the slope is uniform. (7) A single contour can not intervene between two higher or two lower contours; that is, the maximum contours of ridge and minimum contours of valley must run in pairs if they do not close. (8) No contour should be drawn across a stream. As it approaches the stream it should turn up-stream and disappear in coincidence with the line defining the water's edge. Its re-appearance should be at a point directly across the stream. (9) The sharpest bends in contours occur at their intersection with ridge or valley lines.

Procedure.—(1) Interpolate points on contours assuming that the slope between adjacent known elevations is uniform. Give each interpolated point its elevation. (2) After numerous points have been located in this manner, draw in the contours, observing the preceding facts concerning them. Make each fifth or tenth contour heavier than the rest, and mark its elevation as explained in Problem 87, page 217.

Hints and Precautions.—(1) The interpolated position of points on contours may be computed by proportion. For

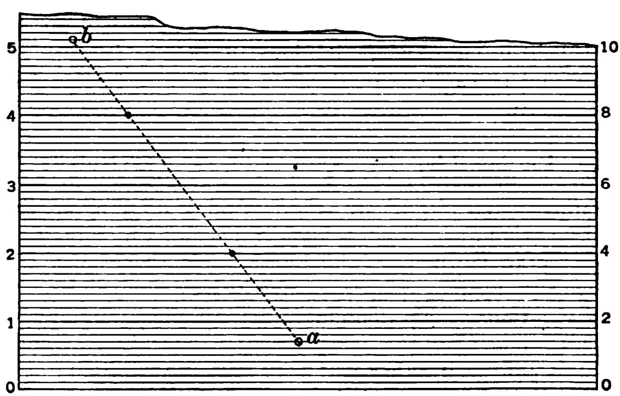


FIG. 51.

example, if *a* and *b* with elevations of 53 ft. and 63 ft. are 120 ft. apart, the 55-ft. contour will cross the line *ab* at a distance of 2/10 × 120 ft. = 24 ft. from *a*; the 60-ft. contour, at a distance of 7/10 × 120 ft. = 84 ft. from *a*. (2) There are a number of mechanical contrivances whereby contour points may be interpolated with greater rapidity and with less mental labor on the part of the draftsman than with the arithmetical method. One of these is shown in Fig. 51. A number of parallel lines are drawn at equal intervals on tracing cloth, each fifth or tenth line being made heavier or of a different color than the rest and numbered as shown. Now if it be desired to interpolate the position of 52-ft. and 54-ft. contours between *a* with elevation of 50.7 and *b* with elevation of 55.1, the line corresponding to 0.7 ft. (left end) is placed over *a* and the tracing turned about *a* as a center until the line corresponding to 5.1 ft. (left end) covers *b*. The interpolated points are at the intersections of lines representing their elevations and the line *ab*, and may be pricked through the tracing cloth. Had the known points been much closer together, the figures at the right end of the tracing would have been used, or in other words, the value of each space would have been doubled; and if the scale were small, the contour interval large, and the topography rugged, each space might represent 1 ft. Thus it will be seen that, by assigning different values to the spaces, a single piece of tracing cloth prepared in this way can be made to suit a variety of conditions.

Problem 90. Topographic Map Construction

Object.—To construct a complete topographic map from field notes, relief being represented by contours. (The data of Problems 59, 60, 61, and 64, pages 153, 160, and 167 may be used.)

Procedure.—(1) If the skeleton of the survey is a traverse, plot by method of tangents (Problem 84, p. 206), chords (Problem 85, p. 211), or total latitudes and departures (Problem 86, p. 213); if the horizontal control is in the form of rectangles or squares, plot by methods used in Problem 86 for constructing meridians and parallels. (2) Plot the details of the map as in Problem 88, p. 225, using conventional signs (Problem 87, p. 217). (3) Mark each ground point by a dot and mark each elevation in such position that there will be no doubt as to which point it refers. (4) Interpolate contours as in the preceding problem. (5) Show the direction of the true or magnetic meridian by a neat but not elaborate meridian needle (Art. 26, page 21). (6) Place necessary notes so that they will not interfere with the map. (7) Make a neat and appropriate title (Art. 28, page 23). (8) Ink in the map, observing the hints and precautions of Problem 87.

Problem 91. Base-line Reduction

Object.—To compute the true length of a base-line from given field notes. The data of Problem 68, page 176, may be used.

Formulas.—

(1) *Correction for Temperature (for one tape length)* = $+ Lk(T - T_0)$.

(2) *Correction for Pull (for one tape length)* = $+ L\dfrac{(P - P_0)}{aE}$.

(3) *Correction for Sag (for one tape length)* =

$$+ \frac{L}{24}\left(\frac{w}{PP_0}\right)^2 (l_0{}^2P^2 - l^2P_0{}^2).$$

(4) *Correction for Grade (between grade intersections)* =

$$-\left(\frac{h^2}{2L'} + \frac{h^4}{8L'^3}\right), \text{ (approximate)}.$$

(4′) *Correction for Grade (between grade intersections* =

$$- 2L' \sin^2 \tfrac{1}{2}\theta, \text{ (exact)}$$

(5) *Correction to Reduce to Sea-level (for whole base-line)* =

$$- L''\frac{A}{R}$$

L is length of tape in feet.
L' is length of uniform grade.
L'' is the length of the base-line in feet.
T_0 is the standard temperature.
T is the mean of the observed temperatures.
k is the coefficient of expansion of the tape for 1° F.
a is the cross-sectional area of the tape *in square inches.*
E is the modulus of elasticity of the material of the tape in pounds/in.^{2}
w is the weight of the tape *per linear foot.*
l_0 is standard distance in feet between supports.
l is the distance in feet between supports during observations.
P_0 is the standard pull.
P is the observed pull.
h is the difference in elevation between grade points at ends of length L'.
θ is the angle of inclination of base-line between grade intersections, $\sin \theta = \dfrac{h}{L'}$.

A is the mean altitude of the base above sea-level.

R is the radius of the earth in feet. (Mean radius equals 20,890,600 ft., Log R equals 7.31995.)

All the corrections are in feet.

Procedure.—(1) Transpose the necessary data to the computation book. (2) Decide upon a neat and systematic form for computations. (3) Compute the mean of the observed temperatures. (4) Calculate the correction for temperature, pull, sag, and erroneous length of tape for one tape length according to the preceding formulas. (5) Find the algebraic sum of the above corrections, and multiply by the number of tape lengths in the base-line. (6) Compute the total correction for grade. (7) Apply the total corrections for sag, pull, temperature, and grade to the measured length of the base. This gives its correct horizontal length. (8) Reduce this to the corresponding length at sea-level. (9) Check all work.

Hints and Precautions.—(1) The computer should be very careful to apply corrections with their proper signs which will not necessarily be the same as those prefixed to the correction in the formulas. For example, if the observed pull or temperature is less than the pull or temperature at which the tape was standardized, the corresponding correction as applied to the measured base will be negative. The fact to be always borne in mind is that conditions which tend to make the tape too long will necessitate a positive correction to the measured length, and *vice versa.* This applies to corrections for erroneous length as well as sag, pull, and temperature. The corrections for grade and sea-level will always be negative. (2) Tapes standardized by the Bureau of Standards at Washington are accompanied by a Certificate of Comparison, the contents of which depend upon the specifications for standardization made by the owner of the tape and the amount of the fee charged by the Bureau of Standards, but in all cases it will contain the length of the tape at a given temperature and pull either flat or supported at given intervals. A complete Certificate of Comparison also includes the coefficient of expansion and modulus of elasticity of the tape. These latter quantities are exceedingly important, if other than the standard pull is used and if the temperature at which measurements are taken is greatly different from that at which the tape was standardized, because assumed values may contain a considerable error. However, if the tape is steel, and these values are unknown, the coefficient of expansion may be taken as 0.0000065 for 1° F., and the modulus of elasticity as 28,000,000. (3) The correction for pull becomes zero if $P = P_0$ in Formula (2). (4) If the tape was standardized flat, the correction for sag becomes $-\frac{L}{24}\left(\frac{wl}{P}\right)^2$. If the distance between supports and the pull are the same as they were during the standardization test, the corrections for sag and pull become zero. (5) The sea-level correction is usually omitted from the base-line calculations of the smaller triangulation systems, but in geodetic work it is always considered. (6) For ordinary grades, Formula (4) may be used without introducing appreciable error; for very steep grades Formula (4′) should be used. For grades of less than 2 per cent, Formula (4) may be reduced to $C_g = -\frac{h^2}{2L'}$ unless the required precision is very high. If the distances between succeeding grade intersections are equal, as when changes of grade occur at each tape length, Formula (4) may be written in the form:

Total correction for grade =

$$\left[\frac{(h_1^2+h_2^2+h_3^2+\ldots h_n^2)}{2L'}+\frac{(h_1^4+h_2^4+h_3^4+\ldots h_n^4)}{8L'^3}\right]$$

in which h_1, h_2, h_3, etc., are the differences in elevation between the ends of succeeding uniform grades. (7) The cross-sectional area of the tape can best be found by first computing the volume with weight of tape as basis (3.56 cu. in. of steel to the pound); the cross-sectional area is then determined by dividing the volume by the length of the tape. Since E (Formula (2)) is in pounds/in.², it is necessary that a be in *square inches*.

Problem 92. Triangulation Adjustments and Computations

Object.—To adjust the angles of a triangulation system, and to compute the lengths of sides and co-ordinates of triangulation points.

It is assumed that the angles about each point in the triangulation system have been accurately determined either with repeating or direction theodolite, that the length of at least one line is known, that the true azimuth of one line is known, and that the relation between the area of the system and the required precision is such that the spheroidity of the earth may be neglected.

Procedure.—(1) Adjust the angles about each triangulation station so that their sum shall equal 360°. If all the angles have been taken with the same precision, which will usually be the case, the angular error of closure should be divided equally among the angles regardless of size of angle; if the field notes indicate that the conditions under which observations of certain angles were taken were unfavorable, or if a greater number of observations were taken on some angles than on others, the error of closure should be distributed accordingly. This is called the *Station* adjustment, and the resulting angles will be referred to as "station" angles. (2) Make a rough sketch of the triangulation system, and record the value of each angle according to the station adjustment. (3) Carefully study the figures formed by the various triangles in the system, and divide the system into quadrilaterals, polygons, and single triangles, but as far as possible avoid figures composed of a single triangle. Fig. 52, page 234, shows such subdivision of a triangulation system. The quadrilaterals *ABCD*, *DEFG*, *DGHJ*, and *GHKL* were first determined; this division left the polygon *EPONF* and the triangles *CDE*, *ADJ*, *LGM*, *MGF*, and *OPR*. (4) By inspection adjust each quadrilateral so that the sum of the angles of each of its four triangles is equal to 180° and the sum of its interior angles is 360°. *Change the values of the station angles as little as possible.* (5) In a similar manner adjust each polygon so that the sum of the angles in each of its triangles is 180°, the sum of its interior angles is $(n - 2) \times 180°$ (in which n is the number of sides), and the sum of the angles about the interior station (as Q in Fig. 52, page 234) is 360°. (6) Adjust each figure composed of a single triangle so that the sum of its interior angles is 180°. (7) Re-adjust the angles, if necessary, so that the sum of the angles about each interior station of the triangulation system is equal to 360°, the geometric conditions stated in (4), (5), and (6) are fulfilled, and the adjusted values will be altered the smallest amount. Make this adjustment by altering the values of angles of the weakest figures. In the majority of cases this will mean the alteration of figures composed of single triangles, assuming the angles of adjusted quadrilaterals and polygons as correct.

As an example all angles about D (Fig. 52) except a and d have been determined through quadrilateral adjustments.

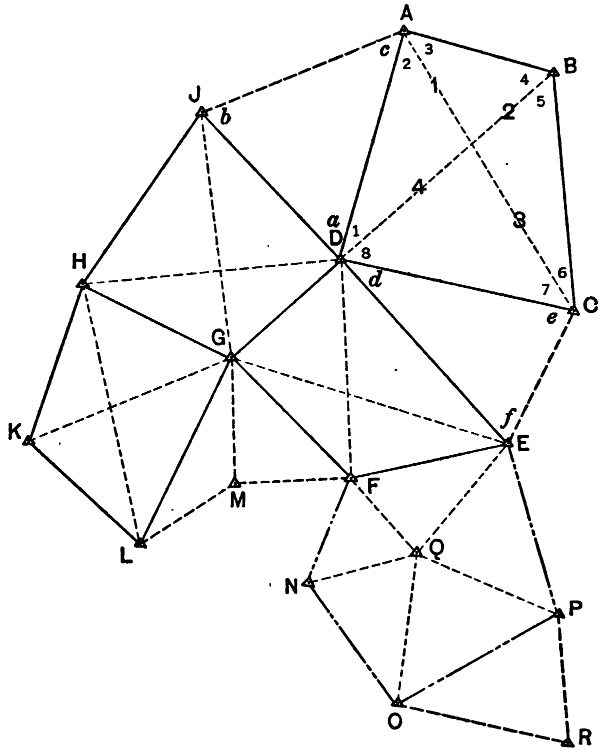


FIG. 52.—TRIANGULATION FIGURES.

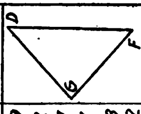
Angles a and d have been determined through the adjustment of single triangles. Suppose the sum of the angles about D is 04″ in excess of 360°. Each of the angles a and d would be decreased by 02″ and angles b, c, e, and f, increased by 01″.

(8) Starting with the base-line or side of known length, compute the lengths of the other sides by the equation $\frac{a}{b} = \frac{\sin A}{\sin B}$.

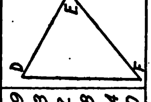
Use logarithms and record in form shown in Fig. 53. Check, not by re-computation, but by making each computation depend upon the preceding until the length of some line has been determined by two independent series of computations. From the preceding statement it follows that each line must be

Quadrilateral DEFG

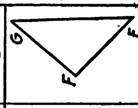
Δ DFG

Lines Stations	Dists. Angles	Logs	①
DF	8778.28	3.943409	
G	104° 44′ 40″	0.014542	
D	34 37 38	9.754527	
F	40 37 42	9.813681	
FG	5157.96	3.712478	
GD	5910.61	3.771632	

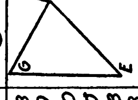
Δ DFE

Lines Stations	Dists. Angles	Logs	②
DF		3.943409	
E	78° 08′ 44″	0.009363	
D	43 00 57	9.833912	
F	58 50 19	9.932328	
FE	6119.05	3.786684	
ED	7675.38	3.885100	

Δ FEG

Lines Stations	Dists. Angles	Logs	③
FE		3.786684	
G	44° 23′ 49″	0.155134	
F	99 28 01	9.994045	
E	36 08 10	9.770635	
GE	8627.06	3.935863	
GF	5157.67	3.712454	

Δ GED

Lines Stations	Dists. Angles	Logs	④
GE		3.935863	
D	77° 38′ 35″	0.010180	
G	60 20 51	9.939040	
E	42 00 34	9.825590	
ED	7675.01	3.885083	
DG	5910.62	3.771633	

FG = 5157.96
5157.67
5157.81 Mean

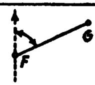
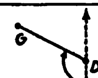
ED = 7675.38
7675.01
7675.19 Mean

GD = 5910.61
5910.62
5910.61 Mean

FIG. 53.—COMPUTATION OF TRIANGLE SIDES.

computed by two independent solutions, or must be computed by one solution and made the basis of computations in another. (This will be impossible if the triangulation system is a chain of simple triangles.) If all conditions had been satisfied in the adjustment of angles, these independent determinations should exactly agree. Since the adjustment of angles was made purely by inspection and all possible conditions are not fulfilled, this is not likely to be the case, but it may be safely assumed that the computations are correct if the discrepancy between these independent determinations is small.

In Fig. 53, page 235, are recorded the computations for each of the triangles of the quadrilateral *DGEF* (Fig. 52, page 234), *DF* being the line of known length. Solutions 1 and 2 are sufficient to determine the sides of the quadrilateral, but none of the lengths is checked. Solution 3 is based upon the computed length of *FE* of solution 2; and through solution 3, *FG* of solution 1 is checked (the discrepancy being 0.29 ft.)

Symbols	G from F 		G from D 	
	Dists. and Azimuths	Logs	Dists. and Azimuths	Logs
l	5157.81	3.712466	5910.61	3.771632
	Log cos Z	9.818659		9.748214
Z	48° 48′ 09″		124° 03′ 29″	
	Log sin Z	9.876474		9.918277
Lat.	+3397.23	3.531125	+3310.21	3.519846
Dep.	+3880.97	3.588940	−4896.77	3.689909
L_1	− 87.20		0.00	
M_1	−8777.85		0.00	
Tl. Lat. G	+3310.03		+3310.21	
Tl. Dep. G	−4896.88		−4896.77	

Mean Tl. Lat. G = +3310.12
" " Dep. G = −4896.82

l = length
Z = azimuth
L_1 = total lat. of known point
M_1 = total dep. " " "

FIG. 54.—COMPUTATION OF CO-ORDINATES.

and *GE* is computed. Solution 4 is based on the computed length *GE*, and in the determination of *ED* and *DG* not only are the corresponding distances in solutions 1 and 2 checked very closely but the fact that this check is performed shows that the length of the line *GE* is correct as far as computations are concerned.

(9) Starting with the line of known azimuth, compute the azimuths of the other lines of the system (about one triangulation point at a time) using the *adjusted* angles. By so doing

the azimuth of every line will be computed from each end. Check by noting that opposite azimuths differ by 180°. Tabulate the results. (10) Calculate the latitudes and departures (Problem 73, page 183) of a sufficient number of the lines of the triangulation system so that the co-ordinates of each point may be determined by two solutions. When possible utilize lines the lengths of which have been determined by two solutions, and use the mean of the lengths. Fig. 54 illustrates a convenient form and shows computations for latitudes and departures of the lines *FG* and *DG*. (11) Choose some triangulation point as an origin, and compute the co-ordinates of each of the other stations (by two routes when possible) using the method of total latitudes and departures (Problem 86, page 213). If the latitudes and departures are computed correctly and no mistake has been made in the totals, the co-ordinates of a station determined by one route should agree very closely with the co-ordinates determined by any other route. Check by noting this agreement, and assume the correct co-ordinates as the mean of the corresponding computed values. Fig. 54 also shows computations for co-ordinates.

Hints and Precautions.—(1) The station adjustment is usually, though not always, performed in the field (Problem 18, page 62). This is as it should be, for the observer is in much better position to judge the reliability of the angles at this time than he or anyone else could possibly be later, even though elaborate notes were taken. Other conditions being the same it is common practice to assume that the angular precision varies as the number of observations with the direction instrument or as the number of sets of observations with the repeating instrument, although such assumption is not in accord with the theory of errors. (2) In making the figure adjustment it will be considerably easier to deal with the discrepancies of each figure rather than with the angles themselves, since the former will be much the smaller. These discrepancies may be recorded within the corresponding figures on the sketch and should be prefixed by a positive or negative sign according as the correction must be added to or subtracted from the station angles. Thus in the polygon *FNOPE* (Fig. 52, page 234) the sum of the station angles (not shown) of each triangle will give a discrepancy for that triangle, and the sum of the interior station angles of the polygon will give a discrepancy for the polygon. By inspecting these small quantities the computer may readily estimate the correction that will least change the station angles and will at the same time satisfy the given conditions. (3) Below are given four equations which render the determination of the angle corrections of a quadrilateral very simple and which fulfill for each figure the geometric conditions previously cited:

$$V_1 = V_2 = \tfrac{1}{4}d_4 + \tfrac{1}{8}(d_1 - d_3)$$
$$V_3 = V_4 = \tfrac{1}{4}d_2 + \tfrac{1}{8}(d_1 - d_3)$$
$$V_5 = V_6 = \tfrac{1}{4}d_2 - \tfrac{1}{8}(d_1 - d_3)$$
$$V_7 = V_8 = \tfrac{1}{4}d_4 - \tfrac{1}{8}(d_1 - d_3)$$

In the above equations d_1, d_2, d_3, and d_4 are the discrepancies of the four *triangles* of which the quadrilateral is composed as numbered in *A B C D* (Fig. 52), while the corrections are V_1, V_2, ... etc., the subscripts referring to *angles* numbered as in *A B C D*. Corrections satisfying these geometric conditions may be determined more readily by using these equations than by inspecting the figure. (4) Do not use logarithmic tables with more places than are necessary. In computing lengths of the sides, a six-place table will generally be sufficient if the angles are carried to seconds. See Arts. 12 and 21, pages 8 and 17. (5) It should be noted that the procedure outlined makes appreciable mistakes impossible by providing a check by independent solution of every line, azimuth, and co-ordinate.

Practical Applications.—This method of adjusting the angles and computing the co-ordinates of a triangulation system is sufficiently exact for all surveys except those of the highest precision. It will always furnish sufficiently precise horizontal control for mapping. For very exact or extended triangulation systems such as those established by the U. S. Coast and Geodetic Survey, the Great Lakes Survey, and some of the larger cities, the methods of this problem will not give sufficiently precise results.

A more exact method of adjustment of angles and one in which more conditions must be fulfilled than are here stated is based upon the theory of *least squares*, but because of the intricate and lengthy computations it is not practicable except on such surveys as those just mentioned. In the quadrilateral for which computations are shown (Fig. 53, page 235) the maximum difference between the length of a side computed from angles determined by the exact "least squares" method and computed from angles determined by the four preceding equations occurs in GE and is 0.16 ft. in a length of 8627 ft. or about 1 in 50,000.

Since the measured angles are spherical angles, each triangle will contain more than 180°. The amount greater than 180° is termed the *spherical excess* and is about one second for each 75.5 square miles of area of triangle. It is clear that no correction for spherical excess will be necessary unless the triangles are very large, and then only in the most precise work.

TABLES

TABLE I.*—CORRECTION TO OBSERVED ALTITUDE OF THE SUN FOR REFRACTION AND PARALLAX

App't alt.	Temperature										App't alt.
	−10° C. +14° F.	−5° C. +23° F.	0° C. +32° F.	+5° C. +41° F.	+10° C. +50° F.	+15° C. +59° F.	+20° C. +68° F.	+25° C. +77° F.	+30° C. +86° F.	+35° C. +95° F.	
°	′	′	′	′	′	′	′	′	′	′	°
10	5.52	5.42	5.30	5.20	5.10	5.00	4.92	4.83	4.75	4.67	10
11	5.02	4.92	4.82	4.73	4.63	4.55	4.47	4.38	4.32	4.23	11
12	4.60	4.50	4.42	4.33	4.25	4.17	4.10	4.03	3.97	3.88	12
13	4.23	4.15	4.07	4.00	3.92	3.85	3.78	3.72	3.65	3.58	13
14	3.92	3.83	3.77	3.70	3.62	3.55	3.50	3.45	3.37	3.32	14
15	3.65	3.58	3.50	3.43	3.37	3.32	3.25	3.20	3.13	3.08	15
16	3.43	3.35	3.30	3.23	3.17	3.12	3.07	3.00	2.95	2.90	16
17	3.22	3.15	3.10	3.03	2.98	2.92	2.88	2.82	2.77	2.72	17
18	3.02	2.95	2.90	2.85	2.80	2.75	2.70	2.65	2.60	2.55	18
19	2.83	2.78	2.73	2.68	2.63	2.58	2.53	2.48	2.43	2.40	19
20	2.68	2.63	2.58	2.53	2.48	2.43	2.38	2.33	2.30	2.27	20
21	2.53	2.48	2.43	2.38	2.35	2.30	2.27	2.22	2.17	2.13	21
22	2.38	2.35	2.30	2.25	2.22	2.18	2.13	2.08	2.05	2.02	22
23	2.28	2.25	2.20	2.15	2.12	2.08	2.03	1.98	1.95	1.93	23
24	2.17	2.13	2.08	2.05	2.02	1.98	1.93	1.88	1.87	1.83	24
25	2.07	2.03	1.98	1.95	1.92	1.88	1.83	1.80	1.77	1.75	25
26	1.99	1.95	1.90	1.87	1.83	1.80	1.75	1.72	1.70	1.67	26
27	1.88	1.85	1.82	1.78	1.75	1.72	1.68	1.63	1.62	1.60	27
28	1.80	1.77	1.72	1.70	1.67	1.63	1.60	1.57	1.53	1.52	28
29	1.72	1.68	1.65	1.63	1.60	1.57	1.53	1.50	1.47	1.46	29
30	1.65	1.62	1.58	1.57	1.53	1.50	1.47	1.45	1.42	1.40	30
32	1.53	1.50	1.47	1.45	1.42	1.38	1.35	1.33	1.30	1.28	32
34	1.41	1.37	1.35	1.32	1.30	1.27	1.25	1.23	1.20	1.18	34
36	1.30	1.27	1.25	1.22	1.20	1.18	1.15	1.13	1.10	1.08	36
38	1.20	1.18	1.15	1.13	1.12	1.10	1.07	1.05	1.02	1.02	38
40	1.11	1.10	1.07	1.05	1.03	1.02	0.98	0.97	0.95	0.93	40
42	1.03	1.00	0.98	0.97	0.95	0.93	0.90	0.88	0.87	0.87	42
44	0.96	0.93	0.92	0.90	0.88	0.87	0.85	0.83	0.82	0.80	44
46	0.89	0.88	0.87	0.85	0.83	0.82	0.80	0.78	0.77	0.75	46
48	0.83	0.82	0.80	0.78	0.77	0.75	0.73	0.72	0.70	0.68	48
50	0.77	0.75	0.73	0.72	0.70	0.68	0.67	0.67	0.65	0.63	50
55	0.63	0.62	0.60	0.60	0.58	0.57	0.57	0.55	0.53	0.52	55
60	0.52	0.52	0.50	0.50	0.48	0.47	0.47	0.45	0.45	0.43	60
65	0.42	0.40	0.40	0.40	0.38	0.38	0.37	0.37	0.35	0.33	65
70	0.32	0.32	0.32	0.30	0.30	0.30	0.28	0.28	0.28	0.27	70
75	0.23	0.23	0.23	0.22	0.22	0.22	0.20	0.20	0.20	0.18	75
80	0.15	0.15	0.13	0.13	0.13	0.13	0.13	0.12	0.12	0.12	80
85	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.05	0.05	0.05	85
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90

* From *Principal Facts of Earth's Magnetism*, Coast and Geodetic Survey.

TABLE II.—CORRECTION TO OBSERVED ALTITUDE OF A STAR FOR REFRACTION

App't alt.	Temperature										App't alt.
	−10° C. +14° F.	−5° C. +23° F.	0° C. +32° F.	+5° C. +41° F.	+10° C. +50° F.	+15° C. +59° F.	+20° C. +68° F.	+25° C. +77° F.	+30° C. +86° F.	+35° C. +95° F.	
°	′	′	′	′	′	′	′	′	′	′	°
10	5.67	5.57	5.45	5.35	5.25	5.15	5.07	4.98	4.90	4.82	10
11	5.17	5.07	4.97	4.88	4.78	4.70	4.62	4.53	4.47	4.38	11
12	4.75	4.65	4.57	4.48	4.40	4.32	4.25	4.18	4.12	4.03	12
13	4.38	4.30	4.22	4.15	4.07	4.00	3.93	3.87	3.80	3.73	13
14	4.06	3.97	3.91	3.84	3.76	3.69	3.64	3.59	3.51	3.46	14
15	3.79	3.72	3.64	3.57	3.51	3.46	3.39	3.34	3.27	3.22	15
16	3.57	3.49	3.44	3.37	3.31	3.26	3.21	3.14	3.09	3.04	16
17	3.36	3.29	3.24	3.17	3.12	3.06	3.02	2.96	2.91	2.86	17
18	3.16	3.09	3.04	2.99	2.94	2.89	2.84	2.79	2.74	2.69	18
19	2.97	2.92	2.87	2.82	2.77	2.72	2.67	2.62	2.57	2.54	19
20	2.82	2.77	2.72	2.67	2.62	2.57	2.52	2.47	2.44	2.41	20
21	2.67	2.62	2.57	2.52	2.49	2.44	2.41	2.36	2.31	2.27	21
22	2.52	2.49	2.44	2.39	2.36	2.32	2.27	2.22	2.19	2.16	22
23	2.42	2.39	2.34	2.29	2.26	2.22	2.17	2.12	2.09	2.07	23
24	2.31	2.27	2.22	2.19	2.16	2.12	2.07	2.02	2.01	1.97	24
25	2.21	2.17	2.12	2.09	2.06	2.02	1.97	1.94	1.91	1.89	25
26	2.12	2.08	2.03	2.00	1.96	1.93	1.88	1.85	1.83	1.80	26
27	2.01	1.98	1.95	1.91	1.88	1.85	1.81	1.76	1.75	1.73	27
28	1.93	1.90	1.85	1.83	1.80	1.76	1.73	1.70	1.66	1.65	28
29	1.85	1.81	1.78	1.76	1.73	1.70	1.66	1.63	1.60	1.59	29
30	1.78	1.75	1.71	1.70	1.66	1.63	1.60	1.58	1.55	1.53	30
32	1.65	1.62	1.59	1.57	1.54	1.50	1.47	1.45	1.42	1.40	32
34	1.53	1.49	1.47	1.44	1.42	1.39	1.35	1.35	1.32	1.30	34
36	1.42	1.39	1.37	1.34	1.32	1.30	1.27	1.25	1.22	1.20	36
38	1.32	1.30	1.27	1.25	1.24	1.22	1.19	1.17	1.14	1.14	38
40	1.22	1.21	1.18	1.16	1.14	1.13	1.09	1.08	1.06	1.04	40
42	1.14	1.11	1.09	1.08	1.06	1.04	1.01	0.99	0.98	0.98	42
44	1.07	1.04	1.03	1.01	0.99	0.98	0.96	0.94	0.93	0.91	44
46	0.99	0.98	0.97	0.95	0.93	0.92	0.90	0.88	0.87	0.85	46
48	0.93	0.92	0.90	0.88	0.87	0.85	0.83	0.82	0.80	0.78	48
50	0.86	0.84	0.82	0.81	0.79	0.77	0.76	0.76	0.74	0.72	50
55	0.72	0.71	0.69	0.69	0.67	0.66	0.66	0.64	0.62	0.61	55
60	0.59	0.59	0.57	0.57	0.55	0.54	0.54	0.52	0.52	0.50	60
65	0.48	0.46	0.46	0.46	0.44	0.44	0.43	0.43	0.41	0.39	65
70	0.37	0.37	0.37	0.35	0.35	0.35	0.33	0.33	0.33	0.32	70
75	0.27	0.27	0.27	0.26	0.26	0.26	0.24	0.24	0.24	0.22	75
80	0.18	0.18	0.16	0.16	0.16	0.16	0.16	0.15	0.15	0.15	80
85	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.06	0.06	0.06	85
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90

Table III.—Refraction Corrections to be Applied to Apparent Declinations

(To be used with solar attachment)

January	Hour angle	Refraction correction lat. 40°	February	Hour angle	Refraction correction lat. 40°	March	Hour angle	Refraction correction lat. 40°	April	Hour angle	Refraction correction lat. 40°	May	Hour angle	Refraction correction lat. 40°	June	Hour angle	Refraction correction lat. 40°
		′ ″		See Jan. 30	′ ″			′ ″			′ ″			′ ″			′ ″
1	1	1 58	1			1	1	1 03	1	3	0 57	1	3	0 39	1	4	0 44
2	2	2 16	2	3	2 13	2	2	1 10	2	4	1 19	2	4	0 55	2	5	1 11
3	3	3 04	3	4	3 41	3	3	1 27	3	5	2 18	3	5	1 30	3		
4	4	6 23	4			4	4	2 06	4			4			4	1	0 19
5			5	1	1 26	5			5	1	0 39	5	1	0 26	5	2	0 23
	1	1 54		2	1 37		1	0 59		2	0 44		2	0 30		3	0 30
6	2	2 11	6	3	2 04	6	2	1 06	6	3	0 54	6	3	0 37	6	4	0 43
7	3	2 59	7	4	3 21	7	3	1 21	7	4	1 14	7	4	0 53	7	5	1 10
8	4	6 01	8			8	4	1 56	8	5	2 08	8	5	1 26	8		
9			9	1	1 21	9	5	4 04	9			9			9	1	0 18
10			10	2	1 31	10			10	1	0 36	10	1	0 25	10	2	0 22
	1	1 50		3	1 56		1	0 55		2	0 41		2	0 29		3	0 29
11	2	2 06	11	4	3 04	11	2	1 02	11	3	0 51	11	3	0 36	11	4	0 43
12	3	2 49	12			12	3	1 15	12	4	1 10	12	4	0 51	12	5	1 09
13	4	5 33	13			13	4	1 47	13	5	1 58	13	5	1 22	13		
14			14	1	1 16	14	5	3 34	14			14			14	1	0 18
15			15	2	1 25	15			15	1	0 34	15	1	0 23	15	2	0 22
	1	1 46		3	1 48		1	0 52		2	0 38		2	0 27		3	0 29
16	2	2 01	16	4	2 47	16	2	0 58	16	3	0 48	16	3	0 34	16	4	0 42
17	3	2 40	17	5	8 39	17	3	1 10	17	4	1 06	17	4	0 49	17	5	1 08
18	4	5 00	18			18	4	1 39	18	5	1 49	18	5	1 18	18		
19			19	1	1 12	19	5	3 08	19			19			19	1	0 18
20			20	2	1 20	20			20	1	0 32	20	1	0 22	20	2	0 22
	1	1 42		3	1 40		1	0 48		2	0 36		2	0 26		3	0 29
21	2	1 56	21	4	2 31	21	2	0 54	21	3	0 45	21	3	0 33	21	4	0 42
22	3	2 31	22	5	6 49	22	3	1 05	22	4	1 02	22	4	0 47	22	5	1 08
23	4	4 35	23			23	4	1 32	23	5	1 42	23	5	1 15	23		
24			24	1	1 07	24	5	2 51	24			24			24	1	0 18
25	1	1 37	25	2	1 15	25			25	1	0 30	25	1	0 21	25	2	0 22
	2	1 50		3	1 33		1	0 45		2	0 34		2	0 25		3	0 29
26	3	2 22	26	4	2 18	26	2	0 50	26	3	0 42	26	3	0 32	26	4	0 42
27	4	4 07	27	5	5 29	27	3	1 01	27	4	0 58	27	4	0 46	27	5	1 08
28			28			28	4	1 25	28	5	1 36	28	5	1 13	28		
29			29	See Mar. 1		29	5	2 34	29			29			29	1	0 18
30	1	1 32				30			30	1	0 28	30	1	0 20	30	2	0 22
31	2	1 44				31	1	0 42		2	0 32	31	2	0 24		3	0 29
							2	0 47					3	0 31			

For latitudes other than 40° multiply by latitude coefficient, Table III(a).

TABLE III (*Cont'd*).—REFRACTION CORRECTIONS TO BE APPLIED TO APPARENT DECLINATIONS

(*To be used with solar attachment*)

July	Hour angle	Refraction correction lat. 40°	August	Hour angle	Refraction correction lat. 40°	September	Hour angle	Refraction correction lat. 40°	October	Hour angle	Refraction correction lat. 40°	November	Hour angle	Refraction correction lat. 40°	December	Hour angle	Refraction correction lat. 40°
		′ ″			′ ″			′ ″			′ ″			′ ″			′ ″
1	4	0 43	1	5	1 22	1	1	0 39	1	1	0 56	1			1	1	1 54
2	5	1 09	2	—	—	2	2	0 44	2	2	1 06	2	1	1 26	2	2	2 11
3	—	—	3	1	0 26	3	3	0 54	3	3	1 21	3	2	1 37	3	3	2 59
4	1	0 19	4	2	0 30	4	4	1 14	4	4	1 56	4	3	2 04	4	4	6 01
5	2	0 23	5	3	0 37	5	5	2 08	5	5	4 04	5	4	3 21	5		
	3	0 30		4	0 53												
6	4	0 43	6	5	1 26	6	1	0 42	6	1	1 03	6	1	1 32	6	1	1 58
7	5	1 10	7	—	—	7	2	0 47	7	2	1 10	7	2	1 44	7	2	2 16
8	—	—	8	1	0 28	8	3	0 57	8	3	1 27	8	3	2 13	8	3	3 04
9	1	0 20	9	2	0 32	9	4	1 19	9	4	2 06	9	4	3 41	9	4	6 23
10	2	0 24	10	3	0 39	10	5	2 18	10	5	4 39	10			10		
	3	0 31		4	0 55												
11	4	0 44	11	5	1 30	11	1	0 45	11	1	1 07	11	1	1 37	11	1	2 00
12	5	1 11	12	—	—	12	2	0 50	12	2	1 15	12	2	1 50	12	2	2 19
13	—	—	13	1	0 30	13	3	1 01	13	3	1 33	13	3	2 22	13	3	3 09
14	1	0 21	14	2	0 34	14	4	1 25	14	4	2 18	14	4	4 07	14	4	6 43
15	2	0 25	15	3	0 42	15	5	2 34	15	5	5 29	15			15		
	3	0 32		4	0 58												
16	4	0 46	16	5	1 36	16	1	0 48	16	1	1 12	16	1	1 42	16	1	2 01
17	5	1 13	17	—	—	17	2	0 54	17	2	1 20	17	2	1 56	17	2	2 20
18	—	—	18	1	0 32	18	3	1 05	18	3	1 40	18	3	2 31	18	3	3 11
19	1	0 22	19	2	0 36	19	4	1 32	19	4	2 31	19	4	4 35	19	4	6 47
20	2	0 26	20	3	0 45	20	5	2 51	20	5	6 49	20			20		
	3	0 33		4	1 02												
21	4	0 47	21	5	1 42	21	1	0 52	21	1	1 16	21	1	1 46	21	1	2 01
22	5	1 15	22	—	—	22	2	0 58	22	2	1 25	22	2	2 01	22	2	2 20
23	—	—	23	1	0 34	23	3	1 10	23	3	1 48	23	3	2 40	23	3	3 11
24	1	0 23	24	2	0 38	24	4	1 39	24	4	2 47	24	4	4 59	24	4	6 49
25	2	0 27	25	3	0 48	25	5	3 08	25	5	8 39	25			25		
	3	0 34		4	1 06												
26	4	0 49	26	5	1 49	26	1	0 55	26	1	1 21	26	1	1 50	26	1	2 00
27	5	1 18	27	—	—	27	2	1 02	27	2	1 31	27	2	2 06	27	2	2 19
28	—	—	28	1	0 36	28	3	1 15	28	3	1 56	28	3	2 49	28	3	3 09
29	1	0 25	29	2	0 41	29	4	1 47	29	4	3 04	29	4	5 33	29	4	6 43
30	2	0 29	30	3	0 51	30	5	3 34	30	5	11 01	30			30		
31	3	0 36	31	4	1 10				31	See Nov. 1					31	See Jan. 1	
	4	0 51		5	1 58												

For latitudes other than 40° multiply by latitude coefficient, Table III(a).

TABLE III(a).—LATITUDE COEFFICIENTS

Latitude	Coefficient	Latitude	Coefficient	Latitude	Coefficient
15°	0.30	30°	0.65	45°	1.20
16	0.32	31	0.68	46	1.24
17	0.34	32	0.71	47	1.29
18	0.36	33	0.75	48	1.33
19	0.38	34	0.78	49	1.38
20	0.40	35	0.82	50	1.42
21	0.42	36	0.85	51	1.47
22	0.44	37	0.89	52	1.53
23	0.46	38	0.92	53	1.58
24	0.48	39	0.96	54	1.64
25	0.50	40	1.00	55	1.70
26	0.53	41	1.04	56	1.76
27	0.56	42	1.08	57	1.82
28	0.59	43	1.12	58	1.88
29	0.62	44	1.16	59	1.94

To obtain the refraction correction (to be applied to declination) for any other latitude than 40° multiply the refraction correction for latitude 40° (Table III) by the coefficient corresponding to the latitude of observation.

TABLE IV*.—LOCAL MEAN (ASTRONOMICAL) TIME OF THE CULMINATIONS AND ELONGATIONS OF POLARIS IN THE YEAR 1915

(Computed for latitude 40° north and longitude 90° or 6ʰ west of Greenwich)

Date	East elongation.	Upper culmination	West elongation.	Lower culmination.
1915	h m	h m	h m	h m
January 1	0 51.7	6 46.9	12 42.1	18 44.9
January 15	23 52.5	5 51.6	11 46.8	17 49.6
February 1	22 45.3	4 44.5	10 39.7	16 42.5
February 15	21 50.1	3 49.2	9 44.4	15 47.2
March 1	20 54.8	2 54.0	8 49.2	14 52.0
March 15	19 59.6	1 58.8	7 54.0	13 56.8
April 1	18 52.7	0 51.9	6 47.1	12 49.9
April 15	17 57.7	23 52.9	5 52.0	11 54.8
May 1	16 54.8	22 50.0	4 49.2	10 52.0
May 15	15 59.9	21 55.1	3 54.2	9 57.0
June 1	14 53.3	20 48.5	2 47.6	8 50.4
June 15	13 58.5	19 53.7	1 52.8	7 55.6
July 1	12 55.9	18 51.1	0 50.2	6 53.0
July 15	12 01.1	17 56.3	23 51.5	5 58.2
August 1	10 54.5	16 49.7	22 44.9	4 51.7
August 15	9 59.8	15 55.0	21 50.2	3 56.9
September 1	8 53.2	14 48.4	20 43.6	2 50.3
September 15	7 58.3	13 53.5	19 48.7	1 55.4
October 1	6 55.5	12 50.7	18 45.9	0 52.7
October 15	6 00.6	11 55.8	17 51.0	23 53.8
November 1	4 53.7	10 48.9	16 44.1	22 46.9
November 15	3 58.6	9 53.8	15 49.0	21 51.8
December 1	2 55.6	8 50.8	14 46.0	20 48.8
December 15	2 00.4	7 55.6	13 50.8	19 53.6

A. *To refer the above tabular quantities to years other than* 1915.

For year		m		For year		m	
1914	subtract	1.5		1922	add	3.1	
1916	add	1.6	up to Mar. 1	1923	add	4.5	
	subtract	2.3	on and after Mar. 1	1924	add	5.9	up to Mar. 1
1917	subtract	0.7			add	2.0	on and after Mar. 1
1918	add	0.9		1925	add	3.3	
1919	add	2.5		1926	add	4.6	
1920	add	4.0	up to Mar. 1	1927	add	5.9	
	add	0.1	on and after Mar. 1	1928	add	7.2	up to Mar. 1
1921	add	1.6			add	3.3	on and after Mar. 1

* Table IV was kindly furnished for this publication by Mr. O. H. Tittman, Superintendent of the U. S. Coast and Geodetic Survey.

TABLE IV.— *Concluded*

B. *To refer to any calendar day other than the first and fifteenth of each month* SUBTRACT *the quantities below from the tabular quantity for the* PRECEDING DATE.

Day of month	Minutes	No of days elapsed
2 or 16	3.9	1
3 17	7.8	2
4 18	11.8	3
5 19	15.7	4
6 20	19.6	5
7 21	23.5	6
8 22	27.4	7
9 23	31.4	8
10 24	35.3	9
11 25	39.2	10
12 26	43.1	11
13 27	47.0	12
14 28	51.0	13
29	54.9	14
30	58.8	15
31	62.7	16

C. *To refer the table to Standard time and to the civil or common method of reckoning:*

(*a*) ADD to the tabular quantities four minutes for every degree of longitude the place is west of the Standard meridian and SUBTRACT when the place is east of the Standard meridian.

(*b*) The astronomical day begins twelve hours after the civil day, *i.e.*, begins at noon on the civil day of the same date, and is reckoned from zero to twenty-four hours. Consequently an astronomical time less than twelve hours refers to the same civil day, whereas an astronomical time greater than twelve hours refers to the morning of the next civil day.

It will be noticed that for the tabular year two eastern elongations occur on January 14 and two western elongations on July 13. There are also two upper culminations on April 14 and two lower culminations on October 14. The lower culmination either follows or precedes the upper culmination by $11^h\ 58^m.0$.

D. *To refer to any other than the tabular latitude between the limits of* 10° *and* 50° *north:* ADD to the time of west elongation $0^m.10$ for every degree south of 40° and SUBTRACT from the time of west elongation $0^m.16$ for every degree north of 40°. Reverse these operations for correcting times of east elongation.

E. *To refer to any other than the tabular longitude:* ADD $0^m.16$ for each 15° east of the ninetieth meridian and SUBTRACT $0^m.16$ for each 15° west of the ninetieth meridian.

TABLE V.*—AZIMUTH OF POLARIS WHEN AT ELONGATION FOR ANY YEAR BETWEEN 1914 AND 1928

Latitude	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928
°	° '	° '	° '	° '	° '	° '	° '	° '	° '	° '	° '	° '	° '	° '	° '
10	1 10.3	1 10.0	1 09.6	1 09.3	1 09.0	1 08.7	1 08.4	1 08.1	1 07.8	1 07.4	1 07.2	1 06.8	1 06.5	1 06.2	1 05.9
11	10.5	10.2	09.9	09.6	09.2	08.9	08.6	08.3	08.0	07.7	07.4	07.0	06.7	06.4	06.1
12	10.8	10.4	10.1	09.8	09.5	09.2	08.9	08.6	08.2	07.9	07.6	07.3	07.0	06.7	06.4
13	11.0	10.7	10.4	10.1	09.8	09.4	09.1	08.8	08.5	08.2	07.8	07.6	07.2	06.9	06.6
14	11.3	11.0	10.7	10.4	10.0	09.7	09.4	09.1	08.8	08.5	08.2	07.8	07.5	07.2	06.9
15	11.6	11.3	11.0	10.7	10.4	10.0	09.7	09.4	09.1	08.8	08.5	08.1	07.8	07.5	07.2
16	12.0	11.7	11.4	11.0	10.7	10.4	10.1	09.8	09.4	09.1	08.8	08.5	08.2	07.8	07.5
17	12.4	12.0	11.7	11.4	11.1	10.8	10.4	10.1	09.8	09.5	09.2	08.8	08.5	08.2	07.9
18	12.8	12.4	12.1	11.8	11.5	11.1	10.7	10.5	10.2	09.8	09.5	09.2	08.9	08.6	08.2
19	13.2	12.8	12.5	12.2	11.9	11.6	11.2	10.9	10.6	10.2	09.9	09.6	09.3	09.0	08.6
20	13.6	13.3	13.0	12.7	12.3	12.0	11.7	11.4	11.0	10.7	10.4	10.0	09.7	09.4	09.1
21	14.1	13.8	13.5	13.1	12.8	12.5	12.2	11.8	11.5	11.2	10.8	10.5	10.2	09.8	09.5
22	14.6	14.3	14.0	13.6	13.3	13.0	12.6	12.3	12.0	11.6	11.3	11.0	10.6	10.3	10.0
23	15.2	14.8	14.5	14.2	13.8	13.5	13.2	12.8	12.5	12.2	11.8	11.5	11.2	10.8	10.5
24	15.8	15.4	15.1	14.7	14.4	14.1	13.7	13.4	13.0	12.7	12.4	12.0	11.7	11.4	11.0
25	16.4	16.0	15.7	15.3	15.0	14.7	14.3	14.0	13.6	13.3	13.0	12.6	12.3	11.9	11.6
26	17.0	16.6	16.3	16.0	15.6	15.3	14.9	14.7	14.2	13.9	13.6	13.2	12.9	12.5	12.2
27	17.7	17.3	17.0	16.6	16.3	15.9	15.6	15.2	14.9	14.6	14.2	13.9	13.5	13.2	12.8
28	18.4	18.0	17.7	17.3	17.0	16.6	16.3	15.9	15.6	15.2	14.9	14.6	14.2	13.8	13.5
29	19.1	18.8	18.4	18.1	17.7	17.4	17.0	16.6	16.3	16.0	15.6	15.2	14.9	14.6	14.2
30	19.9	19.6	19.2	18.8	18.5	18.1	17.8	17.4	17.0	16.7	16.4	16.0	15.6	15.3	14.9
31	20.7	20.4	20.0	19.7	19.3	18.9	18.6	18.2	17.9	17.5	17.2	16.8	16.4	16.1	15.7
32	21.6	21.2	20.9	20.5	20.1	19.8	19.4	19.1	18.7	18.3	18.0	17.6	17.2	16.9	16.5
33	22.5	22.1	21.8	21.4	21.0	20.7	20.3	19.9	19.6	19.2	18.8	18.5	18.1	17.8	17.4
34	1 23.5	1 23.1	1 22.7	1 22.4	1 22.0	1 21.6	1 21.2	1 20.9	1 20.5	1 20.1	1 19.8	1 19.4	1 19.0	1 18.6	1 18.3

TABLE V.*—AZIMUTH OF POLARIS WHEN AT ELONGATION FOR ANY YEAR BETWEEN 1914 AND 1928—*Concluded*

Latitude	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
35	1 24.5	1 24.1	1 23.7	1 23.3	1 23.0	1 22.6	1 22.2	1 21.8	1 21.5	1 21.1	1 20.7	1 20.4	1 20.0	1 19.6	1 19.2
36	25.5	25.2	24.8	24.4	24.0	23.6	23.3	22.9	22.5	22.1	21.7	21.4	21.0	20.6	20.2
37	26.7	26.3	25.9	25.3	25.1	24.7	24.3	24.0	23.6	23.2	22.8	22.4	22.0	21.6	21.3
38	27.8	27.4	27.0	26.6	26.2	25.9	25.5	25.1	24.7	24.3	23.9	23.5	23.2	22.8	22.4
39	29.0	28.6	28.2	27.8	27.5	27.1	26.7	26.3	25.8	25.5	25.1	24.7	24.3	23.9	23.5
40	30.3	29.9	29.5	29.1	28.7	28.3	27.9	27.5	27.1	26.7	26.3	25.9	25.5	25.1	24.7
41	31.7	31.3	30.9	30.4	30.0	29.6	29.1	28.8	28.4	28.0	27.6	27.2	26.8	26.4	26.0
42	33.1	32.7	32.3	31.9	31.5	31.0	30.6	30.2	29.8	29.4	29.0	28.6	28.2	27.8	27.3
43	34.6	34.2	33.8	33.4	32.9	32.5	32.1	31.8	31.2	30.8	30.4	30.0	29.6	29.1	28.7
44	36.2	35.8	35.3	34.9	34.5	34.1	33.6	33.2	32.8	32.4	31.9	31.5	31.1	30.6	30.2
45	37.8	37.4	37.0	36.6	36.1	35.7	35.3	34.8	34.4	34.0	33.5	33.1	32.6	32.2	31.8
46	39.6	39.2	38.7	38.3	37.8	37.4	37.0	36.5	36.1	35.6	35.2	34.8	34.3	33.9	33.4
47	41.5	41.0	40.6	40.1	39.7	39.2	38.8	38.3	37.9	37.4	37.0	36.5	36.1	35.6	35.2
48	43.4	43.0	42.5	42.0	41.6	41.1	40.7	40.2	39.8	39.3	38.8	38.4	37.9	37.4	37.0
49	45.5	45.0	44.5	44.1	43.6	43.1	42.7	42.2	41.7	41.3	40.8	40.3	39.9	39.4	38.9
50	1 47.7	1 47.2	1 46.7	1 46.2	1 45.7	1 45.3	1 44.8	1 44.3	1 43.8	1 43.4	1 42.9	1 42.4	1 41.9	1 41.4	1 41.0

* Table V was kindly furnished for this publication by Mr. O. H. Tittman, Superintendent of the U. S. Coast and Geodetic Survey.

The above table was computed with the mean declination of Polaris for each year. A more accurate result will be had by applying to the tabular values the following correction, which depends on the difference of the mean and the apparent place of the star. The deduced azimuth will, in general, be correct within 0′.3.

For middle of	Correction ′	For middle of	Correction ′	For middle of	Correction ′
January	−0.5	May	+0.1	September	−0.1
February	−0.4	June	+0.2	October	−0.4
March	−0.3	July	+0.2	November	−0.6
April	0.0	August	+0.1	December	−0.8

TABLE VI.*—TOTAL SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, ON THE YEARLY AVERAGE, AT PROMINENT PLACES IN NORTH AMERICA

[*A + sign indicates a deflection of the north-seeking end of the magnet toward the east, a − sign the contrary direction*]

Local mean time	1 Key West, Fla.	2 Los Angeles, Cal.	3 Washington, D. C.	4 Philadelphia, Pa.	5 Madison, Wis.	6 Toronto, Canada	7 Sitka, Alaska	8 Uglaamie, Point Barrow	9 Plover Point, Point Barrow	10 Fort Rae, Great Slave Lake	11 Kingua Fjord, Cumberland Sound	12 Fort Conger, Grinnell Land	Average values, stations 1 to 6, inclusive
	′	′	′	′	′	′	′	′	′	′	′	′	′
1 a.m.	+0.0	+0.0	+0.7	+0.6	+0.1	+0.6	+0.2	−12.8	− 8.0	−11.0	+11.7	+43.2	+0.35
2 a.m.	−0.0	+0.1	+0.7	+0.5	0.0	+0.5	+1.0	− 4.9	− 1.9	− 6.6	+15.8	+45.1	+0.05
3 a.m.	+0.1	+0.2	+0.9	+0.6	+0.2	+0.8	+1.4	+ 3.3	+ 3.6	+ 0.8	+18.0	+41.2	+0.07
4 a.m.	+0.2	+0.3	+1.2	+1.0	+0.5	+1.2	+2.0	+ 6.2	+10.9	+ 7.4	+19.1	+25.7	+0.75
5 a.m.	+0.4	+0.6	+1.7	+1.5	+1.0	+1.8	+2.9	+14.3	+16.6	+13.6	+19.3	+31.6	+1.19
6 a.m.	+1.0	+1.3	+2.1	+2.1	+1.4	+2.7	+4.2	+21.6	+19.3	+21.0	+20.1	+19.7	+1.79
7 a.m.	+2.1	+2.4	+2.8	+3.3	+2.6	+3.5	+5.3	+26.1	+27.1	+26.2	+19.9	+26.6	+2.80
8 a.m.	+2.5	+3.1	+3.2	+3.5	+3.2	+3.8	+6.0	+26.7	+27.0	+29.4	+17.4	+18.7	+3.24
9 a.m.	+2.2	+2.6	+2.3	+2.8	+3.0	+3.0	+5.3	+26.1	+19.9	+25.5	+10.8	+ 1.2	+2.67
10 a.m.	+1.1	+1.1	+0.9	+0.8	+1.7	−0.8	+3.0	+ 9.9	+ 9.3	+16.8	+ 3.7	−12.7	+1.09
11 a.m.	−0.2	−0.8	−1.3	−1.6	−0.7	−2.0	+0.6	+ 1.4	− 0.4	+ 8.0	+ 1.3	−21.4	−1.08
Noon	−1.4	−2.2	−3.2	−3.4	−2.5	−4.2	−2.1	− 5.9	− 8.2	− 0.9	− 9.0	−40.7	−2.80
1 p.m.	−2.1	−2.7	−4.3	−4.3	−3.5	−5.0	−3.2	− 7.3	−10.7	− 4.0	−15.1	−45.6	−3.63
2 p.m.	−2.2	−2.6	−4.3	−4.1	−3.5	−4.8	−4.2	− 7.7	− 9.8	− 8.1	−21.2	−49.2	−3.56
3 p.m.	−1.9	−2.0	−3.5	−3.1	−2.6	−3.8	−4.6	− 7.3	− 9.9	−10.6	−20.4	−45.8	−2.80
4 p.m.	−1.3	−1.1	−2.5	−2.2	−1.6	−2.5	−4.6	− 9.1	− 9.8	−11.3	−20.6	−53.7	−1.85
5 p.m.	−0.8	−0.5	−1.5	−1.0	−0.7	−1.3	−3.8	− 9.9	−10.2	−12.1	−23.6	−23.7	−0.95
6 p.m.	−0.4	−0.2	−0.8	−0.4	−0.2	−0.3	−3.2	− 9.9	− 9.7	−12.9	−19.4	−17.3	−0.36
7 p.m.	−0.2	−0.0	0.0	+0.0	+0.2	+0.2	−2.4	− 8.4	− 8.4	−12.5	−16.1	−27.2	+0.05
8 p.m.	+0.1	+0.1	+0.6	+0.8	+0.2	+0.7	−1.4	− 6.0	− 9.0	−11.0	−15.5	− 3.5	+0.44
9 p.m.	+0.2	+0.1	+1.0	+0.6	+0.6	+1.2	−0.8	− 8.1	− 7.5	−12.0	− 8.8	+ 3.5	+0.64
10 p.m.	+0.2	+0.1	+1.1	+1.2	+0.7	+1.3	−0.4	−10.9	− 7.9	−11.9	− 0.6	+22.4	+0.79
11 p.m.	+0.2	+0.1	+1.1	+0.7	+0.2	+1.2	−0.6	− 9.1	−11.5	−11.9	+ 3.9	−30.0	+0.60
Midnight.	+0.1	+0.0	+1.0	+0.6	+0.1	+0.8	−0.6	−13.4	−10.8	−12.0	+ 9.2	+32.6	+0.45
Range	4.7	5.8	7.5	7.8	6.7	8.8	10.6	40.1	38.6	41.4	43.7	98.8	6.9

* From Appendix No. 9, Coast and Geodetic Survey Report of 1890.

Table VII.*—Azimuth of Polaris at any Hour Angle

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′											Correction for 1′ increase in declination of Polaris	
	Latitude 10°	Latitude 11°	Latitude 12°	Latitude 13°	Latitude 14°	Latitude 15°	Latitude 16°	Latitude 17°	Latitude 18°	Latitude 19°	Latitude 20°	Latitude 10°	Latitude 20°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
0 15	0 04 36	0 04 37	0 04 38	0 04 39	0 04 41	0 04 42	0 04 43	0 04 45	0 04 47	0 04 48	0 04 50	− 4	− 4
0 30	0 09 11	0 09 13	0 09 15	0 09 17	0 09 20	0 09 23	0 09 25	0 09 29	0 09 32	0 09 36	0 09 39	− 8	− 8
0 45	0 13 43	0 13 46	0 13 49	0 13 53	0 13 57	0 14 01	0 14 05	0 14 10	0 14 15	0 14 20	0 14 26	−12	−13
1 00	0 18 12	0 18 16	0 18 20	0 18 25	0 18 30	0 18 35	0 18 41	0 18 47	0 18 54	0 19 01	0 19 09	−16	−17
1 15	0 22 36	0 22 41	0 22 46	0 22 52	0 22 58	0 23 05	0 23 12	0 23 20	0 23 28	0 23 37	0 23 46	−20	−21
1 30	0 26 54	0 27 00	0 27 06	0 27 13	0 27 21	0 27 29	0 27 37	0 27 46	0 27 56	0 28 07	0 28 18	−24	−25
1 45	0 31 05	0 31 12	0 31 19	0 31 27	0 31 36	0 31 45	0 31 55	0 32 06	0 32 17	0 32 29	0 32 42	−27	−29
2 00	0 35 09	0 35 16	0 35 24	0 35 33	0 35 43	0 35 53	0 36 04	0 36 16	0 36 29	0 36 43	0 36 57	−31	−32
2 15	0 39 03	0 39 11	0 39 20	0 39 30	0 39 41	0 39 52	0 40 04	0 40 18	0 40 32	0 40 47	0 41 03	−34	−36
2 30	0 42 47	0 42 56	0 43 06	0 43 17	0 43 28	0 43 41	0 43 54	0 44 09	0 44 24	0 44 40	0 44 58	−37	−39
2 45	0 46 19	0 46 29	0 46 40	0 46 52	0 47 04	0 47 18	0 47 32	0 47 48	0 48 04	0 48 22	0 48 41	−40	−42
3 00	0 49 40	0 49 51	0 50 02	0 50 15	0 50 28	0 50 42	0 50 58	0 51 15	0 51 33	0 51 52	0 52 12	−43	−45
3 15	0 52 48	0 52 59	0 53 11	0 53 24	0 53 39	0 53 54	0 54 11	0 54 28	0 54 47	0 55 07	0 55 29	−46	−48
3 30	0 55 43	0 55 54	0 56 07	0 56 21	0 56 36	0 56 52	0 57 09	0 57 28	0 57 47	0 58 09	0 58 31	−49	−51
3 45	0 58 22	0 58 34	0 58 48	0 59 02	0 59 18	0 59 35	0 59 53	1 00 12	1 00 33	1 00 55	1 01 18	−51	−53
4 00	1 00 47	1 01 00	1 01 13	1 01 28	1 01 44	1 02 02	1 02 21	1 02 41	1 03 02	1 03 25	1 03 49	−53	−55
4 15	1 02 56	1 03 09	1 03 23	1 03 38	1 03 55	1 04 13	1 04 33	1 04 53	1 05 15	1 05 39	1 06 04	−55	−57
4 30	1 04 49	1 05 02	1 05 17	1 05 33	1 05 50	1 06 08	1 06 28	1 06 49	1 07 12	1 07 36	1 08 02	−56	−59
4 45	1 06 25	1 06 39	1 06 53	1 07 10	1 07 27	1 07 46	1 08 06	1 08 28	1 08 51	1 09 15	1 09 42	−58	−61
5 00	1 07 44	1 07 58	1 08 13	1 08 29	1 08 47	1 09 06	1 09 26	1 09 48	1 10 12	1 10 37	1 11 03	−59	−62
5 15	1 08 46	1 08 59	1 09 15	1 09 31	1 09 49	1 10 08	1 10 29	1 10 51	1 11 15	1 11 40	1 12 07	−60	−62
5 30	1 09 30	1 09 43	1 09 59	1 10 15	1 10 33	1 10 52	1 11 13	1 11 35	1 11 59	1 12 25	1 12 52	−60	−63
5 45	1 09 56	1 10 09	1 10 25	1 10 41	1 10 59	1 11 18	1 11 39	1 12 01	1 12 25	1 12 51	1 13 18	−61	−63
6 00	1 10 04	1 10 17	1 10 32	1 10 49	1 11 07	1 11 26	1 11 47	1 12 09	1 12 33	1 12 58	1 13 26	−61	−64
6 15	1 09 54	1 10 07	1 10 22	1 10 39	1 10 55	1 11 15	1 11 36	1 11 58	1 12 22	1 12 47	1 13 14	−61	−63
6 30	1 09 26	1 09 39	1 09 54	1 10 10	1 10 28	1 10 46	1 11 07	1 11 29	1 11 52	1 12 17	1 12 44	−60	−63
6 45	1 08 40	1 08 53	1 09 08	1 09 23	1 09 41	1 09 59	1 10 19	1 10 41	1 11 04	1 11 29	1 11 55	−60	−62

* From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′											Correction for 1′ increase in declination of Polaris	
	Latitude 10°	Latitude 11°	Latitude 12°	Latitude 13°	Latitude 14°	Latitude 15°	Latitude 16°	Latitude 17°	Latitude 18°	Latitude 19°	Latitude 20°	Latitude 10°	Latitude 20°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
7 00	1 07 37	1 07 50	1 08 05	1 08 19	1 08 36	1 08 54	1 09 14	1 09 35	1 09 58	1 10 22	1 10 47	−59	−61
7 15	1 06 16	1 06 29	1 06 42	1 06 57	1 07 14	1 07 32	1 07 51	1 08 11	1 08 33	1 08 57	1 09 22	−58	−60
7 30	1 04 39	1 04 51	1 05 04	1 05 19	1 05 35	1 05 52	1 06 10	1 06 30	1 06 52	1 07 15	1 07 39	−56	−58
7 45	1 02 44	1 02 56	1 03 09	1 03 23	1 03 38	1 03 55	1 04 13	1 04 32	1 04 53	1 05 15	1 05 39	−54	−56
8 00	1 00 34	1 00 45	1 00 58	1 01 11	1 01 26	1 01 42	1 01 59	1 02 18	1 02 38	1 02 59	1 03 22	−52	−55
8 15	0 58 09	0 58 19	0 58 31	0 58 44	0 58 58	0 59 13	0 59 30	0 59 47	1 00 06	1 00 27	1 00 49	−50	−53
8 30	0 55 28	0 55 38	0 55 49	0 56 02	0 56 15	0 56 29	0 56 45	0 57 02	0 57 20	0 57 39	0 58 00	−48	−50
8 45	0 52 33	0 52 43	0 52 53	0 53 05	0 53 18	0 53 31	0 53 46	0 54 02	0 54 19	0 54 37	0 54 57	−45	−48
9 00	0 49 25	0 49 33	0 49 44	0 49 55	0 50 07	0 50 19	0 50 33	0 50 48	0 51 04	0 51 21	0 51 40	−42	−45
9 15	0 46 05	0 46 13	0 46 22	0 46 32	0 46 43	0 46 55	0 47 08	0 47 21	0 47 36	0 47 52	0 48 09	−40	−42
9 30	0 42 32	0 42 40	0 42 48	0 42 57	0 43 07	0 43 18	0 43 30	0 43 43	0 43 57	0 44 11	0 44 27	−37	−38
9 45	0 38 49	0 38 56	0 39 03	0 39 12	0 39 21	0 39 31	0 39 42	0 39 53	0 40 06	0 40 19	0 40 33	−34	−35
10 00	0 34 56	0 35 02	0 35 09	0 35 16	0 35 24	0 35 33	0 35 43	0 35 53	0 36 04	0 36 16	0 36 29	−31	−31
10 15	0 30 54	0 30 59	0 31 05	0 31 12	0 31 19	0 31 27	0 31 35	0 31 44	0 31 54	0 32 05	0 32 16	−28	−28
10 30	0 26 44	0 26 48	0 26 54	0 26 59	0 27 06	0 27 12	0 27 20	0 27 28	0 27 36	0 27 45	0 27 55	−24	−24
10 45	0 22 27	0 22 31	0 22 35	0 22 40	0 22 45	0 22 51	0 22 57	0 23 04	0 23 11	0 23 18	0 23 27	−20	−20
11 00	0 18 04	0 18 08	0 18 11	0 18 15	0 18 19	0 18 24	0 18 29	0 18 34	0 18 40	0 18 46	0 18 52	−16	−16
11 15	0 13 37	0 13 40	0 13 42	0 13 45	0 13 48	0 13 52	0 13 56	0 14 00	0 14 04	0 14 09	0 14 13	−12	−12
11 30	0 09 07	0 09 08	0 09 10	0 09 12	0 09 14	0 09 16	0 09 19	0 09 22	0 09 25	0 09 28	0 09 31	− 8	− 8
11 45	0 04 34	0 04 35	0 04 36	0 04 37	0 04 38	0 04 39	0 04 40	0 04 41	0 04 43	0 04 44	0 04 46	− 4	− 4
Elongation: Azimuth	1 10 04	1 10 18	1 10 33	1 10 49	1 11 07	1 11 26	1 11 47	1 12 09	1 12 33	1 12 58	1 13 26	−61	−64
	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	s	s
Hour angle	5 59 11	5 59 06	5 59 02	5 58 56	5 58 51	5 58 46	5 58 41	5 58 36	5 58 30	5 58 25	5 58 19	+ 2	+ 2

* From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′											Correction for 1′ increase in declination of Polaris	
	Latitude 20°	Latitude 21°	Latitude 22°	Latitude 23°	Latitude 24°	Latitude 25°	Latitude 26°	Latitude 27°	Latitude 28°	Latitude 29°	Latitude 30°	Latitude 20°	Latitude 30°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
0 15	0 04 50	0 04 52	0 04 55	0 04 57	0 04 59	0 05 02	0 05 04	0 05 07	0 05 10	0 05 13	0 05 17	− 4	− 5
0 30	0 09 39	0 09 43	0 09 48	0 09 52	0 09 57	0 10 02	0 10 07	0 10 13	0 10 19	0 10 25	0 10 31	− 8	− 9
0 45	0 14 26	0 14 32	0 14 38	0 14 45	0 14 52	0 15 00	0 15 07	0 15 16	0 15 25	0 15 34	0 15 44	−13	−14
1 00	0 19 09	0 19 16	0 19 25	0 19 34	0 19 43	0 19 53	0 20 04	0 20 15	0 20 26	0 20 39	0 20 51	−17	−18
1 15	0 23 46	0 23 56	0 24 07	0 24 18	0 24 29	0 24 42	0 24 55	0 25 08	0 25 23	0 25 38	0 25 54	−21	−23
1 30	0 28 18	0 28 29	0 28 42	0 28 55	0 29 09	0 29 24	0 29 39	0 29 55	0 30 12	0 30 30	0 30 49	−25	−27
1 45	0 32 42	0 32 55	0 33 10	0 33 25	0 33 41	0 33 58	0 34 16	0 34 34	0 34 54	0 35 15	0 35 37	−29	−31
2 00	0 36 57	0 37 12	0 37 29	0 37 46	0 38 04	0 38 23	0 38 43	0 39 04	0 39 26	0 39 50	0 40 15	−32	−35
2 15	0 41 03	0 41 20	0 41 38	0 41 57	0 42 17	0 42 38	0 43 00	0 43 24	0 43 49	0 44 14	0 44 42	−36	−39
2 30	0 44 58	0 45 16	0 45 36	0 45 57	0 46 19	0 46 42	0 47 06	0 47 32	0 47 59	0 48 27	0 48 57	−39	−43
2 45	0 48 41	0 49 01	0 49 22	0 49 45	0 50 08	0 50 33	0 51 00	0 51 27	0 51 57	0 52 27	0 53 00	−42	−46
3 00	0 52 12	0 52 33	0 52 56	0 53 20	0 53 45	0 54 12	0 54 40	0 55 10	0 55 41	0 56 14	0 56 48	−45	−50
3 15	0 55 29	0 55 51	0 56 15	0 56 41	0 57 08	0 57 36	0 58 06	0 58 37	0 59 10	0 59 45	1 00 22	−48	−53
3 30	0 58 31	0 58 55	0 59 20	0 59 47	1 00 15	1 00 45	1 01 16	1 01 49	1 02 24	1 03 01	1 03 40	−51	−56
3 45	1 01 18	1 01 43	1 02 09	1 02 37	1 03 07	1 03 38	1 04 11	1 04 46	1 05 22	1 06 00	1 06 41	−53	−58
4 00	1 03 49	1 04 15	1 04 43	1 05 12	1 05 42	1 06 15	1 06 49	1 07 25	1 08 02	1 08 42	1 09 24	−55	−61
4 15	1 06 04	1 06 31	1 06 59	1 07 29	1 08 01	1 08 34	1 09 09	1 09 46	1 10 25	1 11 06	1 11 49	−57	−63
4 30	1 08 02	1 08 29	1 08 58	1 09 29	1 10 01	1 10 35	1 11 12	1 11 50	1 12 30	1 13 12	1 13 56	−59	−64
4 45	1 09 42	1 10 10	1 10 39	1 11 10	1 11 44	1 12 19	1 12 55	1 13 34	1 14 15	1 14 58	1 15 44	−61	−66
5 00	1 11 03	1 11 32	1 12 02	1 12 34	1 13 07	1 13 43	1 14 20	1 15 00	1 15 41	1 16 25	1 17 11	−62	−68
5 15	1 12 07	1 12 36	1 13 06	1 13 38	1 14 12	1 14 48	1 15 26	1 16 06	1 16 48	1 17 32	1 18 19	−62	−69
5 30	1 12 52	1 13 21	1 13 51	1 14 24	1 14 58	1 15 34	1 16 13	1 16 53	1 17 35	1 18 20	1 19 07	−63	−69
5 45	1 13 18	1 13 47	1 14 18	1 14 50	1 15 25	1 16 01	1 16 39	1 17 19	1 18 02	1 18 47	1 19 34	−63	−69
6 00	1 13 26	1 13 54	1 14 25	1 14 57	1 15 32	1 16 08	1 16 46	1 17 26	1 18 09	1 18 53	1 19 41	−64	−70
6 15	1 13 14	1 13 43	1 14 13	1 14 45	1 15 19	1 15 55	1 16 33	1 17 13	1 17 55	1 18 40	1 19 26	−63	−69
6 30	1 12 44	1 13 12	1 13 42	1 14 14	1 14 48	1 15 23	1 16 01	1 16 40	1 17 22	1 18 06	1 18 52	−63	−68
6 45	1 11 55	1 12 23	1 12 52	1 13 24	1 13 57	1 14 32	1 15 09	1 15 48	1 16 29	1 17 12	1 17 58	−62	−67

* From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′											Correction for 1′ increase in declination of Polaris	
	Latitude 20°	Latitude 21°	Latitude 22°	Latitude 23°	Latitude 24°	Latitude 25°	Latitude 26°	Latitude 27°	Latitude 28°	Latitude 29°	Latitude 30°	Latitude 20°	Latitude 30°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
7 00	1 10 47	1 11 15	1 11 44	1 12 15	1 12 47	1 13 22	1 13 58	1 14 36	1 15 16	1 15 59	1 16 44	−61	−66
7 15	1 09 22	1 09 49	1 10 17	1 10 47	1 11 19	1 11 53	1 12 28	1 13 05	1 13 45	1 14 26	1 15 10	−60	−65
7 30	1 07 39	1 08 05	1 08 32	1 09 02	1 09 33	1 10 05	1 10 40	1 11 16	1 11 54	1 12 35	1 13 17	−58	−64
7 45	1 05 39	1 06 04	1 06 30	1 06 59	1 07 28	1 08 00	1 08 33	1 09 08	1 09 46	1 10 24	1 11 06	−56	−62
8 00	1 03 22	1 03 46	1 04 11	1 04 38	1 05 07	1 05 38	1 06 10	1 06 43	1 07 19	1 07 57	1 08 36	−55	−60
8 15	1 00 49	1 01 11	1 01 36	1 02 02	1 02 30	1 02 59	1 03 29	1 04 02	1 04 36	1 05 12	1 05 49	−53	−57
8 30	0 58 00	0 58 22	0 58 45	0 59 10	0 59 36	1 00 04	1 00 33	1 01 04	1 01 36	1 02 10	1 02 46	−50	−54
8 45	0 54 57	0 55 17	0 55 40	0 56 03	0 56 28	0 56 54	0 57 22	0 57 51	0 58 21	0 58 53	0 59 27	−48	−51
9 00	0 51 40	0 51 59	0 52 20	0 52 41	0 53 04	0 53 29	0 53 55	0 54 22	0 54 51	0 55 21	0 55 53	−45	−48
9 15	0 48 09	0 48 27	0 48 46	0 49 07	0 49 28	0 49 51	0 50 15	0 50 40	0 51 07	0 51 35	0 52 05	−42	−45
9 30	0 44 27	0 44 43	0 45 01	0 45 20	0 45 40	0 46 01	0 46 22	0 46 46	0 47 11	0 47 37	0 48 04	−38	−42
9 45	0 40 33	0 40 48	0 41 04	0 41 21	0 41 39	0 41 58	0 42 19	0 42 40	0 43 02	0 43 26	0 43 51	−35	−38
10 00	0 36 29	0 36 43	0 36 57	0 37 12	0 37 29	0 37 46	0 38 04	0 38 23	0 38 43	0 39 04	0 39 27	−31	−34
10 15	0 32 16	0 32 28	0 32 41	0 32 54	0 33 08	0 33 24	0 33 40	0 33 57	0 34 14	0 34 34	0 34 53	−28	−30
10 30	0 27 55	0 28 05	0 28 16	0 28 28	0 28 40	0 28 53	0 29 07	0 29 22	0 29 37	0 29 53	0 30 10	−24	−26
10 45	0 23 27	0 23 35	0 23 44	0 23 54	0 24 05	0 24 16	0 24 27	0 24 39	0 24 52	0 25 06	0 25 20	−20	−22
11 00	0 18 52	0 18 59	0 19 07	0 19 15	0 19 23	0 19 32	0 19 41	0 19 51	0 20 01	0 20 12	0 20 24	−16	−18
11 15	0 14 13	0 14 19	0 14 24	0 14 30	0 14 37	0 14 43	0 14 50	0 14 58	0 15 05	0 15 14	0 15 22	−12	−13
11 30	0 09 31	0 09 34	0 09 38	0 09 42	0 09 46	0 09 51	0 09 56	0 10 01	0 10 06	0 10 11	0 10 17	− 8	− 9
11 45	0 04 46	0 04 48	0 04 50	0 04 52	0 04 54	0 04 56	0 04 58	0 05 00	0 05 03	0 05 06	0 05 09	− 4	− 4
Elongation: Azimuth	1 13 26	1 13 55	1 14 25	1 14 58	1 15 32	1 16 08	1 16 46	1 17 27	1 18 09	1 18 54	1 19 41	−64	−69
	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	s	s
Hour angle	5 58 19	5 58 14	5 58 08	5 58 03	5 57 57	5 57 51	5 57 45	5 57 39	5 57 33	5 57 27	5 57 21	+ 2	+ 2

* From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′											Correction for 1′ increase in declination of Polaris	
	Latitude 30°	Latitude 31°	Latitude 32°	Latitude 33°	Latitude 34°	Latitude 35°	Latitude 36°	Latitude 37°	Latitude 38°	Latitude 39°	Latitude 40°	Latitude 30°	Latitude 40°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
0 15	0 05 17	0 05 20	0 05 23	0 05 27	0 05 31	0 05 35	0 05 40	0 05 44	0 05 49	0 05 54	0 06 00	− 5	− 5
0 30	0 10 31	0 10 38	0 10 45	0 10 53	0 11 01	0 11 09	0 11 18	0 11 27	0 11 37	0 11 47	0 11 57	− 9	−10
0 45	0 15 44	0 15 54	0 16 04	0 16 16	0 16 27	0 16 40	0 16 53	0 17 07	0 17 21	0 17 36	0 17 52	−14	−16
1 00	0 20 51	0 21 05	0 21 19	0 21 34	0 21 50	0 22 06	0 22 24	0 22 42	0 23 01	0 23 21	0 23 42	−18	−21
1 15	0 25 54	0 26 11	0 26 28	0 26 47	0 27 06	0 27 27	0 27 48	0 28 11	0 28 34	0 28 59	0 29 26	−23	−26
1 30	0 30 49	0 31 09	0 31 30	0 31 52	0 32 15	0 32 40	0 33 05	0 33 32	0 34 00	0 34 30	0 35 01	−27	−31
1 45	0 35 37	0 36 00	0 36 24	0 36 49	0 37 16	0 37 44	0 38 14	0 38 44	0 39 17	0 39 51	0 40 27	−31	−36
2 00	0 40 15	0 40 41	0 41 08	0 41 37	0 42 07	0 42 38	0 43 12	0 43 47	0 44 23	0 45 02	0 45 42	−35	−40
2 15	0 44 42	0 45 11	0 45 41	0 46 13	0 46 46	0 47 21	0 47 58	0 48 37	0 49 18	0 50 00	0 50 45	−39	−45
2 30	0 48 57	0 49 29	0 50 02	0 50 37	0 51 13	0 51 52	0 52 32	0 53 14	0 53 59	0 54 46	0 55 35	−43	−49
2 45	0 53 00	0 53 34	0 54 10	0 54 47	0 55 27	0 56 08	0 56 52	0 57 37	0 58 25	0 59 16	1 00 09	−46	−53
3 00	0 56 48	0 57 25	0 58 03	0 58 43	0 59 25	1 00 10	1 00 56	1 01 45	1 02 37	1 03 31	1 04 28	−50	−57
3 15	1 00 22	1 01 01	1 01 41	1 02 24	1 03 08	1 03 55	1 04 45	1 05 37	1 06 31	1 07 29	1 08 29	−53	−60
3 30	1 03 40	1 04 20	1 05 03	1 05 48	1 06 35	1 07 24	1 08 16	1 09 11	1 10 08	1 11 09	1 12 12	−56	−63
3 45	1 06 41	1 07 23	1 08 08	1 08 54	1 09 44	1 10 35	1 11 30	1 12 27	1 13 27	1 14 30	1 15 36	−58	−66
4 00	1 09 24	1 10 08	1 10 54	1 11 43	1 12 34	1 13 28	1 14 24	1 15 23	1 16 26	1 17 31	1 18 40	−61	−69
4 15	1 11 49	1 12 35	1 13 23	1 14 13	1 15 06	1 16 01	1 16 59	1 18 00	1 19 05	1 20 12	1 21 23	−63	−72
4 30	1 13 56	1 14 43	1 15 32	1 16 23	1 17 18	1 18 14	1 19 14	1 20 17	1 21 23	1 22 32	1 23 45	−64	−74
4 45	1 15 44	1 16 31	1 17 21	1 18 14	1 19 09	1 20 07	1 21 08	1 22 12	1 23 20	1 24 31	1 25 45	−66	−75
5 00	1 17 11	1 18 00	1 18 51	1 19 44	1 20 40	1 21 39	1 22 41	1 23 46	1 24 55	1 26 07	1 27 23	−68	−76
5 15	1 18 19	1 19 08	1 19 59	1 20 54	1 21 50	1 22 50	1 23 53	1 24 59	1 26 08	1 27 21	1 28 38	−69	−77
5 30	1 19 07	1 19 56	1 20 48	1 21 42	1 22 40	1 23 40	1 24 43	1 25 49	1 26 59	1 28 12	1 29 30	−69	−78
5 45	1 19 34	1 20 23	1 21 15	1 22 10	1 23 07	1 24 08	1 25 11	1 26 17	1 27 27	1 28 41	1 29 58	−69	−78
6 00	1 19 41	1 20 30	1 21 22	1 22 16	1 23 13	1 24 14	1 25 17	1 26 23	1 27 33	1 28 47	1 30 04	−70	−78
6 15	1 19 26	1 20 15	1 21 07	1 22 01	1 22 58	1 23 58	1 25 01	1 26 07	1 27 17	1 28 30	1 29 46	−69	−78
6 30	1 18 52	1 19 41	1 20 32	1 21 26	1 22 22	1 23 21	1 24 24	1 25 29	1 26 38	1 27 50	1 29 06	−68	−77
6 45	1 17 58	1 18 46	1 19 36	1 20 29	1 21 25	1 22 23	1 23 24	1 24 29	1 25 37	1 26 48	1 28 03	−67	−76

* From Appendix No. 10, Coast ahd Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′											Correction for 1′ increase in declination of Polaris	
	Latitude 30°	Latitude 31°	Latitude 32°	Latitude 33°	Latitude 34°	Latitude 35°	Latitude 36°	Latitude 37°	Latitude 38°	Latitude 39°	Latitude 40°	Latitude 30°	Latitude 40°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
7 00	1 16 44	1 17 31	1 18 20	1 19 12	1 20 06	1 21 04	1 22 04	1 23 07	1 24 14	1 25 24	1 26 37	−66	−75
7 15	1 15 10	1 15 56	1 16 44	1 17 35	1 18 28	1 19 24	1 20 23	1 21 25	1 22 30	1 23 38	1 24 50	−65	−73
7 30	1 13 17	1 14 02	1 14 49	1 15 38	1 16 30	1 17 24	1 18 21	1 19 21	1 20 25	1 21 31	1 22 41	−64	−72
7 45	1 11 06	1 11 49	1 12 34	1 13 22	1 14 12	1 15 05	1 16 00	1 16 58	1 17 59	1 19 04	1 20 11	−62	−69
8 00	1 08 36	1 09 18	1 10 01	1 10 47	1 11 36	1 12 26	1 13 20	1 14 16	1 15 14	1 16 16	1 17 21	−60	−66
8 15	1 05 49	1 06 29	1 07 11	1 07 55	1 08 41	1 09 30	1 10 21	1 11 14	1 12 11	1 13 10	1 14 12	−57	−64
8 30	1 02 46	1 03 24	1 04 04	1 04 46	1 05 29	1 06 16	1 07 04	1 07 55	1 08 49	1 09 45	1 10 44	−54	−61
8 45	0 59 27	1 00 03	1 00 40	1 01 20	1 02 01	1 02 45	1 03 31	1 04 19	1 05 10	1 06 03	1 06 59	−51	−58
9 00	0 55 53	0 56 26	0 57 02	0 57 39	0 58 18	0 58 59	0 59 43	1 00 27	1 01 14	1 02 04	1 02 57	−48	−54
9 15	0 52 05	0 52 36	0 53 09	0 53 43	0 54 20	0 54 58	0 55 38	0 56 20	0 57 04	0 57 50	0 58 39	−45	−50
9 30	0 48 04	0 48 33	0 49 03	0 49 35	0 50 08	0 50 43	0 51 20	0 51 58	0 52 39	0 53 22	0 54 07	−42	−46
9 45	0 43 51	0 44 17	0 44 44	0 45 13	0 45 44	0 46 16	0 46 49	0 47 24	0 48 01	0 48 40	0 49 21	−38	−42
10 00	0 39 27	0 39 50	0 40 15	0 40 41	0 41 08	0 41 37	0 42 07	0 42 39	0 43 12	0 43 47	0 44 24	−34	−38
10 15	0 34 53	0 35 14	0 35 35	0 35 58	0 36 22	0 36 48	0 37 14	0 37 42	0 38 12	0 38 43	0 39 15	−30	−34
10 30	0 30 10	0 30 28	0 30 47	0 31 07	0 31 28	0 31 50	0 32 12	0 32 37	0 33 02	0 33 29	0 33 57	−26	−29
10 45	0 25 20	0 25 35	0 25 51	0 26 08	0 26 25	0 26 43	0 27 03	0 27 23	0 27 44	0 28 07	0 28 30	−22	−24
11 00	0 20 24	0 20 36	0 20 49	0 21 02	0 21 16	0 21 31	0 21 46	0 22 03	0 22 20	0 22 38	0 22 57	−18	−20
11 15	0 15 22	0 15 31	0 15 41	0 15 51	0 16 02	0 16 13	0 16 24	0 16 37	0 16 50	0 17 03	0 17 17	−13	−15
11 30	0 10 17	0 10 23	0 10 29	0 10 36	0 10 43	0 10 51	0 10 58	0 11 07	0 11 15	0 11 24	0 11 34	− 9	−10
11 45	0 05 09	0 05 12	0 05 15	0 05 19	0 05 22	0 05 26	0 05 30	0 05 34	0 05 38	0 05 43	0 05 47	− 4	− 5
Elongation: Azimuth.	1 19 41	1 20 30	1 21 22	1 22 17	1 23 14	1 24 14	1 25 18	1 26 24	1 27 34	1 28 48	1 30 05	−69	−78
	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	s	s
Hour angle.	5 57 21	5 57 14	5 57 08	5 57 01	5 56 54	5 56 47	5 56 39	5 56 32	5 56 24	5 56 16	5 56 08	+ 2	+ 3

* From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′											Correction for 1′ increase in declination of Polaris	
	Latitude 40°	Latitude 41°	Latitude 42°	Latitude 43°	Latitude 44°	Latitude 45°	Latitude 46°	Latitude 47°	Latitude 48°	Latitude 49°	Latitude 50°	Latitude 40°	Latitude 50°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
0 15	0 06 00	0 06 05	0 06 11	0 06 17	0 06 24	0 06 31	0 06 38	0 06 46	0 06 54	0 07 03	0 07 12	−5	−6
0 30	0 11 57	0 12 09	0 12 21	0 12 33	0 12 46	0 13 00	0 13 14	0 13 30	0 13 46	0 14 03	0 14 21	−10	−13
0 45	0 17 52	0 18 09	0 18 27	0 18 45	0 19 05	0 19 25	0 19 47	0 20 10	0 20 34	0 21 00	0 21 27	−16	−19
1 00	0 23 42	0 24 04	0 24 28	0 24 52	0 25 18	0 25 45	0 26 14	0 26 45	0 27 17	0 27 51	0 28 27	−21	−25
1 15	0 29 26	0 29 53	0 30 22	0 30 53	0 31 25	0 31 59	0 32 34	0 33 12	0 33 52	0 34 34	0 35 18	−26	−32
1 30	0 35 01	0 35 34	0 36 08	0 36 45	0 37 23	0 38 03	0 38 46	0 39 30	0 40 18	0 41 08	0 42 01	−31	−38
1 45	0 40 27	0 41 05	0 41 45	0 42 27	0 43 11	0 43 57	0 44 46	0 45 38	0 46 33	0 47 30	0 48 31	−36	−43
2 00	0 45 42	0 46 25	0 47 10	0 47 57	0 48 47	0 49 39	0 50 35	0 51 33	0 52 35	0 53 40	0 54 49	−40	−49
2 15	0 50 45	0 51 33	0 52 22	0 53 15	0 54 10	0 55 08	0 56 09	0 57 14	0 58 22	0 59 35	1 00 51	−45	−54
2 30	0 55 35	0 56 26	0 57 21	0 58 18	0 59 18	1 00 22	1 01 29	1 02 40	1 03 54	1 05 13	1 06 37	−49	−59
2 45	1 00 09	1 01 05	1 02 04	1 03 06	1 04 11	1 05 20	1 06 32	1 07 48	1 09 09	1 10 34	1 12 04	−53	−64
3 00	1 04 28	1 05 28	1 06 30	1 07 36	1 08 46	1 10 00	1 11 17	1 12 39	1 14 05	1 15 36	1 17 12	−57	−68
3 15	1 08 29	1 09 32	1 10 39	1 11 49	1 13 03	1 14 21	1 15 43	1 17 10	1 18 41	1 20 18	1 22 00	−60	−72
3 30	1 12 12	1 13 19	1 14 29	1 15 43	1 17 00	1 18 22	1 19 49	1 21 20	1 22 56	1 24 37	1 26 25	−63	−76
3 45	1 15 36	1 16 46	1 17 59	1 19 16	1 20 37	1 22 03	1 23 33	1 25 08	1 26 49	1 28 35	1 30 27	−66	−80
4 00	1 18 40	1 19 52	1 21 08	1 22 28	1 23 53	1 25 21	1 26 54	1 28 34	1 30 18	1 32 08	1 34 05	−69	−83
4 15	1 21 23	1 22 38	1 23 56	1 25 19	1 26 46	1 28 17	1 29 54	1 31 33	1 33 24	1 35 17	1 37 17	−72	−86
4 30	1 23 45	1 25 02	1 26 22	1 27 47	1 29 16	1 30 50	1 32 30	1 34 14	1 36 06	1 38 01	1 40 04	−74	−88
4 45	1 25 45	1 27 03	1 28 26	1 29 52	1 31 23	1 32 59	1 34 41	1 36 28	1 38 20	1 40 19	1 42 25	−75	−90
5 00	1 27 23	1 28 42	1 30 06	1 31 34	1 33 06	1 34 44	1 36 27	1 38 16	1 40 10	1 42 11	1 44 19	−76	−91
5 15	1 28 38	1 29 58	1 31 23	1 32 52	1 34 25	1 36 04	1 37 48	1 39 38	1 41 34	1 43 36	1 45 45	−77	−92
5 30	1 29 30	1 30 50	1 32 16	1 33 45	1 35 20	1 36 59	1 38 44	1 40 34	1 42 31	1 44 34	1 46 44	−78	−93
5 45	1 29 58	1 31 20	1 32 45	1 34 15	1 35 50	1 37 29	1 39 14	1 41 05	1 43 02	1 45 05	1 47 16	−78	−94
6 00	1 30 04	1 31 25	1 32 50	1 34 20	1 35 55	1 37 34	1 39 19	1 41 09	1 43 06	1 45 09	1 47 19	−78	−93
6 15	1 29 46	1 31 07	1 32 32	1 34 01	1 35 35	1 37 14	1 38 58	1 40 48	1 42 44	1 44 46	1 46 56	−78	−93
6 30	1 29 06	1 30 26	1 31 50	1 33 18	1 34 51	1 36 29	1 38 12	1 40 01	1 41 56	1 43 57	1 46 04	−77	−92
6 45	1 28 03	1 29 20	1 30 44	1 32 11	1 33 43	1 35 19	1 37 01	1 38 48	1 40 41	1 42 40	1 44 46	−76	−91

* From Appendix No. 9, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Concluded*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′											Correction for 1′ increase in declination of polaris	
	Latitude 40°	Latitude 41°	Latitude 42°	Latitude 43°	Latitude 44°	Latitude 45°	Latitude 46°	Latitude 47°	Latitude 48°	Latitude 49°	Latitude 50°	Latitude 40°	Latitude 50°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
7 00	1 26 37	1 27 54	1 29 15	1 30 41	1 32 11	1 33 45	1 35 25	1 37 10	1 39 01	1 40 58	1 43 02	−75	−89
7 15	1 24 50	1 26 05	1 27 24	1 28 48	1 30 16	1 31 48	1 33 25	1 35 08	1 36 56	1 38 51	1 40 51	−73	−87
7 30	1 22 41	1 23 54	1 25 11	1 26 32	1 27 58	1 29 27	1 31 02	1 32 42	1 34 27	1 36 18	1 38 16	−72	−85
7 45	1 20 11	1 21 22	1 22 36	1 23 55	1 25 17	1 26 44	1 28 16	1 29 52	1 31 34	1 33 22	1 35 15	−69	−82
8 00	1 17 21	1 18 29	1 19 41	1 20 57	1 22 16	1 23 40	1 25 08	1 26 41	1 28 19	1 30 02	1 31 51	−66	−79
8 15	1 14 12	1 15 17	1 16 26	1 17 38	1 18 54	1 20 14	1 21 39	1 23 07	1 24 41	1 26 20	1 28 05	−64	−76
8 30	1 10 44	1 11 46	1 12 52	1 14 00	1 15 12	1 16 29	1 17 49	1 19 14	1 20 43	1 22 17	1 23 56	−61	−72
8 45	1 06 59	1 07 57	1 08 59	1 10 04	1 11 12	1 12 23	1 13 40	1 15 00	1 16 24	1 17 53	1 19 27	−58	−68
9 00	1 02 57	1 03 52	1 04 50	1 05 50	1 06 55	1 08 02	1 09 13	1 10 28	1 11 47	1 13 10	1 14 38	−54	−64
9 15	0 58 39	0 59 30	1 00 24	1 01 21	1 02 20	1 03 23	1 04 29	1 05 39	1 06 52	1 08 10	1 09 32	−50	−59
9 30	0 54 07	0 54 54	0 55 44	0 56 36	0 57 31	0 58 28	0 59 29	1 00 34	1 01 41	1 02 53	1 04 08	−46	−55
9 45	0 49 21	0 50 04	0 50 49	0 51 37	0 52 27	0 53 20	0 54 15	0 55 13	0 56 15	0 57 20	0 58 29	−42	−50
10 00	0 44 24	0 45 02	0 45 43	0 46 25	0 47 10	0 47 58	0 48 47	0 49 40	0 50 35	0 51 34	0 52 35	−38	−45
10 15	0 39 15	0 39 49	0 40 25	0 41 03	0 41 42	0 42 24	0 43 08	0 43 54	0 44 43	0 45 35	0 46 29	−34	−40
10 30	0 33 57	0 34 26	0 34 57	0 35 30	0 36 04	0 36 40	0 37 18	0 37 58	0 38 40	0 39 24	0 40 12	−29	−34
10 45	0 28 30	0 28 55	0 29 21	0 29 48	0 30 17	0 30 47	0 31 19	0 31 53	0 32 28	0 33 05	0 33 45	−24	−29
11 00	0 22 57	0 23 16	0 23 37	0 23 59	0 24 22	0 24 47	0 25 12	0 25 39	0 26 08	0 26 38	0 27 09	−20	−23
11 15	0 17 17	0 17 32	0 17 48	0 18 05	0 18 22	0 18 40	0 19 00	0 19 20	0 19 42	0 20 04	0 20 28	−15	−18
11 30	0 11 34	0 11 44	0 11 54	0 12 06	0 12 17	0 12 29	0 12 42	0 12 56	0 13 10	0 13 25	0 13 43	−10	−12
11 45	0 05 47	0 05 52	0 05 58	0 06 04	0 06 09	0 06 15	0 06 22	0 06 29	0 06 36	0 06 44	0 06 51	− 5	− 6
Elongation: Azimuth.	1 30 05	1 31 26	1 32 51	1 34 21	1 35 56	1 37 35	1 39 20	1 41 11	1 43 08	1 45 11	1 47 21	−78	−93
	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	s	h m s
Hour angle.	5 56 08	5 56 00	5 55 51	5 55 43	5 55 33	5 55 24	5 55 14	5 55 04	5 54 53	5 54 42	5 54 31	+ 3	+ 3

* From Appendix No. 9, Coast and Geodetic Survey Report of 1895.

TABLE VIII.*—POLAR DISTANCE OF POLARIS FOR THE BEGINNING OF EACH YEAR, 1914–1928

Year	Mean polar distance	Year	Mean polar distance
	° ′ ″		° ′ ″
1914	1 09 12.1	1921	1 07 02.5
1915	1 08 53.5	1922	1 06 44.1
1916	1 08 35.0	1923	1 06 25.7
1917	1 08 16.5	1924	1 06 07.3
1918	1 07 57.9	1925	1 05 48.9
1919	1 07 39.5	1926	1 05 30.5
1920	1 07 21.0	1927	1 05 12.2
		1928	1 04 53.8

Mean Polar Distances of Polaris are found by subtracting from 90° the corresponding *Mean* declinations. For precise observations the *Apparent* declination (to be found in the Nautical Almanac) should be used since the variation between Mean and Apparent Declinations is constantly changing, and has as a maximum value about ½′.

* From data furnished by the Superintendent of the U. S. Coast and Geodetic Survey for this book.

TABLE IX.[1] **HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS**

	0°		1°		2°		3°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	0.99	99.93	2.73	99.80	4.48	99.62	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
c = 0.75	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
c = 1.00	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
c = 1.25	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

[1] From "Theory of Practice of Surveying," by Prof. J. B. Johnson, New York; John Wiley & Sons.

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS.—*Continued*

Minutes	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
c = 0.75	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
c = 1.00	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
c = 1.25	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	8°		9°		10°		11°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
c = 0.75	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
c = 1.00	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
c = 1.25	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	12°		13°		14°		15°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
c = 0.75 .	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
c = 1.00 .	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
c = 1.25 .	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	16°		17°		18°		19°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
c = 0.75	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
c = 1.00	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33
c = 1.25	1.20	0.35	1.19	0.38	1.19	0.40	1.18	0.42

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	20°		21°		22°		23°	
Minutes.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
c = 0.75	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
c = 1.00	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
c = 1.25	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	24°		25°		26°		27°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff Elev.	Hor. Dist.	Diff. Elev.
0	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
c = 0.75.	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
c = 1.00.	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
c = 1.25.	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Concluded*

	28°		29°		30°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76.35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
c = 0.75..	0.66	0.36	0.65	0.37	0.65	0.38
c = 1.00..	0.88	0.48	0.87	0.49	0.86	0.51
c = 1.25..	1.10	0.60	1.09	0.62	1.08	0.64

Table X.*—Minutes in Decimals of a Degree

′	0″	10″	15″	20″	30″	40″	45″	50″	′
0	.00000	.00278	.00417	.00556	.00833	.01111	.01250	.01389	0
1	.01667	.01944	.02083	.02222	.02500	.02778	.02917	.03055	1
2	.03333	.03611	.03750	.03889	.04167	.04444	.04583	.04722	2
3	.05000	.05278	.05417	.05556	.05833	.06111	.06250	.06389	3
4	.06667	.06944	.07083	.07222	.07500	.07778	.07917	.08056	4
5	.08333	.08611	.08750	.08889	.09167	.09444	.09583	.09722	5
6	.10000	.10278	.10417	.10556	.10833	.11111	.11250	.11389	6
7	.11667	.11944	.12083	.12222	.12500	.12778	.12917	.13056	7
8	.13333	.13611	.13750	.13889	.14167	.14444	.14583	.14722	8
9	.15000	.15278	.15417	.15556	.15833	.16111	.16250	.16389	9
10	.16667	.16944	.17083	.17222	.17500	.17778	.17917	.18056	10
11	.18333	.18611	.18750	.18889	.19167	.19444	.19583	.19722	11
12	.20000	.20278	.20417	.20556	.20833	.21111	.21250	.21389	12
13	.21667	.21944	.22083	.22222	.22500	.22778	.22917	.23056	13
14	.23333	.23611	.23750	.23889	.24167	.24444	.24583	.24722	14
15	.25000	.25278	.25417	.25556	.25833	.26111	.26250	.26389	15
16	.26667	.26944	.27083	.27222	.27500	.27778	.27917	.28056	16
17	.28333	.28611	.28750	.28889	.29167	.29444	.29583	.29722	17
18	.30000	.30278	.30417	.30556	.30833	.31111	.31250	.31389	18
19	.31667	.31944	.32083	.32222	.32500	.32778	.32917	.33056	19
20	.33333	.33611	.33750	.33889	.34167	.34444	.34583	.34722	20
21	.35000	.35278	.35417	.35556	.35833	.36111	.36250	.36389	21
22	.36667	.36944	.37083	.37222	.37500	.37778	.37917	.38056	22
23	.38333	.38611	.38750	.38889	.39167	.39444	.39583	.39722	23
24	.40000	.40278	.40417	.40556	.40833	.41111	.41250	.41389	24
25	.41667	.41944	.42083	.42222	.42500	.42778	.42917	.43056	25
26	.43333	.43611	.43750	.43889	.44167	.44444	.44583	.44722	26
27	.45000	.45278	.45417	.45556	.45833	.46111	.46250	.46389	27
28	.46667	.46944	.47083	.47222	.47500	.47778	.47917	.48056	28
29	.48333	.48611	.48750	.48889	.49167	.49444	.49583	.49722	29
30	.50000	.50278	.50417	.50556	.50833	.51111	.51250	.51389	30
31	.51667	.51944	.52083	.52222	.52500	.52778	.52917	.53056	31
32	.53333	.53611	.53750	.53889	.54167	.54444	.54583	.54722	32
33	.55000	.55278	.55417	.55556	.55833	.56111	.56250	.56389	33
34	.56667	.56944	.57083	.57222	.57500	.57778	.57917	.58056	34
35	.58333	.58611	.58750	.58889	.59167	.59444	.59583	.59722	35
36	.60000	.60278	.60417	.60556	.60833	.61111	.61250	.61389	36
37	.61667	.61944	.62083	.62222	.62500	.62778	.62917	.63056	37
38	.63333	.63611	.63750	.63889	.64167	.64444	.64583	.64722	38
39	.65000	.65278	.65417	.65556	.65833	.66111	.66250	.66389	39
40	.66667	.66944	.67083	.67222	.67500	.67778	.67917	.68056	40
41	.68333	.68611	.68750	.68889	.69167	.69444	.69583	.69722	41
42	.70000	.70278	.70417	.70556	.70833	.71111	.71250	.71389	42
43	.71667	.71944	.72083	.72222	.72500	.72778	.72917	.73056	43
44	.73333	.73611	.73750	.73889	.74167	.74444	.74583	.74722	44
45	.75000	.75278	.75417	.75556	.75833	.76111	.76250	.76389	45
46	.76667	.76944	.77083	.77222	.77500	.77778	.77917	.78056	46
47	.78333	.78611	.78750	.78889	.79167	.79444	.79583	.79722	47
48	.80000	.80278	.80417	.80556	.80833	.81111	.81250	.81389	48
49	.81667	.81944	.82083	.82222	.82500	.82778	.82917	.83056	49
50	.83333	.83611	.83750	.83889	.84167	.84444	.84583	.84722	50
51	.85000	.85278	.85417	.85556	.85833	.86111	.86250	.86389	51
52	.86667	.86944	.87083	.87222	.87500	.87778	.87917	.88056	52
53	.88333	.88611	.88750	.88889	.89167	.89444	.89583	.89722	53
54	.90000	.90278	.90417	.90556	.90833	.91111	.91250	.91389	54
55	.91667	.91944	.92083	.92222	.92500	.92778	.92917	.93056	55
56	.93333	.93611	.93750	.93889	.94167	.94444	.94583	.94722	56
57	.95000	.95278	.95417	.95556	.95833	.96111	.96250	.96389	57
58	.96667	.96944	.97083	.97222	.97500	.97778	.97917	.98056	58
59	.98333	.98611	.98750	.98889	.99167	.99444	.99583	.99722	59
′	0″	10″	15″	20″	30″	40″	45″	50″	′

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TABLE XI

N.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	*0300	*0724	*1147	*1570	*1993	*2415	424
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
104	7033	7451	7868	8284	8700	9116	9532	9947	*0361	*0775	416
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	*0195	*0600	*1004	*1408	*1812	*2216	*2619	*3021	404
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	*0207	*0602	*0998	397

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	434	43	87	130	174	217	260	304	347	391	434
	433	43	87	130	173	217	260	303	346	390	433
	432	43	86	130	173	216	259	302	346	389	432
	431	43	86	129	172	216	259	302	345	388	431
	430	43	86	129	172	215	258	301	344	387	430
	429	43	86	129	172	215	257	300	343	386	429
	428	43	86	128	171	214	257	300	342	385	428
	427	43	85	128	171	214	256	299	342	384	427
	426	43	85	128	170	213	256	298	341	383	426
	425	43	85	128	170	213	255	298	340	383	425
	424	42	85	127	170	212	254	297	339	382	424
	423	42	85	127	169	212	254	296	338	381	423
	422	42	84	127	169	211	253	295	338	380	422
	421	42	84	126	168	211	253	295	337	379	421
	420	42	84	126	168	210	252	294	336	378	420
	419	42	84	126	168	210	251	293	335	377	419
	418	42	84	125	167	209	251	293	334	376	418
	417	42	83	125	167	209	250	292	334	375	417
	416	42	83	125	166	208	250	291	333	374	416
	415	42	83	125	166	208	249	291	332	374	415
	414	41	83	124	166	207	248	290	331	373	414
	413	41	83	124	165	207	248	289	330	372	413
	412	41	82	124	165	206	247	288	330	371	412
	411	41	82	123	164	206	247	288	329	370	411
	410	41	82	123	164	205	246	287	328	369	410
	409	41	82	123	164	205	245	286	327	368	409
	408	41	82	122	163	204	245	286	326	367	408
	407	41	81	122	163	204	244	285	326	366	407
	406	41	81	122	162	203	244	284	325	365	406
	405	41	81	122	162	203	243	284	324	365	405
	404	40	81	121	162	202	242	283	323	364	404
	403	40	81	121	161	202	242	282	322	363	403
	402	40	80	121	161	201	241	281	322	362	402
	401	40	80	120	160	201	241	281	321	361	401
	400	40	80	120	160	200	240	280	320	360	400
	399	40	80	120	160	200	239	279	319	359	399
	398	40	80	119	159	199	239	279	318	358	398
	397	40	79	119	159	199	238	278	318	357	397
	396	40	79	119	158	198	238	277	317	356	396
	395	40	79	119	158	198	237	277	316	356	395
	394	39	79	118	158	197	236	276	315	355	394
	393	39	79	118	157	197	236	275	314	354	393
	392	39	78	118	157	196	235	274	314	353	392
	391	39	78	117	156	196	235	274	313	352	391
	390	39	78	117	156	195	234	273	312	351	390
	389	39	78	117	156	195	233	272	311	350	389
	388	39	78	116	155	194	233	272	310	349	388
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
112	9218	9606	9993	*0380	*0766	*1153	*1538	*1924	*2309	*2694	386
113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	*0320	379
115	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
117	8186	8557	8928	9298	9668	*0038	*0407	*0776	*1145	*1514	370
118	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	9543	9904	*0266	*0626	*0987	*1347	*1707	*2067	*2426	360
121	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	*0258	*0611	*0963	*1315	*1667	*2018	*2370	*2721	*3071	352
124	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	387	39	77	116	155	194	232	271	310	348	387
	386	39	77	116	154	193	232	270	309	347	386
	385	39	77	116	154	193	231	270	308	347	385
	384	38	77	115	154	192	230	269	307	346	384
	383	38	77	115	153	192	230	268	306	345	383
	382	38	76	115	153	191	229	267	306	344	382
	381	38	76	114	152	191	229	267	305	343	381
	380	38	76	114	152	190	228	266	304	342	380
	379	38	76	114	152	190	227	265	303	341	379
	378	38	76	113	151	189	227	265	302	340	378
	377	38	75	113	151	189	226	264	302	339	377
	376	38	75	113	150	188	226	263	301	338	376
	375	38	75	113	150	188	225	263	300	338	375
	374	37	75	112	150	187	224	262	299	337	374
	373	37	75	112	149	187	224	261	298	336	373
	372	37	74	112	149	186	223	260	298	335	372
	371	37	74	111	148	186	223	260	297	334	371
	370	37	74	111	148	185	222	259	296	333	370
	369	37	74	111	148	185	221	258	295	332	369
	368	37	74	110	147	184	221	258	294	331	368
	367	37	73	110	147	184	220	257	294	330	367
	366	37	73	110	146	183	220	256	293	329	366
	365	37	73	110	146	183	219	256	292	329	365
	364	36	73	109	146	182	218	255	291	328	364
	363	36	73	109	145	182	218	254	290	327	363
	362	36	72	109	145	181	217	253	290	326	362
	361	36	72	108	144	181	217	253	289	325	361
	360	36	72	108	144	180	216	252	288	324	360
	359	36	72	108	144	180	215	251	287	323	359
	358	36	72	107	143	179	215	251	286	322	358
	357	36	71	107	143	179	214	250	286	321	357
	356	36	71	107	142	178	214	249	285	320	356
	355	36	71	107	142	178	213	249	284	320	355
	354	35	71	106	142	177	212	248	283	319	354
	353	35	71	106	141	177	212	247	282	318	353
	352	35	70	106	141	176	211	246	282	317	352
	351	35	70	105	140	176	211	246	281	316	351
	350	35	70	105	140	175	210	245	280	315	350
	349	35	70	105	140	175	209	244	279	314	349
	348	35	70	104	139	174	209	244	278	313	348
	347	35	69	104	139	174	208	243	278	312	347
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
125	096910	7257	7604	7951	8298	8644	8990	9335	9681	*0026	346
126	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	*0253	338
129	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	113943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	*0245	330
132	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	*0012	323
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
138	9879	*0194	*0508	*0822	*1136	*1450	*1763	*2076	*2389	*2702	314
139	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	347	35	69	104	139	174	208	243	278	312	347
	346	35	69	104	138	173	208	242	277	311	346
	345	35	69	104	138	173	207	242	276	311	345
	344	34	69	103	138	172	206	241	275	310	344
	343	34	69	103	137	172	206	240	274	309	343
	342	34	68	103	137	171	205	239	274	308	342
	341	34	68	102	136	171	205	239	273	307	341
	340	34	68	102	136	170	204	238	272	306	340
	339	34	68	102	136	170	203	237	271	305	339
	338	34	68	101	135	169	203	237	270	304	338
	337	34	67	101	135	169	202	236	270	303	337
	336	34	67	101	134	168	202	235	269	302	336
	335	34	67	101	134	168	201	235	268	302	335
	334	33	67	100	134	167	200	234	267	301	334
	333	33	67	100	133	167	200	233	266	300	333
	332	33	66	100	133	166	199	232	266	299	332
	331	33	66	99	132	166	199	232	265	298	331
	330	33	66	99	132	165	198	231	264	297	330
	329	33	66	99	132	165	197	230	263	296	329
	328	33	66	98	131	164	197	230	262	295	328
	327	33	65	98	131	164	196	229	262	294	327
	326	33	65	98	130	163	196	228	261	293	326
	325	33	65	98	130	163	195	228	260	293	325
	324	32	65	97	130	162	194	227	259	292	324
	323	32	65	97	129	162	194	226	258	291	323
	322	32	64	97	129	161	193	225	258	290	322
	321	32	64	96	128	161	193	225	257	289	321
	320	32	64	96	128	160	192	224	256	288	320
	319	32	64	96	128	160	191	223	255	287	319
	318	32	64	95	127	159	191	223	254	286	318
	317	32	63	95	127	159	190	222	254	285	317
	316	32	63	95	126	158	190	221	253	284	316
	315	32	63	95	126	158	189	221	252	284	315
	314	31	63	94	126	157	188	220	251	283	314
	313	31	63	94	125	157	188	219	250	282	313
	312	31	62	94	125	156	187	218	250	281	312
	311	31	62	93	124	156	187	218	249	280	311
	310	31	62	93	124	155	186	217	248	279	310
	309	31	62	93	124	155	185	216	247	278	309
	308	31	62	92	123	154	185	216	246	277	308
	307	31	61	92	123	154	184	215	246	276	307
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
140	146128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	9219	9527	9835	*0142	*0449	*0756	*1063	*1370	*1676	*1982	307
142	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	*0168	*0469	*0769	*1068	301
145	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	8977	9264	9552	9839	*0126	*0413	*0699	*0986	*1272	*1558	287
152	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	*0051	281
155	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755	*0029	*0303	*0577	*0850	*1124	274
159	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272

PROPORTIONAL PARTS

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
	306	31	61	92	122	153	184	214	245	275	306
	305	31	61	92	122	153	183	214	244	275	305
	304	30	61	91	122	152	182	213	243	274	304
	303	30	61	91	121	152	182	212	242	273	303
	302	30	60	91	121	151	181	211	242	272	302
	301	30	60	90	120	151	181	211	241	271	301
	300	30	60	90	120	150	180	210	240	270	300
	299	30	60	90	120	150	179	209	239	269	299
	298	30	60	89	119	149	179	209	238	268	298
	297	30	59	89	119	149	178	208	238	267	297
	296	30	59	89	118	148	178	207	237	266	296
	295	30	59	89	118	148	177	207	236	266	295
	294	29	59	88	118	147	176	206	235	265	294
	293	29	59	88	117	147	176	205	234	264	293
	292	29	58	88	117	146	175	204	234	263	292
	291	29	58	87	116	146	175	204	233	262	291
	290	29	58	87	116	145	174	203	232	261	290
	289	29	58	87	116	145	173	202	231	260	289
	288	29	58	86	115	144	173	202	230	259	288
	287	29	57	86	115	144	172	201	230	258	287
	286	29	57	86	114	143	172	200	229	257	286
	285	29	57	86	114	143	171	200	228	257	285
	284	28	57	85	114	142	170	199	227	256	284
	283	28	57	85	113	142	170	198	226	255	283
	282	28	56	85	113	141	169	197	226	254	282
	281	28	56	84	112	141	169	197	225	253	281
	280	28	56	84	112	140	168	196	224	252	280
	279	28	56	84	112	140	167	195	223	251	279
	278	28	56	83	111	139	167	195	222	250	278
	277	28	55	83	111	139	166	194	222	249	277
	276	28	55	83	110	138	166	193	221	248	276
	275	28	55	83	110	138	165	193	220	248	275
	274	27	55	82	110	137	164	192	219	247	274
	273	27	55	82	109	137	164	191	218	246	273
	272	27	54	82	109	136	163	190	218	245	272
	271	27	54	81	108	136	163	190	217	244	271
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
160	204120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	*0051	*0319	*0586	*0853	*1121	*1388	*1654	*1921	267
163	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	217484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	*0193	256
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	8046	8297	8548	8799	9049	9299	9550	9800	*0050	*0300	250
174	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	243038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	*0176	245
178	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242

PROPORTIONAL PARTS

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
	272	27	54	82	109	136	163	190	218	245	272
	271	27	54	81	108	136	163	190	217	244	271
	270	27	54	81	108	135	162	189	216	243	270
	269	27	54	81	108	135	161	188	215	242	269
	268	27	54	80	107	134	161	188	214	241	268
	267	27	53	80	107	134	160	187	214	240	267
	266	27	53	80	106	133	160	186	213	239	266
	265	27	53	80	106	133	159	186	212	239	265
	264	26	53	79	106	132	158	185	211	238	264
	263	26	53	79	105	132	158	184	210	237	263
	262	26	52	79	105	131	157	183	210	236	262
	261	26	52	78	104	131	157	183	209	235	261
	260	26	52	78	104	130	156	182	208	234	260
	259	26	52	78	104	130	155	181	207	233	259
	258	26	52	77	103	129	155	181	206	232	258
	257	26	51	77	103	129	154	180	206	231	257
	256	26	51	77	102	128	154	179	205	230	256
	255	26	51	77	102	128	153	179	204	230	255
	254	25	51	76	102	127	152	178	203	229	254
	253	25	51	76	101	127	152	177	202	228	253
	252	25	50	76	101	126	151	176	202	227	252
	251	25	50	75	100	126	151	176	201	226	251
	250	25	50	75	100	125	150	175	200	225	250
	249	25	50	75	100	125	149	174	199	224	249
	248	25	50	74	99	124	149	174	198	223	248
	247	25	49	74	99	124	148	173	198	222	247
	246	25	49	74	98	123	148	172	197	221	246
	245	25	49	74	98	123	147	172	196	221	245
	244	24	49	73	98	122	146	171	195	220	244
	243	24	49	73	97	122	146	170	194	219	243
	242	24	48	73	97	121	145	169	194	218	242
	241	24	48	72	96	121	145	169	193	217	241
	240	24	48	72	96	120	144	168	192	216	240
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
180	255273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	267172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	9513	9746	9980	*0213	*0446	*0679	*0912	*1144	*1377	*1609	233
187	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	278754	8982	9211	9439	9667	9895	*0123	*0351	*0578	*0806	228
191	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	8853	9071	9289	9507	9725	9943	*0161	*0378	*0595	*0813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	9630	9843	*0056	*0268	*0481	*0693	*0906	*1118	*1330	*1542	212

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	239	24	48	72	96	120	143	167	191	215	239
	238	24	48	71	95	119	143	167	190	214	238
	237	24	47	71	95	119	142	166	190	213	237
	236	24	47	71	94	118	142	165	189	212	236
	235	24	47	71	94	118	141	165	188	212	235
	234	23	47	70	94	117	140	164	187	211	234
	233	23	47	70	93	117	140	163	186	210	233
	232	23	46	70	93	116	139	162	186	209	232
	231	23	46	69	92	116	139	162	185	208	231
	230	23	46	69	92	115	138	161	184	207	230
	229	23	46	69	92	115	137	160	183	206	229
	228	23	46	68	91	114	137	160	182	205	228
	227	23	45	68	91	114	136	159	182	204	227
	226	23	45	68	90	113	136	158	181	203	226
	225	23	45	68	90	113	135	158	180	203	225
	224	22	45	67	90	112	134	157	179	202	224
	223	22	45	67	89	112	134	156	178	201	223
	222	22	44	67	89	111	133	155	178	200	222
	221	22	44	66	88	111	133	155	177	199	221
	220	22	44	66	88	110	132	154	176	198	220
	219	22	44	66	88	110	131	153	175	197	219
	218	22	44	65	87	109	131	153	174	196	218
	217	22	43	65	87	109	130	152	174	195	217
	216	22	43	65	86	108	130	151	173	194	216
	215	22	43	65	86	108	129	151	172	194	215
	214	21	43	64	86	107	128	150	171	193	214
	213	21	43	64	85	107	128	149	170	192	213
	212	21	42	64	85	106	127	148	170	191	212
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
205	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	322219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	8380	8583	8787	8991	9194	9398	9601	9805	*0008	*0211	203
214	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202
215	332438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	8456	8656	8855	9054	9253	9451	9650	9849	*0047	*0246	199
219	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	342423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	8305	8500	8694	8889	9083	9278	9472	9666	9860	*0054	194
224	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
225	352183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	9835	*0025	*0215	*0404	*0593	*0783	*0972	*1161	*1350	*1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	*0143	*0328	*0513	*0698	*0883	185

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	212	21	42	64	85	106	127	148	170	191	212
	211	21	42	63	84	106	127	148	169	190	211
	210	21	42	63	84	105	126	147	168	189	210
	209	21	42	63	84	105	125	146	167	188	209
	208	21	42	62	83	104	125	146	166	187	208
	207	21	41	62	83	104	124	145	166	186	207
	206	21	41	62	82	103	124	144	165	185	206
	205	21	41	62	82	103	123	144	164	185	205
	204	20	41	61	82	102	122	143	163	184	204
	203	20	41	61	81	102	122	142	162	183	203
	202	20	40	61	81	101	121	141	162	182	202
	201	20	40	60	80	101	121	141	161	181	201
	200	20	40	60	80	100	120	140	160	180	200
	199	20	40	60	80	100	119	139	159	179	199
	198	20	40	59	79	99	119	139	158	178	198
	197	20	39	59	79	99	118	138	158	177	197
	196	20	39	59	78	98	118	137	157	176	196
	195	20	39	59	78	98	117	137	156	176	195
	194	19	39	58	78	97	116	136	155	175	194
	193	19	39	58	77	97	116	135	154	174	193
	192	19	38	58	77	96	115	134	154	173	192
	191	19	38	57	76	96	115	134	153	172	191
	190	19	38	57	76	95	114	133	152	171	190
	189	19	38	57	76	95	113	132	151	170	189
	188	19	38	56	75	94	113	132	150	169	188
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
235	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	*0030	181
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	389166	9343	9520	9698	9875	*0051	*0228	*0405	*0582	*0759	177
246	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	9674	9847	*0020	*0192	*0365	*0538	*0711	*0883	*1056	*1228	173
252	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	406540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	9933	*0102	*0271	*0440	*0609	*0777	*0946	*1114	*1283	*1451	169
258	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	414973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	9956	*0121	*0286	*0451	*0616	*0781	*0945	*1110	*1275	*1439	165
264	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	187	19	37	56	75	94	112	131	150	168	187
	186	19	37	56	74	93	112	130	149	167	186
	185	19	37	56	74	93	111	130	148	167	185
	184	18	37	55	74	92	110	129	147	166	184
	183	18	37	55	73	92	110	128	146	165	183
	182	18	36	55	73	91	109	127	146	164	182
	181	18	36	54	72	91	109	127	145	163	181
	180	18	36	54	72	90	108	126	144	162	180
	179	18	36	54	72	90	107	125	143	161	179
	178	18	36	53	71	89	107	125	142	160	178
	177	18	35	53	71	89	106	124	142	159	177
	176	18	35	53	70	88	106	123	141	158	176
	175	18	35	53	70	88	105	123	140	158	175
	174	17	35	52	70	87	104	122	139	157	174
	173	17	35	52	69	87	104	121	138	156	173
	172	17	34	52	69	86	103	120	138	155	172
	171	17	34	51	68	86	103	120	137	154	171
	170	17	34	51	68	85	102	119	136	153	170
	169	17	34	51	68	85	101	118	135	152	169
	168	17	34	50	67	84	101	118	134	151	168
	167	17	33	50	67	84	100	117	134	150	167
	166	17	33	50	66	83	100	116	133	149	166
	165	17	33	50	66	83	99	116	132	149	165
	164	16	33	49	66	82	98	115	131	148	164
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
265	423246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	9752	9914	*0075	*0236	*0398	*0559	*0720	*0881	*1042	*1203	161
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
271	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	439333	9491	9648	9806	9964	*0122	*0279	*0437	*0594	*0752	158
276	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
279	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	447158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
281	8706	8861	9015	9170	9324	9478	9633	9787	9941	*0095	154
282	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
283	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	454845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
286	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
287	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
288	9392	9543	9694	9845	9995	*0146	*0296	*0447	*0597	*0748	151
289	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	462398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
294	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
295	469822	9969	*0116	*0263	*0410	*0557	*0704	*0851	*0998	*1145	147
296	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	164	16	33	49	66	82	98	115	131	148	164
	163	16	33	49	65	82	98	114	130	147	163
	162	16	32	49	65	81	97	113	130	146	162
	161	16	32	48	64	81	97	113	129	145	161
	160	16	32	48	64	80	96	112	128	144	160
	159	16	32	48	64	80	95	111	127	143	159
	158	16	32	47	63	79	95	111	126	142	158
	157	16	31	47	63	79	94	110	126	141	157
	156	16	31	47	62	78	94	109	125	140	156
	155	16	31	47	62	78	93	109	124	140	155
	154	15	31	46	62	77	92	108	123	139	154
	153	15	31	46	61	77	92	107	122	138	153
	152	15	30	46	61	76	91	106	122	137	152
	151	15	30	45	60	76	91	106	121	136	151
	150	15	30	45	60	75	90	105	120	135	150
	149	15	30	45	60	75	89	104	119	134	149
	148	15	30	44	59	74	89	104	118	133	148
	147	15	29	44	59	74	88	103	118	132	147
	146	15	29	44	58	73	88	102	117	131	146
	145	15	29	44	58	73	87	102	116	131	145
	144	14	29	43	58	72	86	101	115	130	144
	143	14	29	43	57	72	86	100	114	129	143
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

Table XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
301	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
302	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
305	484300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
308	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	9958	*0099	*0239	*0380	*0520	*0661	*0801	*0941	*1081	*1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	498311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
316	9687	9824	9962	*0099	*0236	*0374	*0511	*0648	*0785	*0922	137
317	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
321	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
322	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
323	9203	9337	9471	9606	9740	9874	*0009	*0143	*0277	*0411	134
324	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
325	511883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	518514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
331	9828	9959	*0090	*0221	*0353	*0484	*0615	*0745	*0876	*1007	131
332	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	525045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
336	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
338	8917	9045	9174	9302	9430	9559	9687	9815	9943	*0072	128
339	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROPORTIONAL PARTS	142	14	28	43	57	71	85	99	114	128	142
	141	14	28	42	56	71	85	99	113	127	141
	140	14	28	42	56	70	84	98	112	126	140
	139	14	28	42	56	70	83	97	111	125	139
	138	14	28	41	55	69	83	97	110	124	138
	137	14	27	41	55	69	82	96	110	123	137
	136	14	27	41	54	68	82	95	109	122	136
	135	14	27	41	54	68	81	95	108	122	135
	134	13	27	40	54	67	80	94	107	121	134
	133	13	27	40	53	67	80	93	106	120	133
	132	13	26	40	53	66	79	92	106	119	132
	131	13	26	39	52	66	79	92	105	118	131
	130	13	26	39	52	65	78	91	104	117	130
	129	13	26	39	52	65	77	90	103	116	129
	128	13	26	38	51	64	77	90	102	115	128
	127	13	25	38	51	64	76	89	102	114	127
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI. *Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
341	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
343	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	537819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	9076	9202	9327	9452	9578	9703	9829	9954	*0079	*0204	125
347	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	*0106	123
355	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	9907	*0026	*0146	*0265	*0385	*0504	*0624	*0743	*0863	*0982	119
364	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	562293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	9374	9491	9608	9725	9842	9959	*0076	*0193	*0309	*0426	117
372	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	574031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	128	13	26	38	51	64	77	90	102	115	128
	127	13	25	38	51	64	76	89	102	114	127
	126	13	25	38	50	63	76	88	101	113	126
	125	13	25	38	50	63	75	88	100	113	125
	124	12	25	37	50	62	74	87	99	112	124
	123	12	25	37	49	62	74	86	98	111	123
	122	12	24	37	49	61	73	85	98	110	122
	121	12	24	36	48	61	73	85	97	109	121
	120	12	24	36	48	60	72	84	96	108	120
	119	12	24	36	48	60	71	83	95	107	119
	118	12	24	35	47	59	71	83	94	106	118
	117	12	23	35	47	59	70	82	94	105	117
	116	12	23	35	46	58	70	81	93	104	116
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
380	579784	9898	*0012	*0126	*0241	*0355	*0469	*0583	*0697	*0811	114
381	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	114
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	585461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	9950	*0061	*0173	*0284	*0396	*0507	*0619	*0730	*0842	*0953	112
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	596597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	9883	9992	*0101	*0210	*0319	*0428	*0537	*0646	*0755	*0864	109
399	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	109
400	602060	2169	2277	2386	2494	2603	2711	2819	2928	3036	108
401	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	607455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
407	9594	9701	9808	9914	*0021	*0128	*0234	*0341	*0447	*0554	107
408	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	106
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	618048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	9093	9198	9302	9406	9511	9615	9719	9824	9928	*0032	104
417	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	115	12	23	35	46	58	69	81	92	104	115
	114	11	23	34	46	57	68	80	91	103	114
	113	11	23	34	45	57	68	79	90	102	113
	112	11	22	34	45	56	67	78	90	101	112
	111	11	22	33	44	56	67	78	89	100	111
	110	11	22	33	44	55	66	77	88	99	110
	109	11	22	33	44	55	65	76	87	98	109
	108	11	22	32	43	54	65	76	86	97	108
	107	11	21	32	43	54	64	75	86	96	107
	106	11	21	32	42	53	64	74	85	95	106
	105	11	21	32	42	53	63	74	84	95	105
	104	10	21	31	42	52	62	73	83	94	104
	103	10	21	31	41	52	62	72	82	93	103
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
420	623249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
425	628389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
426	9410	9512	9613	9715	9817	9919	*0021	*0123	*0224	*0326	102
427	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
431	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	101
432	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
434	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
435	638489	8589	8689	8789	8888	8988	9088	9188	9287	9387	100
436	9486	9586	9686	9785	9885	9984	*0084	*0183	*0283	*0382	99
437	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375	99
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
439	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	3551	3650	3749	3847	3946	4044	4143	4242	4340	98
441	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	648360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	9335	9432	9530	9627	9724	9821	9919	*0016	*0113	*0210	97
447	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	97
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	3309	3405	3502	3598	3695	3791	3888	3984	4080	96
451	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	658011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
457	9916	*0011	*0106	*0201	*0296	*0391	*0486	*0581	*0676	*0771	95
458	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
459	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
460	662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	94
461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	104	10	21	31	42	52	62	73	83	94	104
	103	10	21	31	41	52	62	72	82	93	103
	102	10	20	31	41	51	61	71	82	92	102
	101	10	20	30	40	51	61	71	81	91	101
	100	10	20	30	40	50	60	70	80	90	100
	99	10	20	30	40	50	59	69	79	89	99
	98	10	20	29	39	49	59	69	78	88	98
	97	10	19	29	39	49	58	68	78	87	97
	96	10	19	29	38	48	58	67	77	86	96
	95	10	19	29	38	48	57	67	76	86	95
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
465	667453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967	*0060	*0153	93
468	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	676694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	9428	9519	9610	9700	9791	9882	9973	*0063	*0154	*0245	91
479	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	91
480	681241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	685742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930	*0019	*0107	89
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	694605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	9057	9144	9231	9317	9404	9491	9578	9664	9751	87
501	9838	9924	*0011	*0098	*0184	*0271	*0358	*0444	*0531	*0617	87
502	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	703291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	94	9	19	28	38	47	56	66	75	85	94
	93	9	19	28	37	47	56	65	74	84	93
	92	9	18	28	37	46	55	64	74	83	92
	91	9	18	27	36	46	55	64	73	82	91
	90	9	18	27	36	45	54	63	72	81	90
	89	9	18	27	36	45	53	62	71	80	89
	88	9	18	26	35	44	53	62	70	79	88
	87	9	17	26	35	44	52	61	70	78	87
	86	9	17	26	34	43	52	60	69	77	86
	85	9	17	26	34	43	51	60	68	77	85
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
510	707570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	9270	9355	9440	9524	9609	9694	9779	9863	9948	*0033	85
513	710117	0202	0287	0371	0456	0540	0625	0710	0794	0879	85
514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	711807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
519	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
520	716003	6087	6170	6254	6337	6421	6504	6588	6671	6754	83
521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	*0077	83
525	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	728354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	*0055	*0136	*0217	*0298	*0378	*0459	*0540	*0621	*0702	81
538	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
543	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	736397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	9572	9651	9731	9810	9889	9968	*0047	*0126	*0205	*0284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PROP. PARTS	86	9	17	26	34	43	52	60	69	77	86
	85	9	17	26	34	43	51	60	68	77	85
	84	8	17	25	34	42	50	59	67	76	84
	83	8	17	25	33	42	50	58	66	75	83
	82	8	16	25	33	41	49	57	66	74	82
	81	8	16	24	32	41	49	57	65	73	81
	80	8	16	24	32	40	48	56	64	72	80
	79	8	16	24	32	40	47	55	63	71	79
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
555	744293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	*0045	*0123	*0200	*0277	*0354	*0431	77
563	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	752048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	759668	9743	9819	9894	9970	*0045	*0121	*0196	*0272	*0347	75
576	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	75
577	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
579	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
580	763428	3503	3578	3653	3727	3802	3877	3952	4027	4101	75
581	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	767156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	*0042	74
589	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	74
590	770852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	774517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PRO. PARTS	78	8	16	23	31	39	47	55	62	70	78
	77	8	15	23	31	39	46	54	62	69	77
	76	8	15	23	30	38	46	53	61	68	76
	75	8	15	23	30	38	45	53	60	68	75
	74	7	15	22	30	37	44	52	59	67	74
	73	7	15	22	29	37	44	51	58	66	73
	72	7	14	22	29	36	43	50	58	65	72
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
600	778151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
602	9596	9669	9741	9813	9885	9957	*0029	*0101	*0173	*0245	72
603	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	781755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	788875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	9581	9651	9722	9792	9863	9933	*0004	*0074	*0144	*0215	70
617	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
619	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	795880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	69
632	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	68
635	802774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	68
641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	809560	9627	9694	9762	9829	9896	9964	*0031	*0098	*0165	67
646	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PRO. P'TS	73	7	15	22	29	37	44	51	58	66	73
	72	7	14	22	29	36	43	50	58	65	72
	71	7	14	21	28	36	43	50	57	64	71
	70	7	14	21	28	35	42	49	56	63	70
	69	7	14	21	28	35	41	48	55	62	69
	68	7	14	20	27	34	41	48	54	61	68
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
650	812913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	816241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	9610	9676	9741	9807	9873	9939	*0004	*0070	*0136	66
661	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	822822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	829304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	9947	*0011	*0075	*0139	*0204	*0268	*0332	*0396	*0460	*0525	64
677	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832509	2573	2637	2700	2764	2828	2892	2956	3020	3083	64
681	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	835691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	9478	9541	9604	9667	9729	9792	9855	9918	9981	*0043	63
692	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	841985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PRO. P'TS	67	7	13	20	27	34	40	47	54	60	67
	66	7	13	20	26	33	40	46	53	59	66
	65	7	13	20	26	33	39	46	52	59	65
	64	6	13	19	26	32	38	45	51	58	64
	63	6	13	19	25	32	38	44	50	57	63
	62	6	12	19	25	31	37	43	50	56	62
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff
700	845098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	848189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	850033	0095	0156	0217	0279	0340	0401	0462	0524	0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	1320	1381	1442	1503	1564	1625	1686	1747	1809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	854306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	9739	9799	9859	9918	9978	*0038	*0098	*0158	*0218	*0278	60
725	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	3382	3442	3501	3561	3620	3680	3739	3799	3858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	866287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	9290	9349	9408	9466	9525	9584	9642	9701	9760	59
741	9818	9877	9935	9994	*0053	*0111	*0170	*0228	*0287	*0345	59
742	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	872156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff
PRO. P'TS	62	6	12	19	25	31	37	43	50	56	62
	61	6	12	18	24	31	37	43	49	55	61
	60	6	12	18	24	30	36	42	48	54	60
	59	6	12	18	24	30	35	41	47	53	59
	58	6	12	17	23	29	35	41	46	52	58
	Diff.	1	2	3	4	5	6	7	8	9	Diff

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
750	875061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	877947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	9669	9726	9784	9841	9898	9956	*0013	*0070	*0127	*0185	57
759	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	57
760	880814	0871	0928	0985	1042	1099	1156	1213	1271	1328	57
761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	883661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	886491	6547	6604	6660	6716	6773	6829	6885	6942	6998	56
771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	889302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
776	9862	9918	9974	*0030	*0086	*0141	*0197	*0253	*0309	*0365	56
777	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	894870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	9821	9875	9930	9985	*0039	*0094	*0149	*0203	*0258	*0312	55
795	900367	0422	0476	0531	0586	0640	0695	0749	0804	0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PR. PTS.	57	6	11	17	23	29	34	40	46	51	57
	56	6	11	17	22	28	34	39	45	50	56
	55	6	11	17	22	28	33	39	44	50	55
	54	5	11	16	22	27	32	38	43	49	54
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
800	903090	3144	3199	3253	3307	3361	3416	3470	3524	3578	54
801	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	905796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
806	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908485	8539	8592	8646	8699	8753	8807	8860	8914	8967	54
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	9556	9610	9663	9716	9770	9823	9877	9930	9984	*0037	53
813	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	911158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	913814	3867	3920	3973	4026	4079	4132	4184	4237	4290	53
821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	916454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	9601	9653	9706	9758	9810	9862	9914	9967	*0019	*0071	52
832	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	921686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	926857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PR. PTS.	55	6	11	17	22	28	33	39	44	50	55
	54	5	11	16	22	27	32	38	43	49	54
	53	5	11	16	21	27	32	37	42	48	53
	52	5	10	16	21	26	31	36	42	47	52
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
850	929419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
851	9930	9981	*0032	*0083	*0134	*0185	*0236	*0287	*0338	*0389	51
852	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	931966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	937016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	942008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
880	944483	4532	4581	4631	4680	4729	4779	4828	4877	4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	946943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	9878	9926	9975	*0024	*0073	*0121	*0170	*0219	*0267	*0316	49
892	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	951823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PR. PTS.	51	5	10	15	20	26	31	36	41	46	51
	50	5	10	15	20	25	30	35	40	45	50
	49	5	10	15	20	25	29	34	39	44	49
	48	5	10	14	19	24	29	34	38	43	48
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	956649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	*0042	*0090	*0138	*0185	*0233	*0280	*0328	*0376	*0423	48
913	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	48
915	961421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	966142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	8530	8576	8623	8670	8716	8763	8810	8856	8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	9882	9928	9975	*0021	*0068	*0114	*0161	*0207	*0254	*0300	47
934	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
935	970812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
940	973128	3174	3220	3266	3313	3359	3405	3451	3497	3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	975432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PR. PTS.	49	5	10	15	20	25	29	34	39	44	49
	48	5	10	14	19	24	29	34	38	43	48
	47	5	9	14	19	24	28	33	38	42	47
	46	5	9	14	18	23	28	32	37	41	46
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
950	977724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	·275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	984527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	989005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	*0028	*0072	*0117	*0161	*0206	*0250	*0294	44
978	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	993436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	997823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PR. PTS.	46	5	9	14	18	23	28	32	37	41	46
	45	5	9	14	18	23	27	32	36	41	45
	44	4	9	13	18	22	26	31	35	40	44
	43	4	9	13	17	22	26	30	34	39	43
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XI.—*Concluded*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
1000	000000	0043	0087	0130	0174	0217	0260	0304	0347	0391	43
1001	0434	0477	0521	0564	0608	0651	0694	0738	0781	0824	43
1002	0868	0911	0954	0998	1041	1084	1128	1171	1214	1258	43
1003	1301	1344	1388	1431	1474	1517	1561	1604	1647	1690	43
1004	1734	1777	1820	1863	1907	1950	1993	2036	2080	2123	43
1005	002166	2209	2252	2296	2339	2382	2425	2468	2512	2555	43
1006	2598	2641	2684	2727	2771	2814	2857	2900	2943	2986	43
1007	3029	3073	3116	3159	3202	3245	3288	3331	3374	3417	43
1008	3461	3504	3547	3590	3633	3676	3719	3762	3805	3848	43
1009	3891	3934	3977	4020	4063	4106	4149	4192	4235	4278	43
1010	004321	4364	4407	4450	4493	4536	4579	4622	4665	4708	43
1011	4751	4794	4837	4880	4923	4966	5009	5052	5095	5138	43
1012	5181	5223	5266	5309	5352	5395	5438	5481	5524	5567	43
1013	5609	5652	5695	5738	5781	5824	5867	5909	5952	5995	43
1014	6038	6081	6124	6166	6209	6252	6295	6338	6380	6423	43
1015	006466	6509	6552	6594	6637	6680	6723	6765	6808	6851	43
1016	6894	6936	6979	7022	7065	7107	7150	7193	7236	7278	43
1017	7321	7364	7406	7449	7492	7534	7577	7620	7662	7705	43
1018	7748	7790	7833	7876	7918	7961	8004	8046	8089	8132	43
1019	8174	8217	8259	8302	8345	8387	8430	8472	8515	8558	43
1020	008600	8643	8685	8728	8770	8813	8856	8898	8941	8983	43
1021	9026	9068	9111	9153	9196	9238	9281	9323	9366	9408	42
1022	9451	9493	9536	9578	9621	9663	9706	9748	9791	9833	42
1023	9876	9918	9961	*0003	*0045	*0088	*0130	*0173	*0215	*0258	42
1024	010300	0342	0385	0427	0470	0512	0554	0597	0639	0681	42
1025	010724	0766	0809	0851	0893	0936	0978	1020	1063	1105	42
1026	1147	1190	1232	1274	1317	1359	1401	1444	1486	1528	42
1027	1570	1613	1655	1697	1740	1782	1824	1866	1909	1951	42
1028	1993	2035	2078	2120	2162	2204	2247	2289	2331	2373	42
1029	2415	2458	2500	2542	2584	2626	2669	2711	2753	2795	42
1030	012837	2879	2922	2964	3006	3048	3090	3132	3174	3217	42
1031	3259	3301	3343	3385	3427	3469	3511	3553	3596	3638	42
1032	3680	3722	3764	3806	3848	3890	3932	3974	4016	4058	42
1033	4100	4142	4184	4226	4268	4310	4353	4395	4437	4479	42
1034	4521	4563	4605	4647	4689	4730	4772	4814	4856	4898	42
1035	014940	4982	5024	5066	5108	5150	5192	5234	5276	5318	42
1036	5360	5402	5444	5485	5527	5569	5611	5653	5695	5737	42
1037	5779	5821	5863	5904	5946	5988	6030	6072	6114	6156	42
1038	6197	6239	6281	6323	6365	6407	6448	6490	6532	6574	42
1039	6616	6657	6699	6741	6783	6824	6866	6908	6950	6992	42
1040	017033	7075	7117	7159	7200	7242	7284	7326	7367	7409	42
1041	7451	7492	7534	7576	7618	7659	7701	7743	7784	7826	42
1042	7868	7909	7951	7993	8034	8076	8118	8159	8201	8243	42
1043	8284	8326	8368	8409	8451	8492	8534	8576	8617	8659	42
1044	8700	8742	8784	8825	8867	8908	8950	8992	9033	9075	42
1045	019116	9158	9199	9241	9282	9324	9366	9407	9449	9490	42
1046	9532	9573	9615	9656	9698	9739	9781	9822	9864	9905	41
1047	9947	9988	*0030	*0071	*0113	*0154	*0195	*0237	*0278	*0320	41
1048	020361	0403	0444	0486	0527	0568	0610	0651	0693	0734	41
1049	0775	0817	0858	0900	0941	0982	1024	1065	1107	1148	41
1050	021189	1231	1272	1313	1355	1396	1437	1479	1520	1561	41

N.	Diff.	1	2	3	4	5	6	7	8	9	Diff.
PR. PTS.	44	4	9	13	18	22	26	31	35	40	44
	43	4	9	13	17	22	26	30	34	39	43
	42	4	8	13	17	21	25	29	34	38	42
	41	4	8	12	16	21	25	29	33	37	41
	Diff.	1	2	3	4	5	6	7	8	9	Diff.

TABLE XII

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	—∞		10.000000		—∞		—∞	60
1	6.463726	5017.17	.000000	.00	6.463726	5017.17	3.536274	59
2	.764756	2934.85	.000000	.00	.764756	2934.85	.235244	58
3	.940847	2082.32	.000000	.00	.940847	2082.32	.059153	57
4	7.065786	1615.17	.000000	.00	7.065786	1615.17	2.934214	56
5	7.162696	1319.68	10.000000	.02	7.162696	1319.70	2.837304	55
6	.241877	1115.78	9.999999	.00	.241878	1115.78	.758122	54
7	.308824	966.53	.999999	.00	.308825	966.53	.691175	53
8	.366816	852.53	.999999	.00	.366817	852.55	.633183	52
9	.417968	762.63	.999999	.02	.417970	762.62	.582030	51
10	7.463726	689.87	9.999998	.00	7.463727	689.88	2.536273	50
11	.505118	629.80	.999998	.02	.505120	629.82	.494880	49
12	.542906	579.37	.999997	.00	.542909	579.38	.457091	48
13	.577668	536.42	.999997	.02	.577672	536.42	.422328	47
14	.609853	499.38	.999996	.00	.609857	499.38	.390143	46
15	7.639816	467.15	9.999996	.02	7.639820	467.15	2.360180	45
16	.667845	438.80	.999995	.00	.667849	438.83	.332151	44
17	.694173	413.73	.999995	.02	.694179	413.73	.305821	43
18	.718997	391.35	.999994	.02	.719003	391.35	.280997	42
19	.742478	371.27	.999993	.00	.742484	371.28	.257516	41
20	7.764754	353.15	9.999993	.02	7.764761	353.17	2.235239	40
21	.785943	336.72	.999992	.02	.785951	336.73	.214049	39
22	.806146	321.75	.999991	.02	.806155	321.75	.193845	38
23	.825451	308.05	.999990	.02	.825460	308.07	.174540	37
24	.843934	295.47	.999989	.00	.843944	295.50	.156056	36
25	7.861662	283.88	9.999989	.02	7.861674	283.90	2.138326	35
26	.878695	273.17	.999988	.02	.878708	273.18	.121292	34
27	.895085	263.23	.999987	.02	.895099	263.25	.104901	33
28	.910879	254.00	.999986	.02	.910894	254.00	.089106	32
29	.926119	245.38	.999985	.03	.926134	245.40	.073866	31
30	7.940842	237.33	9.999983	.02	7.940858	237.37	2.059142	30
31	.955082	229.80	.999982	.02	.955100	229.82	.044900	29
32	.968870	222.72	.999981	.02	.968889	222.73	.031111	28
33	.982233	216.08	.999980	.02	.982253	216.10	.017747	27
34	.995198	209.82	.999979	.03	.995219	209.83	.004781	26
35	8.007787	203.90	9.999977	.02	8.007809	203.92	1.992191	25
36	.020021	198.30	.999976	.02	.020044	198.35	.979956	24
37	.031919	193.03	.999975	.03	.031945	193.03	.968055	23
38	.043501	188.00	.999973	.02	.043527	188.03	.956473	22
39	.054781	183.25	.999972	.02	.054809	183.28	.945191	21
40	8.065776	178.73	9.999971	.03	8.065806	178.75	1.934194	20
41	.076500	174.42	.999969	.02	.076531	174.43	.923469	19
42	.086965	170.30	.999968	.03	.086997	170.33	.913003	18
43	.097183	166.40	.999966	.03	.097217	166.43	.902783	17
44	.107167	162.65	.999964	.02	.107203	162.67	.892797	16
45	8.116926	159.08	9.999963	.03	8.116963	159.12	1.883037	15
46	.126471	155.65	.999961	.03	.126510	155.68	.873490	14
47	.135810	152.38	.999959	.02	.135851	152.42	.864149	13
48	.144953	149.23	.999958	.03	.144996	149.27	.855004	12
49	.153907	146.23	.999956	.03	.153952	146.25	.846048	11
50	8.162681	143.32	9.999954	.03	8.162727	143.35	1.837273	10
51	.171280	140.55	.999952	.03	.171328	140.58	.828672	9
52	.179713	137.87	.999950	.03	.179763	137.88	.820237	8
53	.187985	135.28	.999948	.03	.188036	135.33	.811964	7
54	.196102	132.80	.999946	.03	.196156	132.83	.803844	6
55	8.204070	130.42	9.999944	.03	8.204126	130.45	1.795874	5
56	.211895	128.10	.999942	.03	.211953	128.13	.788047	4
57	.219581	125.88	.999940	.03	219641	125.90	.780359	3
58	.227134	123.72	.999938	.03	.227195	123.77	.772805	2
59	.234557	121.63	.999936	.03	.234621	121.67	.765379	1
60	8.241855		9.999934		8.241921		1.758079	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.241855	119.63	9.999934	.03	8.241921	119.68	1.758079	60
1	.249033	117.68	.999932	.05	.249102	117.72	.750898	59
2	.256094	115.80	.999929	.03	.256165	115.83	.743835	58
3	.263042	113.98	.999927	.03	.263115	114.02	.736885	57
4	.269881	112.22	.999925	.05	.269956	112.25	.730044	56
5	8.276614	110.48	9.999922	.03	8.276691	110.53	1.723309	55
6	.283243	108.83	.999920	.03	.283323	108.88	.716677	54
7	.289773	107.23	.999918	.05	.289856	107.27	.710144	53
8	.296207	105.65	.999915	.03	.296292	105.70	.703708	52
9	.302546	104.13	.999913	.05	.302634	104.17	.697366	51
10	8.308794	102.67	9.999910	.05	8.308884	102.70	1.691116	50
11	.314954	101.22	.999907	.03	.315046	101.27	.684954	49
12	.321027	99.82	.999905	.05	.321122	99.87	.678878	48
13	.327016	98.47	.999902	.05	.327114	98.52	.672886	47
14	.332924	97.15	.999899	.03	.333025	97.18	.666975	46
15	8.338753	95.85	9.999897	.05	8.338856	95.90	1.661144	45
16	.344504	94.62	.999894	.05	.344610	94.65	.655390	44
17	.350181	93.37	.999891	.05	.350289	93.43	.649711	43
18	.355783	92.20	.999888	.05	.355895	92.25	.644105	42
19	.361315	91.03	.999885	.05	.361430	91.08	.638570	41
20	8.366777	89.90	9.999882	.05	8.366895	89.95	1.633105	40
21	.372171	88.80	.999879	.05	.372292	88.83	.627708	39
22	.377499	87.72	.999876	.05	.377622	87.78	.622378	38
23	.382762	86.67	.999873	.05	.382889	86.72	.617111	37
24	.387962	85.65	.999870	.05	.388092	85.70	.611908	36
25	8.393101	84.63	9.999867	.05	8.393234	84.68	1.606766	35
26	.398179	83.67	.999864	.05	.398315	83.72	.601685	34
27	.403199	82.70	.999861	.05	.403338	82.77	.596662	33
28	.408161	81.78	.999858	.07	.408304	81.82	.591696	32
29	.413068	80.85	.999854	.05	.413213	80.92	.586787	31
30	8.417919	79.97	9.999851	.05	8.418068	80.02	1.581932	30
31	.422717	79.08	.999848	.07	.422869	79.15	.577131	29
32	.427462	78.23	.999844	.05	.427618	78.28	.572382	28
33	.432156	77.40	.999841	.05	.432315	77.45	.567685	27
34	.436800	76.57	.999838	.07	.436962	76.63	.563038	26
35	8.441394	75.78	9.999834	.05	8.441560	75.83	1.558440	25
36	.445941	74.98	.999831	.07	.446110	75.05	.553890	24
37	.450440	74.22	.999827	.05	.450613	74.28	.549387	23
38	.454893	73.47	.999824	.07	.455070	73.52	.544930	22
39	.459301	72.73	.999820	.07	.459481	72.80	.540519	21
40	8.463665	72.00	9.999816	.05	8.463849	72.05	1.536151	20
41	.467985	71.30	.999813	.07	.468172	71.37	.531828	19
42	.472263	70.58	.999809	.07	.472454	70.65	.527546	18
43	.476498	69.92	.999805	.07	.476693	69.98	.523307	17
44	.480693	69.25	.999801	.07	.480892	69.30	.519108	16
45	8.484848	68.58	9.999797	.05	8.485050	68.67	1.514950	15
46	.488963	67.95	.999794	.07	.489170	68.00	.510830	14
47	.493040	67.30	.999790	.07	.493250	67.38	.506750	13
48	.497078	66.70	.999786	.07	.497293	66.75	.502707	12
49	.501080	66.08	.999782	.07	.501298	66.15	.498702	11
50	8.505045	65.48	9.999778	.07	8.505267	65.55	1.494733	10
51	.508974	64.88	.999774	.08	.509200	64.97	.490800	9
52	.512867	64.32	.999769	.07	.513098	64.38	.486902	8
53	.516726	63.75	.999765	.07	.516961	63.82	.483039	7
54	.520551	63.20	.999761	.07	.520790	63.27	.479210	6
55	8.524343	62.65	9.999757	.07	8.524586	62.72	1.475414	5
56	.528102	62.10	.999753	.08	.528349	62.18	.471651	4
57	.531828	61.58	.999748	.07	.532080	61.65	.467920	3
58	.535523	61.05	.999744	.07	.535779	61.13	.464221	2
59	.539186	60.55	.999740	.08	.539447	60.62	.460553	1
60	8.542819		9.999735		8.543084		1.456916	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.542819	60.05	9.999735	.07	8.543084	60.12	1.456916	60
1	.546422	59.55	.999731	.08	.546691	59.62	.453309	59
2	.549995	59.07	.999726	.07	.550268	59.15	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.65	.446183	57
4	.557054	58.10	.999717	.07	.557336	58.20	.442664	56
5	8.560540	57.65	9.999713	.08	8.560828	57.72	1.439172	55
6	.563999	57.20	.999708	.07	.564291	57.27	.435709	54
7	.567431	56.75	.999704	.08	.567727	56.83	.432273	53
8	.570836	56.30	.999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	8.577566	55.43	9.999689	.07	8.577877	55.52	1.422123	50
11	.580892	55.02	.999685	.08	.581208	55.10	.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.20	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.78	.999670	.08	.591051	53.87	.408949	46
15	8.593948	53.40	9.999665	.08	8.594283	53.48	1.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.62	.999655	.08	.600677	52.70	.399323	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.85	.999645	.08	.606978	51.93	.393022	41
20	8.609734	51.48	9.999640	.08	8.610094	51.58	1.389906	40
21	.612823	51.13	.999635	.10	.613189	51.22	.386811	39
22	.615891	50.77	.999629	.08	.616262	50.85	.383738	38
23	.618937	50.42	.999624	.08	.619313	50.50	.380687	37
24	.621962	50.05	.999619	.08	.622343	50.15	.377657	36
25	8.624965	49.72	9.999614	.10	8.625352	49.80	1.374648	35
26	.627948	49.38	.999608	.08	.628340	49.47	.371660	34
27	.630911	49.05	.999603	.10	.631308	49.13	.368692	33
28	.633854	48.70	.999597	.08	.634256	48.80	.365744	32
29	.636776	48.40	.999592	.10	.637184	48.48	.362816	31
30	8.639680	48.05	9.999586	.08	8.640093	48.15	1.359907	30
31	.642563	47.75	.999581	.10	.642982	47.85	.357018	29
32	.645428	47.43	.999575	.08	.645853	47.52	.354147	28
33	.648274	47.13	.999570	.10	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.10	.651537	46.92	.348463	26
35	8.653911	46.52	9.999558	.08	8.654352	46.62	1.345648	25
36	.656702	46.22	.999553	.10	.657149	46.32	.342851	24
37	.659475	45.92	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.45	.334567	21
40	8.667689	45.07	9.999529	.08	8.668160	45.17	1.331840	20
41	.670393	44.78	.999524	.10	.670870	44.88	.329130	19
42	.673080	44.52	.999518	.10	.673563	44.60	.326437	18
43	.675751	44.23	.999512	.10	.676239	44.35	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.07	.321100	16
45	8.681043	43.70	9.999500	.12	8.681544	43.80	1.318456	15
46	.683665	43.45	.999493	.10	.684172	43.53	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.691438	42.67	.999475	.10	.691963	42.77	.308037	11
50	8.693998	42.42	9.999469	.10	8.694529	42.53	1.305471	10
51	.696543	42.17	.999463	.12	.697081	42.27	.302919	9
52	.699073	41.93	.999456	.10	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.12	.702139	41.78	.297861	7
54	.704090	41.45	.999443	.10	.704646	41.57	.295354	6
55	8.706577	41.20	9.999437	.10	8.707140	41.30	1.292860	5
56	.709049	40.97	.999431	.12	.709618	41.08	.290382	4
57	.711507	40.75	.999424	.10	.712083	40.85	.287917	3
58	.713952	40.52	.999418	.12	.714534	40.63	.285466	2
59	.716383	40.28	.999411	.12	.716972	40.40	.283028	1
60	8.718800		9.999404		8.719396		1.280604	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.718800	40.07	9.999404	.10	8.719396	40.17	1.280604	60
1	.721204	39.85	.999398	.12	.721806	39.97	.278194	59
2	.723595	39.62	.999391	.12	.724204	39.73	.275796	58
3	.725972	39.42	.999384	.10	.726588	39.52	.273412	57
4	.728337	39.18	.999378	.12	.728959	39.30	.271041	56
5	8.730688	38.98	9.999371	.12	8.731317	39.10	1.268683	55
6	.733027	38.78	.999364	.12	.733663	38.88	.266337	54
7	.735354	38.55	.999357	.12	.735996	38.68	.264004	53
8	.737667	38.37	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.17	.999343	.12	.740626	38.27	.259374	51
10	8.742259	37.95	9.999336	.12	8.742922	38.08	1.257078	50
11	.744536	37.77	.999329	.12	.745207	37.87	.254793	49
12	.746802	37.55	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.48	.250260	47
14	.751297	37.18	.999308	.12	.751989	37.30	.248011	46
15	8.753528	36.98	9.999301	.12	8.754227	37.10	1.245773	45
16	.755747	36.80	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.60	.999287	.13	.758668	36.73	.241332	43
18	.760151	36.43	.999279	.12	.760872	36.55	.239128	42
19	.762337	36.23	.999272	.12	.763065	36.35	.236935	41
20	8.764511	36.07	9.999265	.13	8.765246	36.18	1.234754	40
21	.766675	35.88	.999257	.12	.767417	36.02	.232583	39
22	.768828	35.70	.999250	.13	.769578	35.82	.230422	38
23	.770970	35.52	.999242	.12	.771727	35.65	.228273	37
24	.773101	35.37	.999235	.13	.773866	35.48	.226134	36
25	8.775223	35.17	9.999227	.12	8.775995	35.32	1.224005	35
26	.777333	35.02	.999220	.13	.778114	35.13	.221886	34
27	.779434	34.83	.999212	.12	.780222	34.97	.219778	33
28	.781524	34.68	.999205	.13	.782320	34.80	.217680	32
29	.783605	34.50	.999197	.13	.784408	34.63	.215592	31
30	8.785675	34.35	9.999189	.13	8.786486	34.47	1.213514	30
31	.787736	34.18	.999181	.12	.788554	34.32	.211446	29
32	.789787	34.02	.999174	.13	.790613	34.15	.209387	28
33	.791828	33.85	.999166	.13	.792662	33.98	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	8.795881	33.55	9.999150	.13	8.796731	33.68	1.203269	25
36	.797894	33.38	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.25	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.07	.999126	.13	.802765	33.22	.197235	22
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	8.805852	32.78	9.999110	.13	8.806742	32.92	1.193258	20
41	.807819	32.63	.999102	.13	.808717	32.77	.191283	19
42	.809777	32.48	.999094	.13	.810683	32.63	.189317	18
43	.811726	32.35	.999086	.15	.812641	32.47	.187359	17
44	.813667	32.20	.999077	.13	.814589	32.33	.185411	16
45	8.815599	32.05	9.999069	.13	8.816529	32.20	1.183471	15
46	.817522	31.90	.999061	.13	.818461	32.05	.181539	14
47	.819436	31.78	.999053	.15	.820384	31.90	.179616	13
48	.821343	31.62	.999044	.13	.822298	31.78	.177702	12
49	.823240	31.50	.999036	.15	.824205	31.63	.175795	11
50	8.825130	31.35	9.999027	.13	8.826103	31.48	1.173897	10
51	.827011	31.22	.999019	.15	.827992	31.37	.172008	9
52	.828884	31.08	.999010	.13	.829874	31.23	.170126	8
53	.830749	30.97	.999002	.15	.831748	31.08	.168252	7
54	.832607	30.82	.998993	.15	.833613	30.97	.166387	6
55	8.834456	30.68	9.998984	.13	8.835471	30.83	1.164529	5
56	.836297	30.55	.998976	.15	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.58	.160837	3
58	.839956	30.30	.998958	.13	.840998	30.45	.159002	2
59	.841774	30.18	.998950	.15	.842825	30.32	.157175	1
60	8.843585		9.998941		8.844644		1.155356	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.843585	30.03	9.998941	.15	8.844644	30.18	1.155356	60
1	.845387	29.93	.998932	.15	.846455	30.08	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	57
4	.850751	29.57	.998905	.15	.851846	29.70	.148154	56
5	8.852525	29.43	9.998896	.15	8.853628	29.58	1.146372	55
6	.854291	29.30	.998887	.15	.855403	29.47	.144597	54
7	.856049	29.20	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.08	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.95	.998860	.15	.860686	29.12	.139314	51
10	8.861283	28.85	9.998851	.17	8.862433	29.00	1.137567	50
11	.863014	28.73	.998841	.15	.864173	28.88	.135827	49
12	.864738	28.62	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.17	.867632	28.65	.132368	47
14	.868165	28.38	.998813	.15	.869351	28.55	.130649	46
15	8.869868	28.28	9.998804	.15	8.871064	28.43	1.128936	45
16	.871565	28.17	.998795	.17	.872770	28.32	.127230	44
17	.873255	28.05	.998785	.15	.874469	28.22	.125531	43
18	.874938	27.95	.998776	.17	.876162	28.12	.123838	42
19	.876615	27.83	.998766	.15	.877849	28.00	.122151	41
20	8.878285	27.73	9.998757	.17	8.879529	27.88	1.120471	40
21	.879949	27.63	.998747	.15	.881202	27.78	.118798	39
22	.881607	27.52	.998738	.17	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.17	.884530	27.58	.115470	37
24	.884903	27.32	.998718	.17	.886185	27.47	.113815	36
25	8.886542	27.20	9.998708	.15	8.887833	27.38	1.112167	35
26	.888174	27.12	.998699	.17	.889476	27.27	.110524	34
27	.889801	27.00	.998689	.17	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.17	.892742	27.07	.107258	32
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	8.894643	26.72	9.998659	.17	8.895984	26.87	1.104016	30
31	.896246	26.60	.998649	.17	.897596	26.78	.102404	29
32	.897842	26.50	.998639	.17	.899203	26.67	.100797	28
33	.899432	26.42	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.32	.998619	.17	.902398	26.48	.097602	26
35	8.902596	26.22	9.998609	.17	8.903987	26.38	1.096013	25
36	.904169	26.12	.998599	.17	.905570	26.28	.094430	24
37	.905736	26.02	.998589	.18	.907147	26.20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.85	.998568	.17	.910285	26.02	.089715	21
40	8.910404	25.75	9.998558	.17	8.911846	25.92	1.088154	20
41	.911949	25.65	.998548	.18	.913401	25.83	.086599	19
42	.913488	25.57	.998537	.17	.914951	25.73	.085049	18
43	.915022	25.47	.998527	.18	.916495	25.65	.083505	17
44	.916550	25.38	.998516	.17	.918034	25.57	.081966	16
45	8.918073	25.30	9.998506	.18	8.919568	25.47	1.080432	15
46	.919591	25.20	.998495	.17	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.18	.922619	25.28	.077381	13
48	.922610	25.03	.998474	.17	.924136	25.22	.075864	12
49	.924112	24.95	.998464	.18	.925649	25.12	.074351	11
50	8.925609	24.85	9.998453	.18	8.927156	25.03	1.072844	10
51	.927100	24.78	.998442	.18	.928658	24.95	.071342	9
52	.928587	24.68	.998431	.17	.930155	24.87	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	8.933015	24.43	9.998399	.18	8.934616	24.62	1.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	3
58	.937398	24.20	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.10	.998355	.18	.940494	24.30	.059506	1
60	8.940296		9.998344		8.941952		1.058048	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.940296	24.03	9.998344	.18	8.941952	24.20	1.058048	60
1	.941738	23.93	.998333	.18	.943404	24.13	.056596	59
2	.943174	23.87	.998322	.18	.944852	24.05	.055148	58
3	.944606	23.80	.998311	.18	.946295	23.98	.053705	57
4	.946034	23.70	.998300	.18	.947734	23.90	.052266	56
5	8.947456	23.63	9.998289	.20	8.949168	23.82	1.050832	55
6	.948874	23.55	.998277	.18	.950597	23.73	.049403	54
7	.950287	23.48	.998266	.18	.952021	23.67	.047979	53
8	.951696	23.40	.998255	.20	.953441	23.58	.046559	52
9	.953100	23.32	.998243	.18	.954856	23.52	.045144	51
10	8.954499	23.25	9.998232	.20	8.956267	23.45	1.043733	50
11	.955894	23.17	.998220	.18	.957674	23.35	.042326	49
12	.957284	23.10	.998209	.20	.959075	23.30	.040925	48
13	.958670	23.03	.998197	.18	.960473	23.22	.039527	47
14	.960052	22.95	.998186	.20	.961866	23.15	.038134	46
15	8.961429	22.87	9.998174	.18	8.963255	23.07	1.036745	45
16	.962801	22.82	.998163	.20	.964639	23.00	.035361	44
17	.964170	22.73	.998151	.20	.966019	22.92	.033981	43
18	.965534	22.65	.998139	.18	.967394	22.87	.032606	42
19	.966893	22.60	.998128	.20	.968766	22.78	.031234	41
20	8.968249	22.52	9.998116	.20	8.970133	22.72	1.029867	40
21	.969600	22.45	.998104	.20	.971496	22.65	.028504	39
22	.970947	22.37	.998092	.20	.972855	22.57	.027145	38
23	.972289	22.32	.998080	.20	.974209	22.52	.025791	37
24	.973628	22.23	.998068	.20	.975560	22.43	.024440	36
25	8.974962	22.18	9.998056	.20	8.976906	22.37	1.023094	35
26	.976293	22.10	.998044	.20	.978248	22.30	.021752	34
27	.977619	22.03	.998032	.20	.979586	22.25	.020414	33
28	.978941	21.97	.998020	.20	.980921	22.17	.019079	32
29	.980259	21.90	.998008	.20	.982251	22.10	.017749	31
30	8.981573	21.83	9.997996	.20	8.983577	22.03	1.016423	30
31	.982883	21.77	.997984	.20	.984899	21.97	.015101	29
32	.984189	21.70	.997972	.22	.986217	21.92	.013783	28
33	.985491	21.63	.997959	.20	.987532	21.83	.012468	27
34	.986789	21.57	.997947	.20	.988842	21.78	.011158	26
35	8.988083	21.52	9.997935	.22	8.990149	21.70	1.009851	25
36	.989374	21.43	.997922	.20	.991451	21.65	.008549	24
37	.990660	21.38	.997910	.22	.992750	21.58	.007250	23
38	.991943	21.32	.997897	.20	.994045	21.53	.005955	22
39	.993222	21.25	.997885	.22	.995337	21.45	.004663	21
40	8.994497	21.18	9.997872	.20	8.996624	21.40	1.003376	20
41	.995768	21.13	.997860	.22	.997908	21.33	.002092	19
42	.997036	21.05	.997847	.20	.999188	21.28	.000812	18
43	.998299	21.02	.997835	.22	9.000465	21.22	0.999535	17
44	.999560	20.93	.997822	.22	.001738	21.15	.998262	16
45	9.000816	20.88	9.997809	.20	9.003007	21.08	0.996993	15
46	.002069	20.82	.997797	.22	.004272	21.03	.995728	14
47	.003318	20.75	.997784	.22	.005534	20.97	.994466	13
48	.004563	20.70	.997771	.22	.006792	20.92	.993208	12
49	.005805	20.65	.997758	.22	.008047	20.85	.991953	11
50	9.007044	20.57	9.997745	.22	9.009298	20.80	0.990702	10
51	.008278	20.53	.997732	.22	.010546	20.73	.989454	9
52	.009510	20.45	.997719	.22	.011790	20.68	.988210	8
53	.010737	20.42	.997706	.22	.013031	20.62	.986969	7
54	.011962	20.33	.997693	.22	.014268	20.57	.985732	6
55	9.013182	20.30	9.997680	.22	9.015502	20.50	0.984498	5
56	.014400	20.22	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.18	.997654	.22	.017959	20.40	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.33	.980817	2
59	.018031	20.07	.997628	.23	.020403	20.28	.979597	1
60	9.019235		9.997614		9.021620		0.978380	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.019235	20.00	9.997614	.22	9.021620	20.23	0.978380	60
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.88	.997588	.23	.024044	20.12	.975956	58
3	.022825	19.85	.997574	.22	.025251	20.07	.974749	57
4	.024016	19.78	.997561	.23	.026455	20.00	.973545	56
5	9.025203	19.72	9.997547	.22	9.027655	19.95	0.972345	55
6	.026386	19.68	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.22	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.80	.968763	52
9	.029918	19.52	.997493	.22	.032425	19.73	.967575	51
10	9.031089	19.47	9.997480	.23	9.033609	19.70	0.966391	50
11	.032257	19.40	.997466	.23	.034791	19.63	.965209	49
12	.033421	19.35	.997452	.22	.035969	19.58	.964031	48
13	.034582	19.32	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	9.036896	19.20	9.997411	.23	9.039485	19.43	0.960515	45
16	.038048	19.15	.997397	.23	.040651	19.37	.959349	44
17	.039197	19.08	.997383	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	9.042625	18.95	9.997341	.23	9.045284	19.17	0.954716	40
21	.043762	18.88	.997327	.23	.046434	19.13	.953566	39
22	.044895	18.85	.997313	.23	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.23	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.23	.049869	18.98	.950131	36
25	9.048279	18.68	9.997271	.23	9.051008	18.93	0.948992	35
26	.049400	18.65	.997257	.25	.052144	18.88	.947856	34
27	.050519	18.60	.997242	.23	.053277	18.83	.946723	33
28	.051635	18.57	.997228	.23	.054407	18.80	.945593	32
29	.052749	18.50	.997214	.25	.055535	18.73	.944465	31
30	9.053859	18.45	9.997199	.23	9.056659	18.70	0.943341	30
31	.054966	18.42	.997185	.25	.057781	18.65	.942219	29
32	.056071	18.35	.997170	.23	.058900	18.60	.941100	28
33	.057172	18.32	.997156	.25	.060016	18.57	.939984	27
34	.058271	18.27	.997141	.23	.061130	18.50	.938870	26
35	9.059367	18.22	9.997127	.25	9.062240	18.47	0.937760	25
36	.060460	18.18	.997112	.23	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.25	.064453	18.38	.935547	23
38	.062639	18.08	.997083	.25	.065556	18.32	.934444	22
39	.063724	18.03	.997068	.25	.066655	18.28	.933345	21
40	9.064806	17.98	9.997053	.23	9.067752	18.23	0.932248	20
41	.065885	17.95	.997039	.25	.068846	18.20	.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.85	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.82	.996994	.25	.072113	18.07	.927887	16
45	9.070176	17.77	9.996979	.25	9.073197	18.02	0.926803	15
46	.071242	17.73	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.67	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.63	.996934	.25	.076432	17.88	.923568	12
49	.074424	17.60	.996919	.25	.077505	17.85	.922495	11
50	9.075480	17.55	9.996904	.25	9.078576	17.80	0.921424	10
51	.076533	17.50	.996889	.25	.079644	17.77	.920356	9
52	.077583	17.47	.996874	.27	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	.25	.082833	17.63	.917167	6
55	9.080719	17.33	9.996828	.27	9.083891	17.60	0.916109	5
56	.081759	17.30	.996812	.25	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.25	.086000	17.50	.914000	3
58	.083832	17.20	.996782	.27	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.25	.088098	17.43	.911902	1
60	9.085894		9.996751		9.089144		0.910856	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.085894	17.13	9.996751	.27	9.089144	17.38	0.910856	60
1	.086922	17.08	.996735	.25	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.27	.091228	17.30	.908772	58
3	.088970	17.00	.996704	.27	.092266	17.27	.907734	57
4	.089990	16.97	.996688	.25	.093302	17.23	.906698	56
5	9.091008	16.93	9.996673	.27	9.094336	17.18	0.905664	55
6	.092024	16.88	.996657	.27	.095367	17.13	.904633	54
7	.093037	16.83	.996641	.27	.096395	17.12	.903605	53
8	.094047	16.82	.996625	.25	.097422	17.07	.902578	52
9	.095056	16.77	.996610	.27	.098446	17.03	.901554	51
10	9.096062	16.72	9.996594	.27	9.099468	16.98	0.900532	50
11	.097065	16.68	.996578	.27	.100487	16.95	.899513	49
12	.098066	16.65	.996562	.27	.101504	16.92	.898496	48
13	.099065	16.62	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.83	.896468	46
15	9.101056	16.53	9.996514	.27	9.104542	16.80	0.895458	45
16	.102048	16.48	.996498	.27	.105550	16.77	.894450	44
17	.103037	16.47	.996482	.28	.106556	16.72	.893444	43
18	.104025	16.42	.996465	.27	.107559	16.68	.892441	42
19	.105010	16.37	.996449	.27	.108560	16.65	.891440	41
20	9.105992	16.35	9.996433	.27	9.109559	16.62	0.890441	40
21	.106973	16.30	.996417	.28	.110556	16.58	.889444	39
22	.107951	16.27	.996400	.27	.111551	16.53	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.20	.996368	.28	.113533	16.47	.886467	36
25	9.110873	16.15	9.996351	.27	9.114521	16.43	0.885479	35
26	.111842	16.12	.996335	.28	.115507	16.40	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.35	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.33	.882528	32
29	.114737	16.02	.996285	.27	.118452	16.28	.881548	31
30	9.115698	15.97	9.996269	.28	9.119429	16.25	0.880571	30
31	.116656	15.95	.996252	.28	.120404	16.22	.879596	29
32	.117613	15.90	.996235	.27	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.12	.876683	26
35	9.120469	15.80	9.996185	.28	9.124284	16.08	0.875716	25
36	.121417	15.75	.996168	.28	.125249	16.03	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.02	.873789	23
38	.123306	15.70	.996134	.28	.127172	15.97	.872828	22
39	.124248	15.65	.996117	.28	.128130	15.95	.871870	21
40	9.125187	15.63	9.996100	.28	9.129087	15.90	0.870913	20
41	.126125	15.58	.996083	.28	.130041	15.88	.869959	19
42	.127060	15.55	.996066	.28	.130994	15.83	.869006	18
43	.127993	15.53	.996049	.28	.131944	15.82	.868056	17
44	.128925	15.48	.996032	.28	.132893	15.77	.867107	16
45	9.129854	15.45	9.996015	.28	9.133839	15.75	0.866161	15
46	.130781	15.42	.995998	.30	.134784	15.70	.865216	14
47	.131706	15.40	.995980	.28	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.28	.136667	15.63	.863333	12
49	.133551	15.32	.995946	.30	.137605	15.62	.862395	11
50	9.134470	15.28	9.995928	.28	9.138542	15.57	0.861458	10
51	.135387	15.27	.995911	.28	.139476	15.55	.860524	9
52	.136303	15.22	.995894	.30	.140409	15.52	.859591	8
53	.137216	15.20	.995876	.28	.141340	15.48	.858660	7
54	.138128	15.15	.995859	.30	.142269	15.45	.857731	6
55	9.139037	15.12	9.995841	.30	9.143196	15.42	0.856804	5
56	.139944	15.10	.995823	.28	.144121	15.38	.855879	4
57	.140850	15.07	.995806	.30	.145044	15.37	.854956	3
58	.141754	15.02	.995788	.28	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.30	.146885	15.30	.853115	1
60	9.143555		9.995753		9.147803		0.852197	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.143555	14.97	9.995753	.30	9.147803	15.25	0.852197	60
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.88	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.83	.995681	.28	.151454	15.15	.848546	56
5	9.148026	14.82	9.995664	.30	9.152363	15.10	0.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.73	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.32	.155077	15.02	.844923	52
9	.151569	14.70	.995591	.30	.155978	14.98	.844022	51
10	9.152451	14.65	9.995573	.30	9.156877	14.97	0.843123	50
11	.153330	14.63	.995555	.30	.157775	14.93	.842225	49
12	.154208	14.58	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.55	.995501	.32	.160457	14.83	.839543	46
15	9.156830	14.50	9.995482	.30	9.161347	14.82	0.838653	45
16	.157700	14.48	.995464	.30	.162236	14.78	.837764	44
17	.158569	14.43	.995446	.32	.163123	14.75	.836877	43
18	.159435	14.43	.995427	.30	.164008	14.73	.835992	42
19	.160301	14.38	.995409	.32	.164892	14.70	.835108	41
20	9.161164	14.35	9.995390	.30	9.165774	14.67	0.834226	40
21	.162025	14.33	.995372	.32	.166654	14.63	.833346	39
22	.162885	14.30	.995353	.32	.167532	14.62	.832468	38
23	.163743	14.28	.995334	.30	.168409	14.58	.831591	37
24	.164600	14.23	.995316	.32	.169284	14.55	.830716	36
25	9.165454	14.22	9.995297	.32	9.170157	14.53	0.829843	35
26	.166307	14.20	.995278	.30	.171029	14.50	.828971	34
27	.167159	14.15	.995260	.32	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.32	.172767	14.45	.827233	32
29	.168856	14.10	.995222	.32	.173634	14.42	.826366	31
30	9.169702	14.08	9.995203	.32	9.174499	14.38	0.825501	30
31	.170547	14.03	.995184	.32	.175362	14.37	.824638	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	14.00	.995146	.32	.177084	14.30	.822916	27
34	.173070	13.97	.995127	.32	.177942	14.28	.822058	26
35	9.173908	13.93	9.995108	.32	9.178799	14.27	0.821201	25
36	.174744	13.90	.995089	.32	.179655	14.22	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.85	.995051	.32	.181360	14.18	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.13	.817789	21
40	9.178072	13.80	9.995013	.33	9.183059	14.13	0.816941	20
41	.178900	13.77	.994993	.32	.183907	14.08	.816093	19
42	.179726	13.75	.994974	.32	.184752	14.08	.815248	18
43	.180551	13.72	.994955	.33	.185597	14.03	.814403	17
44	.181374	13.70	.994935	.32	.186439	14.02	.813561	16
45	9.182196	13.67	9.994916	.33	9.187280	14.00	0.812720	15
46	.183016	13.63	.994896	.32	.188120	13.97	.811880	14
47	.183834	13.62	.994877	.33	.188958	13.93	.811042	13
48	.184651	13.58	.994857	.32	.189794	13.92	.810206	12
49	.185466	13.57	.994838	.33	.190629	13.88	.809371	11
50	9.186280	13.53	9.994818	.33	9.191462	13.87	0.808538	10
51	.187092	13.52	.994798	.32	.192294	13.83	.807706	9
52	.187903	13.48	.994779	.33	.193124	13.82	.806876	8
53	.188712	13.45	.994759	.33	.193953	13.78	.806047	7
54	.189519	13.43	.994739	.32	.194780	13.77	.805220	6
55	9.190325	13.42	9.994720	.33	9.195606	13.73	0.804394	5
56	.191130	13.38	.994700	.33	.196430	13.72	.803570	4
57	.191933	13.35	.994680	.33	.197253	13.68	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.67	.801926	2
59	.193534	13.30	.994640	.33	.198894	13.65	.801106	1
60	9.194332		9.994620		9.199713		0.800287	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.194332	13.28	9.994620	.33	9.199713	13.60	0.800287	60
1	.195129	13.27	.994600	.33	.200529	13.60	.799471	59
2	.195925	13.23	.994580	.33	.201345	13.57	.798655	58
3	.196719	13.20	.994560	.33	.202159	13.53	.797841	57
4	.197511	13.18	.994540	.35	.202971	13.52	.797029	56
5	9.198302	13.15	9.994519	.33	9.203782	13.50	0.796218	55
6	.199091	13.13	.994499	.33	.204592	13.47	.795408	54
7	.199879	13.12	.994479	.33	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.35	.206207	13.43	.793793	52
9	.201451	13.05	.994438	.33	.207013	13.40	.792987	51
10	9.202234	13.05	9.994418	.33	9.207817	13.37	0.792183	50
11	.203017	13.00	.994398	.35	.208619	13.35	.791381	49
12	.203797	13.00	.994377	.33	.209420	13.33	.790580	48
13	.204577	12.95	.994357	.35	.210220	13.30	.789780	47
14	.205354	12.95	.994336	.33	.211018	13.28	.788982	46
15	9.206131	12.92	9.994316	.35	9.211815	13.27	0.788185	45
16	.206906	12.88	.994295	.35	.212611	13.23	.787389	44
17	.207679	12.88	.994274	.33	.213405	13.22	.786595	43
18	.208452	12.83	.994254	.35	.214198	13.18	.785802	42
19	.209222	12.83	.994233	.35	.214989	13.18	.785011	41
20	9.209992	12.80	9.994212	.35	9.215780	13.13	0.784220	40
21	.210760	12.77	.994191	.33	.216568	13.13	.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.07	.781858	37
24	.213055	12.72	.994129	.35	.218926	13.07	.781074	36
25	9.213818	12.68	9.994108	.35	9.219710	13.03	0.780290	35
26	.214579	12.65	.994087	.35	.220492	13.00	.779508	34
27	.215338	12.65	.994066	.35	.221272	13.00	.778728	33
28	.216097	12.62	.994045	.35	.222052	12.97	.777948	32
29	.216854	12.58	.994024	.35	.222830	12.95	.777170	31
30	9.217609	12.57	9.994003	.35	9.223607	12.92	0.776393	30
31	.218363	12.55	.993982	.37	.224382	12.90	.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	.225929	12.85	.774071	27
34	.220618	12.48	.993918	.35	.226700	12.85	.773300	26
35	9.221367	12.47	9.993897	.37	9.227471	12.80	0.772529	25
36	.222115	12.43	.993875	.35	.228239	12.80	.771761	24
37	.222861	12.42	.993854	.37	.229007	12.77	.770993	23
38	.223606	12.38	.993832	.35	.229773	12.77	.770227	22
39	.224349	12.38	.993811	.37	.230539	12.72	.769461	21
40	9.225092	12.35	9.993789	.35	9.231302	12.72	0.768698	20
41	.225833	12.33	.993768	.37	.232065	12.68	.767935	19
42	.226573	12.30	.993746	.35	.232826	12.67	.767174	18
43	.227311	12.28	.993725	.37	.233586	12.65	.766414	17
44	.228048	12.27	.993703	.37	.234345	12.63	.765655	16
45	9.228784	12.23	9.993681	.35	9.235103	12.60	0.764897	15
46	.229518	12.23	.993660	.37	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.37	.236614	12.57	.763386	13
48	.230984	12.18	.993616	.37	.237368	12.53	.762632	12
49	.231715	12.15	.993594	.37	.238120	12.53	.761880	11
50	9.232444	12.13	9.993572	.37	9.238872	12.50	0.761128	10
51	.233172	12.12	.993550	.37	.239622	12.48	.760378	9
52	.233899	12.10	.993528	.37	.240371	12.45	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.45	.758882	7
54	.235349	12.07	.993484	.37	.241865	12.42	.758135	6
55	9.236073	12.03	9.993462	.37	9.242610	12.40	0.757390	5
56	.236795	12.00	.993440	.37	.243354	12.38	.756646	4
57	.237515	12.00	.993418	.37	.244097	12.37	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.33	.755161	2
59	.238953	11.95	.993374	.38	.245579	12.33	.754421	1
60	9.239670		9.993351		9.246319		0.753681	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.239670	11.93	9.993351	.37	9.246319	12.30	0.753681	60
1	.240386	11.92	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.88	.993307	.38	.247794	12.27	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.23	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.23	.750736	56
5	9.243237	11.83	9.993240	.38	9.249998	12.20	0.750002	55
6	.243947	11.82	.993217	.37	.250730	12.18	.749270	54
7	.244656	11.78	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.77	.993149	.37	.252920	12.13	.747080	51
10	9.246775	11.72	9.993127	.38	9.253648	12.10	0.746352	50
11	.247478	11.72	.993104	.38	.254374	12.10	.745626	49
12	.248181	11.70	.993081	.37	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	9.250282	11.63	9.993013	.38	9.257269	12.02	0.742731	45
16	.250980	11.62	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.60	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.57	.992944	.38	.259429	11.95	.740571	42
19	.253067	11.57	.992921	.38	.260146	11.95	.739854	41
20	9.253761	11.53	9.992898	.38	9.260863	11.92	0.739137	40
21	.254453	11.52	.992875	.38	.261578	11.90	.738422	39
22	.255144	11.50	.992852	.38	.262292	11.88	.737708	38
23	.255834	11.48	.992829	.38	.263005	11.87	.736995	37
24	.256523	11.47	.992806	.38	.263717	11.85	.736283	36
25	9.257211	11.45	9.992783	.40	9.264428	11.83	0.735572	35
26	.257898	11.42	.992759	.38	.265138	11.82	.734862	34
27	.258583	11.42	.992736	.38	.265847	11.80	.734153	33
28	.259268	11.38	.992713	.38	.266555	11.77	.733445	32
29	.259951	11.37	.992690	.40	.267261	11.77	.732739	31
30	9.260633	11.35	9.992666	.38	9.267967	11.73	0.732033	30
31	.261314	11.33	.992643	.40	.268671	11.73	.731329	29
32	.261994	11.32	.992619	.38	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.40	.270077	11.70	.729923	27
34	.263351	11.27	.992572	.38	.270779	11.67	.729221	26
35	9.264027	11.27	9.992549	.40	9.271479	11.65	0.728521	25
36	.264703	11.23	.992525	.40	.272178	11.63	.727822	24
37	.265377	11.23	.992501	.38	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.20	.992454	.40	.274269	11.58	.725731	21
40	9.267395	11.17	9.992430	.40	9.274964	11.57	0.725036	20
41	.268065	11.15	.992406	.40	.275658	11.55	.724342	19
42	.268734	11.13	.992382	.38	.276351	11.53	.723649	18
43	.269402	11.12	.992359	.40	.277043	11.52	.722957	17
44	.270069	11.10	.992335	.40	.277734	11.50	.722266	16
45	9.270735	11.08	9.992311	.40	9.278424	11.48	0.721576	15
46	.271400	11.07	.992287	.40	.279113	11.47	.720887	14
47	.272064	11.03	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.42	.280488	11.43	.719512	12
49	.273388	11.02	.992214	.40	.281174	11.40	.718826	11
50	9.274049	10.98	9.992190	.40	9.281858	11.40	0.718142	10
51	.274708	10.98	.992166	.40	.282542	11.38	.717458	9
52	.275367	10.97	.992142	.40	.283225	11.37	.716775	8
53	.276025	10.93	.992118	.42	.283907	11.35	.716093	7
54	.276681	10.93	.992093	.40	.284588	11.33	.715412	6
55	9.277337	10.90	9.992069	.42	9.285268	11.32	0.714732	5
56	.277991	10.90	.992044	.40	.285947	11.28	.714053	4
57	.278645	10.87	.992020	.40	.286624	11.28	.713376	3
58	.279297	10.85	.991996	.42	.287301	11.27	.712699	2
59	.279948	10.85	.991971	.40	.287977	11.25	.712023	1
60	9.280599		9.991947		9 288652		0.711348	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

Table XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.280599	10.82	9.991947	.42	9.288652	11.23	0.711348	60
1	.281248	10.82	.991922	.42	.289326	11.22	.710674	59
2	.281897	10.78	.991897	.40	.289999	11.20	.710001	58
3	.282544	10.77	.991873	.42	.290671	11.18	.709329	57
4	.283190	10.77	.991848	.42	.291342	11.18	.708658	56
5	9.283836	10.73	9.991823	.40	9.292013	11.15	0.707987	55
6	.284480	10.73	.991799	.42	.292682	11.13	.707318	54
7	.285124	10.70	.991774	.42	.293350	11.12	.706650	53
8	.285766	10.70	.991749	.42	.294017	11.12	.705983	52
9	.286408	10.67	.991724	.42	.294684	11.08	.705316	51
10	9.287048	10.67	9.991699	.42	9.295349	11.07	0.704651	50
11	.287688	10.63	.991674	.42	.296013	11.07	.703987	49
12	.288326	10.63	.991649	.42	.296677	11.03	.703323	48
13	.288964	10.60	.991624	.42	.297339	11.03	.702661	47
14	.289600	10.60	.991599	.42	.298001	11.02	.701999	46
15	9.290236	10.57	9.991574	.42	9.298662	11.00	0.701338	45
16	.290870	10.57	.991549	.42	.299322	10.97	.700678	44
17	.291504	10.55	.991524	.43	.299980	10.97	.700020	43
18	.292137	10.52	.991498	.42	.300638	10.95	.699362	42
19	.292768	10.52	.991473	.42	.301295	10.93	.698705	41
20	9.293399	10.50	9.991448	.43	9.301951	10.93	0.698049	40
21	.294029	10.48	.991422	.42	.302607	10.90	.697393	39
22	.294658	10.47	.991397	.42	.303261	10.88	.696739	38
23	.295286	10.45	.991372	.43	.303914	10.88	.696086	37
24	.295913	10.43	.991346	.42	.304567	10.85	.695433	36
25	9.296539	10.42	9.991321	.43	9.305218	10.85	0.694782	35
26	.297164	10.40	.991295	.42	.305869	10.83	.694131	34
27	.297788	10.40	.991270	.43	.306519	10.82	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10.35	.991218	.42	.307816	10.78	.692184	31
30	9.299655	10.35	9.991193	.43	9.308463	10.77	0.691537	30
31	.300276	10.32	.991167	.43	.309109	10.75	.690891	29
32	.300895	10.32	.991141	.43	.309754	10.75	.690246	28
33	.301514	10.30	.991115	.42	.310399	10.72	.689601	27
34	.302132	10.27	.991090	.43	.311042	10.72	.688958	26
35	9.302748	10.27	9.991064	.43	9.311685	10.70	0.688315	25
36	.303364	10.25	.991038	.43	.312327	10.68	.687673	24
37	.303979	10.23	.991012	.43	.312968	10.67	.687032	23
38	.304593	10.23	.990986	.43	.313608	10.65	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.63	.685753	21
40	9.305819	10.18	9.990934	.43	9.314885	10.63	0.685115	20
41	.306430	10.18	.990908	.43	.315523	10.60	.684477	19
42	.307041	10.15	.990882	.45	.316159	10.60	.683841	18
43	.307650	10.15	.990855	.43	.316795	10.58	.683205	17
44	.308259	10.13	.990829	.43	.317430	10.57	.682570	16
45	9.308867	10.12	9.990803	.43	9.318064	10.55	0.681936	15
46	.309474	10.10	.990777	.45	.318697	10.55	.681303	14
47	.310080	10.08	.990750	.43	.319330	10.52	.680670	13
48	.310685	10.07	.990724	.45	.319961	10.52	.680039	12
49	.311289	10.07	.990697	.43	.320592	10.50	.679408	11
50	9.311893	10.03	9.990671	.43	9.321222	10.48	0.678778	10
51	.312495	10.03	.990645	.45	.321851	10.47	.678149	9
52	.313097	10.02	.990618	.45	.322479	10.45	.677521	8
53	.313698	9.98	.990591	.43	.323106	10.45	.676894	7
54	.314297	10.00	.990565	.45	.323733	10.42	.676267	6
55	9.314897	9.97	9.990538	.45	9.324358	10.42	0.675642	5
56	.315495	9.95	.990511	.43	.324983	10.40	.675017	4
57	.316092	9.95	.990485	.45	.325607	10.40	.674393	3
58	.316689	9.92	.990458	.45	.326231	10.37	.673769	2
59	.317284	9.92	.990431	.45	.326853	10.37	.673147	1
60	9.317879		9.990404		9.327475		0.672525	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.317879	9.90	9.990404	.43	9.327475	10.33	0.672525	60
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.85	.990324	.45	.329334	10.32	.670666	57
4	.320249	9.85	.990297	.45	.329953	10.28	.670047	56
5	9.320840	9.83	9.990270	.45	9.330570	10.28	0.669430	55
6	.321430	9.82	.990243	.47	.331187	10.27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.78	.990188	.45	.332418	10.25	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.22	.666967	51
10	9.323780	9.77	9.990134	.45	9.333646	10.22	0.666354	50
11	.324366	9.73	.990107	.47	.334259	10.20	.665741	49
12	.324950	9.73	.990079	.45	.334871	10.18	.665129	48
13	.325534	9.72	.990052	.45	.335482	10.18	.664518	47
14	.326117	9.72	.990025	.47	.336093	10.15	.663907	46
15	9.326700	9.68	9.989997	.45	9.336702	10.15	0.663298	45
16	.327281	9.68	.989970	.47	.337311	10.13	.662689	44
17	.327862	9.67	.989942	.45	.337919	10.13	.662081	43
18	.328442	9.65	.989915	.47	.338527	10.10	.661473	42
19	.329021	9.63	.989887	.45	.339133	10.10	.660867	41
20	9.329599	9.62	9.989860	.47	9.339739	10.08	0.660261	40
21	.330176	9.62	.989832	.47	.340344	10.07	.659656	39
22	.330753	9.60	.989804	.45	.340948	10.07	.659052	38
23	.331329	9.57	.989777	.47	.341552	10.05	.658448	37
24	.331903	9.58	.989749	.47	.342155	10.03	.657845	36
25	9.332478	9.55	9.989721	.47	9.342757	10.02	0.657243	35
26	.333051	9.55	.989693	.47	.343358	10.00	.656642	34
27	.333624	9.52	.989665	.47	.343958	10.00	.656042	33
28	.334195	9.53	.989637	.45	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	9.335337	9.48	9.989582	.48	9.345755	9.97	0.654245	30
31	.335906	9.48	.989553	.47	.346353	9.93	.653647	29
32	.336475	9.47	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.93	.652455	27
34	.337610	9.43	.989469	.47	.348141	9.90	.651859	26
35	9.338176	9.43	9.989441	.47	9.348735	9.90	0.651265	25
36	.338742	9.42	.989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	.989385	.48	.349922	9.87	.650078	23
38	.339871	9.38	.989356	.47	.350514	9.87	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	9.340996	9.37	9.989300	.48	9.351697	9.83	0.648303	20
41	.341558	9.35	.989271	.47	.352287	9.82	.647713	19
42	.342119	9.33	.989243	.48	.352876	9.82	.647124	18
43	.342679	9.33	.989214	.47	.353465	9.80	.646535	17
44	.343239	9.30	.989186	.48	.354053	9.78	.645947	16
45	9.343797	9.30	9.989157	.48	9.354640	9.78	0.645360	15
46	.344355	9.28	.989128	.47	.355227	9.77	.644773	14
47	.344912	9.28	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.25	.989071	.48	.356398	9.73	.643602	12
49	.346024	9.25	.989042	.47	.356982	9.73	.643018	11
50	9.346579	9.25	9.989014	.48	9.357566	9.72	0.642434	10
51	.347134	9.22	.988985	.48	.358149	9.70	.641851	9
52	.347687	9.22	.988956	.48	.358731	9.70	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.67	.640687	7
54	.348792	9.18	.988898	.48	.359893	9.68	.640107	6
55	9.349343	9.17	9.988869	.48	9.360474	9.65	0.639526	5
56	.349893	9.17	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.48	.361632	9.63	.638368	3
58	.350992	9.13	.988782	.48	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.48	.362787	9.62	.637213	1
60	9.352088		9.988724		9.363364		0.636636	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.352088	9.12	9.988724	.48	9.363364	9.60	0.636636	60
1	.352635	9.10	.988695	.48	.363940	9.58	.636060	59
2	.353181	9.08	.988666	.50	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.48	.365090	9.57	.634910	57
4	.354271	9.07	.988607	.48	.365664	9.55	.634336	56
5	9.354815	9.05	9.988578	.50	9.366237	9.55	0.633763	55
6	.355358	9.05	.988548	.48	.366810	9.53	.633190	54
7	.355901	9.03	.988519	.50	.367382	9.52	.632618	53
8	.356443	9.02	.988489	.48	.367953	9.52	.632047	52
9	.356984	9.00	.988460	.50	.368524	9.50	.631476	51
10	9.357524	9.00	9.988430	.48	9.369094	9.48	0.630906	50
11	.358064	8.98	.988401	.50	.369663	9.48	.630337	49
12	.358603	8.97	.988371	.48	.370232	9.45	.629768	48
13	.359141	8.95	.988342	.50	.370799	9.47	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.43	.628633	46
15	9.360215	8.95	9.988282	.50	9.371933	9.43	0.628067	45
16	.360752	8.92	.988252	.48	.372499	9.42	.627501	44
17	.361287	8.92	.988223	.50	.373064	9.42	.626936	43
18	.361822	8.90	.988193	.50	.373629	9.40	.626371	42
19	.362356	8.88	.988163	.50	.374193	9.38	.625807	41
20	9.362889	8.88	9.988133	.50	9.374756	9.38	0.625244	40
21	.363422	8.87	.988103	.50	.375319	9.37	.624681	39
22	.363954	8.85	.988073	.50	.375881	9.35	.624119	38
23	.364485	8.85	.988043	.50	.376442	9.35	.623558	37
24	.365016	8.83	.988013	.50	.377003	9.33	.622997	36
25	9.365546	8.82	9.987983	.50	9.377563	9.32	0.622437	35
26	.366075	8.82	.987953	.52	.378122	9.32	.621878	34
27	.366604	8.78	.987922	.50	.378681	9.30	.621319	33
28	.367131	8.80	.987892	.50	.379239	9.30	.620761	32
29	.367659	8.77	.987862	.50	.379797	9.28	.620203	31
30	9.368185	8.77	9.987832	.52	9.380354	9.27	0.619646	30
31	.368711	8.75	.987801	.50	.380910	9.27	.619090	29
32	.369236	8.75	.987771	.52	.381466	9.23	.618534	28
33	.369761	8.73	.987740	.50	.382020	9.25	.617980	27
34	.370285	8.72	.987710	.52	.382575	9.23	.617425	26
35	9.370808	8.70	9.987679	.50	9.383129	9.22	0.616871	25
36	.371330	8.70	.987649	.52	.383682	9.20	.616318	24
37	.371852	8.68	.987618	.50	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.52	.384786	9.18	.615214	22
39	.372894	8.67	.987557	.52	.385337	9.18	.614663	21
40	9.373414	8.65	9.987526	.50	9.385888	9.17	0.614112	20
41	.373933	8.65	.987496	.52	.386438	9.15	.613562	19
42	.374452	8.63	.987465	.52	.386987	9.15	.613013	18
43	.374970	8.62	.987434	.52	.387536	9.13	.612464	17
44	.375487	8.60	.987403	.52	.388084	9.12	.611916	16
45	9.376003	8.60	9.987372	.52	9.388631	9.12	0.611369	15
46	.376519	8.60	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.57	.987310	.52	.389724	9.10	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.57	.987248	.52	.390815	9.08	.609185	11
50	9.378577	8.53	9.987217	.52	9.391360	9.05	0.608640	10
51	.379089	8.53	.987186	.52	.391903	9.07	.608097	9
52	.379601	8.53	.987155	.52	.392447	9.03	.607553	8
53	.380113	8.52	.987124	.53	.392989	9.03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.03	.606469	6
55	9.381134	8.48	9.987061	.52	9.394073	9.02	0.605927	5
56	.381643	8.48	.987030	.53	.394614	9.00	.605386	4
57	.382152	8.48	.986998	.52	.395154	9.00	.604846	3
58	.382661	8.45	.986967	.52	.395694	8.98	.604306	2
59	.383168	8.45	.986936	.53	.396233	8.97	.603767	1
60	9.383675		9.986904		9.396771		0.603229	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.383675	8.45	9.986904	.52	9.396771	8.97	0.603229	60
1	.384182	8.42	.986873	.53	.397309	8.95	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.42	.986809	.52	.398383	8.93	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	9.386201	8.38	9.986746	.53	9.399455	8.92	0.600545	55
6	.386704	8.38	.986714	.52	.399990	8.90	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.35	.986651	.53	.401058	8.88	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	9.388711	8.33	9.986587	.53	9.402124	8.87	0.597876	50
11	.389211	8.33	.986555	.53	.402656	8.85	.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.30	.986491	.53	.403718	8.85	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.82	.595751	46
15	9.391206	8.28	9.986427	.53	9.404778	8.83	0.595222	45
16	.391703	8.27	.986395	.53	.405308	8.80	.594692	44
17	.392199	8.27	.986363	.53	.405836	8.80	.594164	43
18	.392695	8.27	.986331	.53	.406364	8.80	.593636	42
19	.393191	8.23	.986299	.55	.406892	8.78	.593108	41
20	9.393685	8.23	9.986266	.53	9.407419	8.77	0.592581	40
21	.394179	8.23	.986234	.53	.407945	8.77	.592055	39
22	.394673	8.22	.986202	.55	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.53	.408996	8.75	.591004	37
24	.395658	8.20	.986137	.55	.409521	8.73	.590479	36
25	9.396150	8.18	9.986104	.53	9.410045	8.73	0.589955	35
26	.396641	8.18	.986072	.55	.410569	8.72	.589431	34
27	.397132	8.15	.986039	.53	.411092	8.72	.588908	33
28	.397621	8.17	.986007	.55	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.53	.412137	8.68	.587863	31
30	9.398600	8.13	9.985942	.55	9.412658	8.68	0.587342	30
31	.399088	8.12	.985909	.55	.413179	8.67	.586821	29
32	.399575	8.12	.985876	.55	.413699	8.67	.586301	28
33	.400062	8.12	.985843	.53	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	9.401035	8.08	9.985778	.55	9.415257	8.63	0.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.05	.985679	.55	.416810	8.60	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	9.403455	8.05	9.985613	.55	9.417842	8.60	0.582158	20
41	.403938	8.03	.985580	.55	.418358	8.58	.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.02	.985514	.57	.419387	8.57	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.57	.580099	16
45	9.405862	7.98	9.985447	.55	9.420415	8.53	0.579585	15
46	.406341	7.98	.985414	.55	.420927	8.55	.579073	14
47	.406820	7.98	.985381	.57	.421440	8.53	.578560	13
48	.407299	7.97	.985347	.55	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.57	.422463	8.52	.577537	11
50	9.408254	7.95	9.985280	.55	9.422974	8.50	0.577026	10
51	.408731	7.93	.985247	.57	.423484	8.48	.576516	9
52	.409207	7.92	.985213	.55	.423993	8.50	.576007	8
53	.409682	7.92	.985180	.57	.424503	8.47	.575497	7
54	.410157	7.92	.985146	.55	.425011	8.47	.574989	6
55	9.410632	7.90	9.985113	.57	9.425519	8.47	0.574481	5
56	.411106	7.88	.985079	.57	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.57	.426534	8.45	.573466	3
58	.412052	7.87	.985011	.55	.427041	8.43	.572959	2
59	.412524	7.87	.984978	.57	.427547	8.42	.572453	1
60	9.412996		9.984944		9.428052		0.571948	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.412996	7.85	9.984944	.57	9.428052	8.43	0.571948	60
1	.413467	7.85	.984910	.57	.428558	8.40	.571442	59
2	.413938	7.83	.984876	.57	.429062	8.40	.570938	58
3	.414408	7.83	.984842	.57	.429566	8.40	.570434	57
4	.414878	7.82	.984808	.57	.430070	8.38	.569930	56
5	9.415347	7.80	9.984774	.57	9.430573	8.37	0.569427	55
6	.415815	7.80	.984740	.57	.431075	8.37	.568925	54
7	.416283	7.80	.984706	.57	.431577	8.37	.568423	53
8	.416751	7.77	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.78	.984638	.58	.432580	8.33	.567420	51
10	9.417684	7.77	9.984603	.57	9.433080	8.33	0.566920	50
11	.418150	7.75	.984569	.57	.433580	8.33	.566420	49
12	.418615	7.73	.984535	.58	.434080	8.32	.565920	48
13	.419079	7.75	.984500	.57	.434579	8.32	.565421	47
14	.419544	7.72	.984466	.57	.435078	8.30	.564922	46
15	9.420007	7.72	9.984432	.58	9.435576	8.28	0.564424	45
16	.420470	7.72	.984397	.57	.436073	8.28	.563927	44
17	.420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.70	.984328	.57	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.27	.562437	41
20	9.422318	7.67	9.984259	.58	9.438059	8.25	0.561941	40
21	.422778	7.67	.984224	.57	.438554	8.23	.561446	39
22	.423238	7.65	.984190	.58	.439048	8.25	.560952	38
23	.423697	7.65	.984155	.58	.439543	8.22	.560457	37
24	.424156	7.65	.984120	.58	.440036	8.22	.559964	36
25	9.424615	7.63	9.984085	.58	9.440529	8.22	0.559471	35
26	.425073	7.62	.984050	.58	.441022	8.20	.558978	34
27	.425530	7.62	.984015	.57	.441514	8.20	.558486	33
28	.425987	7.60	.983981	.58	.442006	8.18	.557994	32
29	.426443	7.60	.983946	.58	.442497	8.18	.557503	31
30	9.426899	7.58	9.983911	.60	9.442988	8.18	0.557012	30
31	.427354	7.58	.983875	.58	.443479	8.15	.556521	29
32	.427809	7.57	.983840	.58	.443968	8.17	.556032	28
33	.428263	7.57	.983805	.58	.444458	8.15	.555542	27
34	.428717	7.55	.983770	.58	.444947	8.13	.555053	26
35	9.429170	7.55	9.983735	.58	9.445435	8.13	0.554565	25
36	.429623	7.53	.983700	.60	.445923	8.13	.554077	24
37	.430075	7.53	.983664	.58	.446411	8.12	.553589	23
38	.430527	7.52	.983629	.58	.446898	8.10	.553102	22
39	.430978	7.52	.983594	.60	.447384	8.10	.552616	21
40	9.431429	7.50	9.983558	.58	9.447870	8.10	0.552130	20
41	.431879	7.50	.983523	.60	.448356	8.08	.551644	19
42	.432329	7.48	.983487	.58	.448841	8.08	.551159	18
43	.432778	7.47	.983452	.60	.449326	8.07	.550674	17
44	.433226	7.48	.983416	.58	.449810	8.07	.550190	16
45	9.433675	7.45	9.983381	.60	9.450294	8.05	0.549706	15
46	.434122	7.45	.983345	.60	.450777	8.05	.549223	14
47	.434569	7.45	.983309	.60	.451260	8.05	.548740	13
48	.435016	7.43	.983273	.58	.451743	8.03	.548257	12
49	.435462	7.43	.983238	.60	.452225	8.02	.547775	11
50	9.435908	7.42	9.983202	.60	9.452706	8.02	0.547294	10
51	.436353	7.42	.983166	.60	.453187	8.02	.546813	9
52	.436798	7.40	.983130	.60	.453668	8.00	.546332	8
53	.437242	7.40	.983094	.60	.454148	8.00	.545852	7
54	.437686	7.38	.983058	.60	.454628	7.98	.545372	6
55	9.438129	7.38	9.983022	.60	9.455107	7.98	0.544893	5
56	.438572	7.37	.982986	.60	.455586	7.97	.544414	4
57	.439014	7.37	.982950	.60	.456064	7.97	.543936	3
58	.439456	7.35	.982914	.60	.456542	7.95	.543458	2
59	.439897	7.35	.982878	.60	.457019	7.95	.542981	1
60	9.440338		9.982842		9.457496		0.542504	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.440338	7.33	9.982842	.62	9.457496	7.95	0.542504	60
1	.440778	7.33	.982805	.60	.457973	7.93	.542027	59
2	.441218	7.33	.982769	.60	.458449	7.93	.541551	58
3	.441658	7.30	.982733	.62	.458925	7.92	.541075	57
4	.442096	7.32	.982696	.60	.459400	7.92	.540600	56
5	9.442535	7.30	9.982660	.60	9.459875	7.90	0.540125	55
6	.442973	7.28	.982624	.62	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.60	.460823	7.90	.539177	53
8	.443847	7.28	.982551	.62	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.62	.461770	7.87	.538230	51
10	9.444720	7.25	9.982477	.60	9.462242	7.88	0.537758	50
11	.445155	7.25	.982441	.62	.462715	7.85	.537285	49
12	.445590	7.25	.982404	.62	.463186	7.87	.536814	48
13	.446025	7.23	.982367	.60	.463658	7.83	.536342	47
14	.446459	7.23	.982331	.62	.464128	7.85	.535872	46
15	9.446893	7.22	9.982294	.62	9.464599	7.83	0.535401	45
16	.447326	7.22	.982257	.62	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.82	.533992	42
19	.448623	7.18	.982146	.62	.466477	7.80	.533523	41
20	9.449054	7.18	9.982109	.62	9.466945	7.80	0.533055	40
21	.449485	7.17	.982072	.62	.467413	7.78	.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.17	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	9.451204	7.13	9.981924	.63	9.469280	7.77	0.530720	35
26	.451632	7.13	.981886	.62	.469746	7.75	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.63	.470676	7.75	.529324	32
29	.452915	7.12	.981774	.62	.471141	7.73	.528859	31
30	9.453342	7.10	9.981737	.62	9.471605	7.73	0.528395	30
31	.453768	7.10	.981700	.63	.472069	7.72	.527931	29
32	.454194	7.08	.981662	.62	.472532	7.72	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.70	.527005	27
34	.455044	7.08	.981587	.63	.473457	7.70	.526543	26
35	9.455469	7.07	9.981549	.62	9.473919	7.70	0.526081	25
36	.455893	7.05	.981512	.63	.474381	7.68	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.05	.981436	.62	.475303	7.67	.524697	22
39	.457162	7.03	.981399	.63	.475763	7.67	.524237	21
40	9.457584	7.03	9.981361	.63	9.476223	7.67	0.523777	20
41	.458006	7.02	.981323	.63	.476683	7.65	.523317	19
42	.458427	7.02	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.00	.981247	.63	.477601	7.63	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	9.459688	7.00	9.981171	.63	9.478517	7.63	0.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.63	.479432	7.62	.520568	13
48	.460946	6.97	.981057	.63	.479889	7.60	.520111	12
49	.461364	6.97	.981019	.63	.480345	7.60	.519655	11
50	9.461782	6.95	9.980981	.65	9.480801	7.60	0.519199	10
51	.462199	6.95	.980942	.63	.481257	7.58	.518743	9
52	.462616	6.93	.980904	.63	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.65	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.63	.482621	7.57	.517379	6
55	9.463864	6.92	9.980789	.65	9.483075	7.57	0.516925	5
56	.464279	6.92	.980750	.63	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.65	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.63	.484435	7.53	.515565	2
59	.465522	6.88	.980635	.65	.484887	7.53	.515113	1
60	9.465935		9.980596		9.485339		0.514661	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.465935	6.88	9.980596	.63	9.485339	7.53	0.514661	60
1	.466348	6.88	.980558	.65	.485791	7.52	.514209	59
2	.466761	6.87	.980519	.65	.486242	7.52	.513758	58
3	.467173	6.87	.980480	.63	.486693	7.50	.513307	57
4	.467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	9.467996	6.85	9.980403	.65	9.487593	7.50	0.512407	55
6	.468407	6.83	.980364	.65	.488043	7.48	.511957	54
7	.468817	6.83	.980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.48	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	9.470046	6.82	9.980208	.65	9.489838	7.47	0.510162	50
11	.470455	6.80	.980169	.65	.490286	7.45	.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.80	.980091	.65	.491180	7.45	.508820	47
14	.471679	6.78	.980052	.67	.491627	7.43	.508373	46
15	9.472086	6.77	9.980012	.65	9.492073	7.43	0.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.77	.979934	.65	.492965	7.42	.507035	43
18	.473304	6.77	.979895	.67	.493410	7.40	.506590	42
19	.473710	6.75	.979855	.65	.493854	7.42	.506146	41
20	9.474115	6.73	9.979816	.67	9.494299	7.40	0.505701	40
21	.474519	6.73	.979776	.65	.494743	7.38	.505257	39
22	.474923	6.73	.979737	.67	.495186	7.40	.504814	38
23	.475327	6.72	.979697	.65	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.67	.496073	7.37	.503927	36
25	9.476133	6.72	9.979618	.65	9.496515	7.37	0.503485	35
26	.476536	6.70	.979579	.67	.496957	7.37	.503043	34
27	.476938	6.70	.979539	.67	.497399	7.37	.502601	33
28	.477340	6.68	.979499	.67	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.65	.498282	7.33	.501718	31
30	9.478142	6.67	9.979420	.67	9.498722	7.35	0.501278	30
31	.478542	6.67	.979380	.67	.499163	7.33	.500837	29
32	.478942	6.67	.979340	.67	.499603	7.32	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.32	.499519	26
35	9.480140	6.65	9.979220	.67	9.500920	7.32	0.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.62	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.68	.502235	7.28	.497765	22
39	.481731	6.62	.979059	.67	.502672	7.28	.497328	21
40	9.482128	6.62	9.979019	.67	9.503109	7.28	0.496891	20
41	.482525	6.60	.978979	.67	.503546	7.27	.496454	19
42	.482921	6.58	.978939	.68	.503982	7.27	.496018	18
43	.483316	6.60	.978898	.67	.504418	7.27	.495582	17
44	.483712	6.58	.978858	.68	.504854	7.25	.495146	16
45	9.484107	6.57	9.978817	.67	9.505289	7.25	0.494711	15
46	.484501	6.57	.978777	.67	.505724	7.25	.494276	14
47	.484895	6.57	.978737	.68	.506159	7.23	.493841	13
48	.485289	6.55	.978696	.68	.506593	7.23	.493407	12
49	.485682	6.55	.978655	.67	.507027	7.22	.492973	11
50	9.486075	6.53	9.978615	.68	9.507460	7.22	0.492540	10
51	.486467	6.55	.978574	.68	.507893	7.22	.492107	9
52	.486860	6.52	.978533	.67	.508326	7.22	.491674	8
53	.487251	6.53	.978493	.68	.508759	7.20	.491241	7
54	.487643	6.52	.978452	.68	.509191	7.18	.490809	6
55	9.488034	6.50	9.978411	.68	9.509622	7.20	0.490378	5
56	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	.978329	.68	.510485	7.18	.489515	3
58	.489204	6.48	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
60	9.489982		9.978206		9.511776		0.488224	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
9.489982	6.48	9.978206	.68	9.511776	7.17	0.488224	60
.490371	6.47	.978165	.68	.512206	7.15	.487794	59
.490759	6.47	.978124	.68	.512635	7.15	.487365	58
.491147	6.47	.978083	.68	.513064	7.15	.486936	57
.491535	6.45	.978042	.68	.513493	7.13	.486507	56
9.491922	6.43	9.978001	.70	9.513921	7.13	0.486079	55
.492308	6.45	.977959	.68	.514349	7.13	.485651	54
.492695	6.43	.977918	.68	.514777	7.12	.485223	53
.493081	6.42	.977877	.70	.515204	7.12	.484796	52
.493466	6.42	.977835	.68	.515631	7.10	.484369	51
9.493851	6.42	9.977794	.70	9.516057	7.12	0.483943	50
.494236	6.42	.977752	.68	.516484	7.10	.483516	49
.494621	6.40	.977711	.70	.516910	7.08	.483090	48
.495005	6.38	.977669	.68	.517335	7.10	.482665	47
.495388	6.40	.977628	.70	.517761	7.08	.482239	46
9.495772	6.37	9.977586	.70	9.518186	7.07	0.481814	45
.496154	6.38	.977544	.68	.518610	7.07	.481390	44
.496537	6.37	.977503	.70	.519034	7.07	.480966	43
.496919	6.37	.977461	.70	.519458	7.07	.480542	42
.497301	6.35	.977419	.70	.519882	7.05	.480118	41
9.497682	6.37	9.977377	.70	9.520305	7.05	0.479695	40
.498064	6.33	.977335	.70	.520728	7.05	.479272	39
.498444	6.35	.977293	.70	.521151	7.03	.478849	38
.498825	6.32	.977251	.70	.521573	7.03	.478427	37
.499204	6.33	.977209	.70	.521995	7.03	.478005	36
9.499584	6.32	9.977167	.70	9.522417	7.02	0.477583	35
.499963	6.32	.977125	.70	.522838	7.02	.477162	34
.500342	6.32	.977083	.70	.523259	7.02	.476741	33
.500721	6.30	.977041	.70	.523680	7.00	.476320	32
.501099	6.28	.976999	.70	.524100	7.00	.475900	31
9.501476	6.30	9.976957	.72	9.524520	7.00	0.475480	30
.501854	6.28	.976914	.70	.524940	6.98	.475060	29
.502231	6.27	.976872	.70	.525359	6.98	.474641	28
.502607	6.28	.976830	.72	.525778	6.98	.474222	27
.502984	6.27	.976787	.70	.526197	6.97	.473803	26
9.503360	6.25	9.976745	.72	9.526615	6.97	0.473385	25
.503735	6.25	.976702	.70	.527033	6.97	.472967	24
.504110	6.25	.976660	.72	.527451	6.95	.472549	23
.504485	6.25	.976617	.72	.527868	6.95	.472132	22
.504860	6.23	.976574	.70	.528285	6.95	.471715	21
9.505234	6.23	9.976532	.72	9.528702	6.95	0.471298	20
.505608	6.22	.976489	.72	.529119	6.93	.470881	19
.505981	6.22	.976446	.70	.529535	6.93	.470465	18
.506354	6.22	.976404	.72	.529951	6.92	.470049	17
.506727	6.20	.976361	.72	.530366	6.92	.469634	16
9.507099	6.20	9.976318	.72	9.530781	6.92	0.469219	15
.507471	6.20	.976275	.72	.531196	6.92	.468804	14
.507843	6.18	.976232	.72	.531611	6.90	.468389	13
.508214	6.18	.976189	.72	.532025	6.90	.467975	12
.508585	6.18	.976146	.72	.532439	6.90	.467561	11
9.508956	6.17	9.976103	.72	9.532853	6.88	0.467147	10
.509326	6.17	.976060	.72	.533266	6.88	.466734	9
.509696	6.15	.976017	.72	.533679	6.88	.466321	8
.510065	6.15	.975974	.73	.534092	6.87	.465908	7
.510434	6.15	.975930	.72	.534504	6.87	.465496	6
9.510803	6.15	9.975887	.72	9.534916	6.87	0.465084	5
.511172	6.13	.975844	.73	.535328	6.85	.464672	4
.511540	6.12	.975800	.72	.535739	6.85	.464261	3
.511907	6.13	.975757	.72	.536150	6.85	.463850	2
.512275	6.12	.975714	.73	.536561	6.85	.463439	1
9.512642		9.975670		9.536972		0.463028	0
Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.512642	6.12	9.975670	.72	9.536972	6.83	0.463028	60
1	.513009	6.10	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.10	.975583	.73	.537792	6.83	.462208	58
3	.513741	6.10	.975539	.72	.538202	6.82	.461798	57
4	.514107	6.08	.975496	.73	.538611	6.82	.461389	56
5	9.514472	6.08	9.975452	.73	9.539020	6.82	0.460980	55
6	.514837	6.08	.975408	.72	.539429	6.80	.460571	54
7	.515202	6.07	.975365	.73	.539837	6.80	.460163	53
8	.515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.07	.975277	.73	.540653	6.80	.459347	51
10	9.516294	6.05	9.975233	.73	9.541061	6.78	0.458939	50
11	.516657	6.05	.975189	.73	.541468	6.78	.458532	49
12	.517020	6.03	.975145	.73	.541875	6.77	.458125	48
13	.517382	6.05	.975101	.73	.542281	6.78	.457719	47
14	.517745	6.03	.975057	.73	.542688	6.77	.457312	46
15	9.518107	6.02	9.975013	.73	9.543094	6.75	0.456906	45
16	.518468	6.02	.974969	.73	.543499	6.77	.456501	44
17	.518829	6.02	.974925	.75	.543905	6.75	.456095	43
18	.519190	6.02	.974880	.73	.544310	6.75	.455690	42
19	.519551	6.00	.974836	.73	.544715	6.73	.455285	41
20	9.519911	6.00	9.974792	.73	9.545119	6.75	0.454881	40
21	.520271	6.00	.974748	.75	.545524	6.73	.454476	39
22	.520631	5.98	.974703	.73	.545928	6.72	.454072	38
23	.520990	5.98	.974659	.75	.546331	6.73	.453669	37
24	.521349	5.97	.974614	.73	.546735	6.72	.453265	36
25	9.521707	5.98	9.974570	.75	9.547138	6.70	0.452862	35
26	.522066	5.97	.974525	.73	.547540	6.72	.452460	34
27	.522424	5.95	.974481	.75	.547943	6.70	.452057	33
28	.522781	5.95	.974436	.75	.548345	6.70	.451655	32
29	.523138	5.95	.974391	.73	.548747	6.70	.451253	31
30	9.523495	5.95	9.974347	.75	9.549149	6.68	0.450851	30
31	.523852	5.93	.974302	.75	.549550	6.68	.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.92	.974167	.75	.550752	6.68	.449248	26
35	9.525275	5.92	9.974122	.75	9.551153	6.65	0.448847	25
36	.525630	5.90	.974077	.75	.551552	6.67	.448448	24
37	.525984	5.92	.974032	.75	.551952	6.65	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.88	.973942	.75	.552750	6.65	.447250	21
40	9.527046	5.90	9.973897	.75	9.553149	6.65	0.446851	20
41	.527400	5.88	.973852	.75	.553548	6.63	.446452	19
42	.527753	5.87	.973807	.77	.553946	6.63	.446054	18
43	.528105	5.88	.973761	.75	.554344	6.62	.445656	17
44	.528458	5.87	.973716	.75	.554741	6.63	.445259	16
45	9.528810	5.85	9.973671	.77	9.555139	6.62	0.444861	15
46	.529161	5.87	.973625	.75	.555536	6.62	.444464	14
47	.529513	5.85	.973580	.75	.555933	6.60	.444067	13
48	.529864	5.85	.973535	.77	.556329	6.60	.443671	12
49	.530215	5.83	.973489	.75	.556725	6.60	.443275	11
50	9.530565	5.83	9.973444	.77	9.557121	6.60	0.442879	10
51	.530915	5.83	.973398	.77	.557517	6.60	.442483	9
52	.531265	5.82	.973352	.75	.557913	6.58	.442087	8
53	.531614	5.82	.973307	.77	.558308	6.58	.441692	7
54	.531963	5.82	.973261	.77	.558703	6.57	.441297	6
55	9.532312	5.82	9.973215	.77	9.559097	6.57	0.440903	5
56	.532661	5.80	.973169	.75	.559491	6.57	.440509	4
57	.533009	5.80	.973124	.77	.559885	6.57	.440115	3
58	.533357	5.78	.973078	.77	.560279	6.57	.439721	2
59	.533704	5.80	.973032	.77	.560673	6.55	.439327	1
60	9.534052		9.972986		9.561066		0.438934	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.534052	5.78	9.972986	.77	9.561066	6.55	0.438934	60
1	.534399	5.77	.972940	.77	.561459	6.53	.438541	59
2	.534745	5.78	.972894	.77	.561851	6.55	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.53	.437756	57
4	.535438	5.75	.972802	.78	.562636	6.53	.437364	56
5	9.535783	5.77	9.972755	.77	9.563028	6.52	0.436972	55
6	.536129	5.75	.972709	.77	.563419	6.53	.436581	54
7	.536474	5.73	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.75	.972617	.78	.564202	6.52	.435798	52
9	.537163	5.73	.972570	.77	.564593	6.50	.435407	51
10	9.537507	5.73	9.972524	.77	9.564983	6.50	0.435017	50
11	.537851	5.72	.972478	.78	.565373	6.50	.434627	49
12	.538194	5.73	.972431	.77	.565763	6.50	.434237	48
13	.538538	5.70	.972385	.78	.566153	6.48	.433847	47
14	.538880	5.72	.972338	.78	.566542	6.50	.433458	46
15	9.539223	5.70	9.972291	.77	9.566932	6.47	0.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.70	.972198	.78	.567709	6.48	.432291	43
18	.540249	5.68	.972151	.77	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.45	.431514	41
20	9.540931	5.68	9.972058	.78	9.568873	6.47	0.431127	40
21	.541272	5.68	.972011	.78	.569261	6.45	.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.67	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.65	.971870	.78	.570422	6.45	.429578	36
25	9.542632	5.65	9.971823	.78	9.570809	6.43	0.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.65	.971729	.78	.571581	6.43	.428419	33
28	.543649	5.63	.971682	.78	.571967	6.42	.428033	32
29	.543987	5.63	.971635	.78	.572352	6.43	.427648	31
30	9.544325	5.63	9.971588	.80	9.572738	6.42	0.427262	30
31	.544663	5.62	.971540	.78	.573123	6.40	.426877	29
32	.545000	5.63	.971493	.78	.573507	6.42	.426493	28
33	.545338	5.60	.971446	.80	.573892	6.40	.426108	27
34	.545674	5.62	.971398	.78	.574276	6.40	.425724	26
35	9.546011	5.60	9.971351	.80	9.574660	6.40	0.425340	25
36	.546347	5.60	.971303	.78	.575044	6.38	.424956	24
37	.546683	5.60	.971256	.80	.575427	6.38	.424573	23
38	.547019	5.58	.971208	.78	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.80	.576193	6.38	.423807	21
40	9.547689	5.58	9.971113	.78	9.576576	6.38	0.423424	20
41	.548024	5.58	.971066	.80	.576959	6.37	.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.57	.970970	.80	.577723	6.35	.422277	17
44	.549027	5.55	.970922	.80	.578104	6.37	.421896	16
45	9.549360	5.55	9.970874	.78	9.578486	6.35	0.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5.55	.970779	.80	.579248	6.35	.420752	13
48	.550359	5.55	.970731	.80	.579629	6.33	.420371	12
49	.550692	5.53	.970683	.80	.580009	6.33	.419991	11
50	9.551024	5.53	9.970635	.82	9.580389	6.33	0.419611	10
51	.551356	5.52	.970586	.80	.580769	6.33	.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.52	.970442	.80	.581907	6.32	.418093	6
55	9.552680	5.50	9.970394	.82	9.582286	6.32	0.417714	5
56	.553010	5.52	.970345	.80	.582665	6.32	.417335	4
57	.553341	5.48	.970297	.80	.583044	6.30	.416956	3
58	.553670	5.50	.970249	.82	.583422	6.30	.416578	2
59	.554000	5.48	.970200	.80	.583800	6.28	.416200	1
60	9.554329		9.970152		9.584177		0.415823	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.554329	5.48	9.970152	.82	9.584177	6.30	0.415823	60
1	.554658	5.48	.970103	.80	.584555	6.28	.415445	59
2	.554987	5.47	.970055	.82	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.82	.585309	6.28	.414691	57
4	.555643	5.47	.969957	.80	.585686	6.27	.414314	56
5	9.555971	5.47	9.969909	.82	9.586062	6.28	0.413938	55
6	.556299	5.45	.969860	.82	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.82	.586815	6.25	.413185	53
8	.556953	5.45	.969762	.80	.587190	6.27	.412810	52
9	.557280	5.43	.969714	.82	.587566	6.25	.412434	51
10	9.557606	5.43	9.969665	.82	9.587941	6.25	0.412059	50
11	.557932	5.43	.969616	.82	.588316	6.25	.411684	49
12	.558258	5.42	.969567	.82	.588691	6.25	.411309	48
13	.558583	5.43	.969518	.82	.589066	6.23	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.23	.410560	46
15	9.559234	5.40	9.969420	.83	9.589814	6.23	0.410186	45
16	.559558	5.42	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.40	.969223	.83	.591308	6.22	.408692	41
20	9.560855	5.38	9.969173	.82	9.591681	6.22	0.408319	40
21	.561178	5.38	.969124	.82	.592054	6.20	.407946	39
22	.561501	5.38	.969075	.83	.592426	6.22	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.20	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.18	.406829	36
25	9.562468	5.37	9.968926	.82	9.593542	6.20	0.406458	35
26	.562790	5.37	.968877	.83	.593914	6.18	.406086	34
27	.563112	5.35	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.37	.968777	.82	.594656	6.18	.405344	32
29	.563755	5.33	.968728	.83	.595027	6.18	.404973	31
30	9.564075	5.35	9.968678	.83	9.595398	6.17	0.404602	30
31	.564396	5.33	.968628	.83	.595768	6.17	.404232	29
32	.564716	5.33	.968578	.83	.596138	6.17	.403862	28
33	.565036	5.33	.968528	.82	.596508	6.17	.403492	27
34	.565356	5.33	.968479	.83	.596878	6.15	.403122	26
35	9.565676	5.32	9.968429	.83	9.597247	6.15	0.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.30	.968329	.85	.597985	6.15	.402015	23
38	.566632	5.32	.968278	.83	.598354	6.13	.401646	22
39	.566951	5.30	.968228	.83	.598722	6.15	.401278	21
40	9.567269	5.30	9.968178	.83	9.599091	6.13	0.400909	20
41	.567587	5.28	.968128	.83	.599459	6.13	.400541	19
42	.567904	5.30	.968078	.85	.599827	6.12	.400173	18
43	.568222	5.28	.968027	.83	.600194	6.13	.399806	17
44	.568539	5.28	.967977	.83	.600562	6.12	.399438	16
45	9.568856	5.27	9.967927	.85	9.600929	6.12	0.399071	15
46	.569172	5.27	.967876	.83	.601296	6.12	.398704	14
47	.569488	5.27	.967826	.85	.601663	6.10	.398337	13
48	.569804	5.27	.967775	.83	.602029	6.10	.397971	12
49	.570120	5.25	.967725	.85	.602395	6.10	.397605	11
50	9.570435	5.27	9.967674	.83	9.602761	6.10	0.397239	10
51	.570751	5.25	.967624	.85	.603127	6.10	.396873	9
52	.571066	5.23	.967573	.85	.603493	6.08	.396507	8
53	.571380	5.25	.967522	.85	.603858	6.08	.396142	7
54	.571695	5.23	.967471	.83	.604223	6.08	.395777	6
55	9.572009	5.23	9.967421	.85	9.604588	6.08	0.395412	5
56	.572323	5.22	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.23	.967319	.85	.605317	6.08	.394683	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.20	.967217	.85	.606046	6.07	.393954	1
60	9.573575		9.967166		9.606410		0.393590	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
9.573575	5.22	9.967166	.85	9.606410	6.05	0.393590	60
.573888	5.20	.967115	.85	.606773	6.07	.393227	59
.574200	5.20	.967064	.85	.607137	6.05	.392863	58
.574512	5.20	.967013	.87	.607500	6.05	.392500	57
.574824	5.20	.966961	.85	.607863	6.03	.392137	56
9.575136	5.18	9.966910	.85	9.608225	6.05	0.391775	55
.575447	5.18	.966859	.85	.608588	6.03	.391412	54
.575758	5.18	.966808	.87	.608950	6.03	.391050	53
.576069	5.17	.966756	.85	.609312	6.03	.390688	52
.576379	5.17	.966705	.87	.609674	6.03	.390326	51
9.576689	5.17	9.966653	.85	9.610036	6.02	0.389964	50
.576999	5.17	.966602	.87	.610397	6.03	.389603	49
.577309	5.15	.966550	.85	.610759	6.02	.389241	48
.577618	5.15	.966499	.87	.611120	6.00	.388880	47
.577927	5.15	.966447	.87	.611480	6.02	.388520	46
9.578236	5.15	9.966395	.85	9.611841	6.00	0.388159	45
.578545	5.13	.966344	.87	.612201	6.00	.387799	44
.578853	5.15	.966292	.87	.612561	6.00	.387439	43
.579162	5.13	.966240	.87	.612921	6.00	.387079	42
.579470	5.12	.966188	.87	.613281	6.00	.386719	41
9.579777	5.13	9.966136	.85	9.613641	5.98	0.386359	40
.580085	5.12	.966085	.87	.614000	5.98	.386000	39
.580392	5.12	.966033	.87	.614359	5.98	.385641	38
.580699	5.10	.965981	.87	.614718	5.98	.385282	37
.581005	5.12	.965929	.88	.615077	5.97	.384923	36
9.581312	5.10	9.965876	.87	9.615435	5.97	0.384565	35
.581618	5.10	.965824	.87	.615793	5.97	.384207	34
.581924	5.08	.965772	.87	.616151	5.97	.383849	33
.582229	5.10	.965720	.87	.616509	5.97	.383491	32
.582535	5.08	.965668	.88	.616867	5.95	.383133	31
9.582840	5.08	9.965615	.87	9.617224	5.97	0.382776	30
.583145	5.07	.965563	.87	.617582	5.95	.382418	29
.583449	5.08	.965511	.88	.617939	5.93	.382061	28
.583754	5.07	.965458	.87	.618295	5.95	.381705	27
.584058	5.05	.965406	.88	.618652	5.93	.381348	26
9.584361	5.07	9.965353	.87	9.619008	5.93	0.380992	25
.584665	5.05	.965301	.88	.619364	5.93	.380636	24
.584968	5.07	.965248	.88	.619720	5.93	.380280	23
.585272	5.03	.965195	.87	.620076	5.93	.379924	22
.585574	5.05	.965143	.88	.620432	5.92	.379568	21
9.585877	5.03	9.965090	.88	9.620787	5.92	0.379213	20
.586179	5.05	.965037	.88	.621142	5.92	.378858	19
.586482	5.02	.964984	.88	.621497	5.92	.378503	18
.586783	5.03	.964931	.87	.621852	5.92	.378148	17
.587085	5.02	.964879	.88	.622207	5.90	.377793	16
9.587386	5.03	9.964826	.88	9.622561	5.90	0.377439	15
.587688	5.02	.964773	.88	.622915	5.90	.377085	14
.587989	5.00	.964720	.90	.623269	5.90	.376731	13
.588289	5.02	.964666	.88	.623623	5.88	.376377	12
.588590	5.00	.964613	.88	.623976	5.90	.376024	11
9.588890	5.00	9.964560	.88	9.624330	5.88	0.375670	10
.589190	4.98	.964507	.88	.624683	5.88	.375317	9
.589489	5.00	.964454	.90	.625036	5.87	.374964	8
.589789	4.98	.964400	.88	.625388	5.88	.374612	7
.590088	4.98	.964347	.88	.625741	5.87	.374259	6
9.590387	4.98	9.964294	.90	9.626093	5.87	0.373907	5
.590686	4.97	.964240	.88	.626445	5.87	.373555	4
.590984	4.97	.964187	.90	.626797	5.87	.373203	3
.591282	4.97	.964133	.88	.627149	5.87	.372851	2
.591580	4.97	.964080	.90	.627501	5.85	.372499	1
9.591878		9.964026		9.627852		0.372148	0
Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.
0	9.591878	4.97	9.964026	.90	9.627852	5.85	0.372148
1	.592176	4.95	.963972	.88	.628203	5.85	.371797
2	.592473	4.95	.963919	.90	.628554	5.85	.371446
3	.592770	4.95	.963865	.90	.628905	5.83	.371095
4	.593067	4.93	.963811	.90	.629255	5.85	.370745
5	9.593363	4.93	9.963757	.88	9.629606	5.83	0.370394
6	.593659	4.93	.963704	.90	.629956	5.83	.370044
7	.593955	4.93	.963650	.90	.630306	5.83	.369694
8	.594251	4.93	.963596	.90	.630656	5.82	.369344
9	.594547	4.92	.963542	.90	.631005	5.83	.368995
10	9.594842	4.92	9.963488	.90	9.631355	5.82	0.368645
11	.595137	4.92	.963434	.92	.631704	5.82	.368296
12	.595432	4.92	.963379	.90	.632053	5.82	.367947
13	.595727	4.90	.963325	.90	.632402	5.80	.367598
14	.596021	4.90	.963271	.90	.632750	5.82	.367250
15	9.596315	4.90	9.963217	.90	9.633099	5.80	0.366901
16	.596609	4.90	.963163	.92	.633447	5.80	.366553
17	.596903	4.88	.963108	.90	.633795	5.80	.366205
18	.597196	4.90	.963054	.92	.634143	5.78	.365857
19	.597490	4.88	.962999	.90	.634490	5.80	.365510
20	9.597783	4.87	9.962945	.92	9.634838	5.78	0.365162
21	.598075	4.88	.962890	.90	.635185	5.78	.364815
22	.598368	4.87	.962836	.92	.635532	5.78	.364468
23	.598660	4.87	.962781	.90	.635879	5.78	.364121
24	.598952	4.87	.962727	.92	.636226	5.77	.363774
25	9.599244	4.87	9.962672	.92	9.636572	5.78	0.363428
26	.599536	4.85	.962617	.92	.636919	5.77	.363081
27	.599827	4.85	.962562	.90	.637265	5.77	.362735
28	.600118	4.85	.962508	.92	.637611	5.75	.362389
29	.600409	4.85	.962453	.92	.637956	5.77	.362044
30	9.600700	4.83	9.962398	.92	9.638302	5.75	0.361698
31	.600990	4.83	.962343	.92	.638647	5.75	.361353
32	.601280	4.83	.962288	.92	.638992	5.75	.361008
33	.601570	4.83	.962233	.92	.639337	5.75	.360663
34	.601860	4.83	.962178	.92	.639682	5.75	.360318
35	9.602150	4.82	9.962123	.93	9.640027	5.73	0.359973
36	.602439	4.82	.962067	.92	.640371	5.75	.359629
37	.602728	4.82	.962012	.92	.640716	5.73	.359284
38	.603017	4.80	.961957	.92	.641060	5.73	.358940
39	.603305	4.82	.961902	.93	.641404	5.72	.358596
40	9.603594	4.80	9.961846	.92	9.641747	5.73	0.358253
41	.603882	4.80	.961791	.93	.642091	5.72	.357909
42	.604170	4.78	.961735	.92	.642434	5.72	.357566
43	.604457	4.80	.961680	.93	.642777	5.72	.357223
44	.604745	4.78	.961624	.92	.643120	5.72	.356880
45	9.605032	4.78	9.961569	.93	9.643463	5.72	0.356537
46	.605319	4.78	.961513	.92	.643806	5.70	.356194
47	.605606	4.77	.961458	.93	.644148	5.70	.355852
48	.605892	4.78	.961402	.93	.644490	5.70	.355510
49	.606179	4.77	.961346	.93	.644832	5.70	.355168
50	9.606465	4.77	9.961290	.92	9.645174	5.70	0.354826
51	.606751	4.75	.961235	.93	.645516	5.68	.354484
52	.607036	4.77	.961179	.93	.645857	5.70	.354143
53	.607322	4.75	.961123	.93	.646199	5.68	.353801
54	.607607	4.75	.961067	.93	.646540	5.68	.353460
55	9.607892	4.75	9.961011	.93	9.646881	5.68	0.353119
56	.608177	4.73	.960955	.93	.647222	5.67	.352778
57	.608461	4.73	.960899	.93	.647562	5.68	.352438
58	.608745	4.73	.960843	.95	.647903	5.67	.352097
59	.609029	4.73	.960786	.93	.648243	5.67	.351757
60	9.609313		9.960730		9.648583		0.351417
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.609313	4.73	9.960730	.93	9.648583	5.67	0.351417	60
1	.609597	4.72	.960674	.93	.648923	5.67	.351077	59
2	.609880	4.73	.960618	.95	.649263	5.65	.350737	58
3	.610164	4.72	.960561	.93	.649602	5.67	.350398	57
4	.610447	4.70	.960505	.95	.649942	5.65	.350058	56
5	9.610729	4.72	9.960448	.93	9.650281	5.65	0.349719	55
6	.611012	4.70	.960392	.95	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.93	.650959	5.63	.349041	53
8	.611576	4.70	.960279	.95	.651297	5.65	.348703	52
9	.611858	4.70	.960222	.95	.651636	5.63	.348364	51
10	9.612140	4.68	9.960165	.93	9.651974	5.63	0.348026	50
11	.612421	4.68	.960109	.95	.652312	5.63	.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.93	.653326	5.62	.346674	46
15	9.613545	4.67	9.959882	.95	9.653663	5.62	0.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.67	.959711	.95	.654674	5.62	.345326	42
19	.614665	4.65	.959654	.97	.655011	5.62	.344989	41
20	9.614944	4.65	9.959596	.95	9.655348	5.60	0.344652	40
21	.615223	4.65	.959539	.95	.655684	5.60	.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.65	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.63	.959368	.97	.656692	5.60	.343308	36
25	9.616338	4.63	9.959310	.95	9.657028	5.60	0.342972	35
26	.616616	4.63	.959253	.97	.657364	5.58	.342636	34
27	.616894	4.63	.959195	.95	.657699	5.58	.342301	33
28	.617172	4.63	.959138	.97	.658034	5.58	.341966	32
29	.617450	4.62	.959080	.95	.658369	5.58	.341631	31
30	9.617727	4.62	9.959023	.97	9.658704	5.58	0.341296	30
31	.618004	4.62	.958965	.95	.659039	5.57	.340961	29
32	.618281	4.62	.958908	.97	.659373	5.58	.340627	28
33	.618558	4.60	.958850	.97	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.97	.660042	5.57	.339958	26
35	9.619110	4.60	9.958734	.95	9.660376	5.57	0.339624	25
36	.619386	4.60	.958677	.97	.660710	5.55	.339290	24
37	.619662	4.60	.958619	.97	.661043	5.57	.338957	23
38	.619938	4.58	.958561	.97	.661377	5.55	.338623	22
39	.620213	4.58	.958503	.97	.661710	5.55	.338290	21
40	9.620488	4.58	9.958445	.97	9.662043	5.55	0.337957	20
41	.620763	4.58	.958387	.97	.662376	5.55	.337624	19
42	.621038	4.58	.958329	.97	.662709	5.55	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.55	.336958	17
44	.621587	4.57	.958213	.98	.663375	5.53	.336625	16
45	9.621861	4.57	9.958154	.97	9.663707	5.53	0.336293	15
46	.622135	4.57	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.55	.958038	.98	.664371	5.53	.335629	13
48	.622682	4.57	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.52	.334965	11
50	9.623229	4.55	9.957863	.98	9.665366	5.53	0.334634	10
51	.623502	4.53	.957804	.97	.665698	5.52	.334302	9
52	.623774	4.55	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.53	.957687	.98	.666360	5.52	.333640	7
54	.624319	4.53	.957628	.97	.666691	5.50	.333309	6
55	9.624591	4.53	9.957570	.98	9.667021	5.52	0.332979	5
56	.624863	4.53	.957511	.98	.667352	5.50	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.52	.332318	3
58	.625406	4.52	.957393	.97	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60	9.625948		9.957276		9.668673		0.331327	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.625948	4.52	9.957276	.98	9.668673	5.48	0.331327	60
1	.626219	4.52	.957217	.98	.669002	5.50	.330998	59
2	.626490	4.50	.957158	.98	.669332	5.48	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.50	.330339	57
4	.627030	4.50	.957040	.98	.669991	5.48	.330009	56
5	9.627300	4.50	9.956981	1.00	9.670320	5.48	0.329680	55
6	.627570	4.50	.956921	.98	.670649	5.47	.329351	54
7	.627840	4.48	.956862	.98	.670977	5.48	.329023	53
8	.628109	4.48	.956803	.98	.671306	5.48	.328694	52
9	.628378	4.48	.956744	1.00	.671635	5.47	.328365	51
10	9.628647	4.48	9.956684	.98	9.671963	5.47	0.328037	50
11	.628916	4.48	.956625	.98	.672291	5.47	.327709	49
12	.629185	4.47	.956566	1.00	.672619	5.47	.327381	48
13	.629453	4.47	.956506	.98	.672947	5.45	.327053	47
14	.629721	4.47	.956447	1.00	.673274	5.47	.326726	46
15	9.629989	4.47	9.956387	1.00	9.673602	5.45	0.326398	45
16	.630257	4.45	.956327	.98	.673929	5.47	.326071	44
17	.630524	4.47	.956268	1.00	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416	42
19	.631059	4.45	.956148	.98	.674911	5.43	.325089	41
20	9.631326	4.45	9.956089	1.00	9.675237	5.45	0.324763	40
21	.631593	4.43	.956029	1.00	.675564	5.43	.324436	39
22	.631859	4.43	.955969	1.00	.675890	5.45	.324110	38
23	.632125	4.45	.955909	1.00	.676217	5.43	.323783	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	9.632658	4.42	9.955789	1.00	9.676869	5.42	0.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.43	.322480	33
28	.633454	4.42	.955609	1.02	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	9.633984	4.42	9.955488	1.00	9.678496	5.42	0.321504	30
31	.634249	4.42	.955428	1.00	.678821	5.42	.321179	29
32	.634514	4.40	.955368	1.02	.679146	5.42	.320854	28
33	.634778	4.40	.955307	1.00	.679471	5.40	.320529	27
34	.635042	4.40	.955247	1.02	.679795	5.42	.320205	26
35	9.635306	4.40	9.955186	1.00	9.680120	5.40	0.319880	25
36	.635570	4.40	.955126	1.02	.680444	5.40	.319556	24
37	.635834	4.38	.955065	1.00	.680768	5.40	.319232	23
38	.636097	4.38	.955005	1.02	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.02	.681416	5.40	.318584	21
40	9.636623	4.38	9.954883	1.00	9.681740	5.38	0.318260	20
41	.636886	4.37	.954823	1.02	.682063	5.40	.317937	19
42	.637148	4.38	.954762	1.02	.682387	5.38	.317613	18
43	.637411	4.37	.954701	1.02	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	9.637935	4.37	9.954579	1.02	9.683356	5.38	0.316644	15
46	.638197	4.35	.954518	1.02	.683679	5.37	.316321	14
47	.638458	4.37	.954457	1.02	.684001	5.38	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	9.639242	4.35	9.954274	1.02	9.684968	5.37	0.315032	10
51	.639503	4.35	.954213	1.02	.685290	5.37	.314710	9
52	.639764	4.33	.954152	1.03	.685612	5.37	.314388	8
53	.640024	4.33	.954090	1.02	.685934	5.35	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.37	.313745	6
55	9.640544	4.33	9.953968	1.03	9.686577	5.35	0.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.33	.953845	1.03	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.02	.687540	5.35	.312460	2
59	.641583	4.32	.953722	1.03	.687861	5.35	.312139	1
60	9.641842		9.953660		9.688182		0.311818	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.641842	4.32	9.953660	1.02	9.688182	5.33	0.311818	60
1	.642101	4.32	.953599	1.03	.688502	5.35	.311498	59
2	.642360	4.30	.953537	1.03	.688823	5.33	.311177	58
3	.642618	4.32	.953475	1.03	.689143	5.33	.310857	57
4	.642877	4.30	.953413	1.02	.689463	5.33	.310537	56
5	9.643135	4.30	9.953352	1.03	9.689783	5.33	0.310217	55
6	.643393	4.28	.953290	1.03	.690103	5.33	.309897	54
7	.643650	4.30	.953228	1.03	.690423	5.32	.309577	53
8	.643908	4.28	.953166	1.03	.690742	5.33	.309258	52
9	.644165	4.30	.953104	1.03	.691062	5.32	.308938	51
10	9.644423	4.28	9.953042	1.03	9.691381	5.32	0.308619	50
11	.644680	4.27	.952980	1.03	.691700	5.32	.308300	49
12	.644936	4.28	.952918	1.05	.692019	5.32	.307981	48
13	.645193	4.28	.952855	1.03	.692338	5.30	.307662	47
14	.645450	4.27	.952793	1.03	.692656	5.32	.307344	46
15	9.645706	4.27	9.952731	1.03	9.692975	5.30	0.307025	45
16	.645962	4.27	.952669	1.05	.693293	5.32	.306707	44
17	.646218	4.27	.952606	1.03	.693612	5.30	.306388	43
18	.646474	4.25	.952544	1.05	.693930	5.30	.306070	42
19	.646729	4.25	.952481	1.03	.694248	5.30	.305752	41
20	9.646984	4.27	9.952419	1.05	9.694566	5.28	0.305434	40
21	.647240	4.23	.952356	1.03	.694883	5.30	.305117	39
22	.647494	4.25	.952294	1.05	.695201	5.28	.304799	38
23	.647749	4.25	.952231	1.05	.695518	5.30	.304482	37
24	.648004	4.23	.952168	1.03	.695836	5.28	.304164	36
25	9.648258	4.23	9.952106	1.05	9.696153	5.28	0.303847	35
26	.648512	4.23	.952043	1.05	.696470	5.28	.303530	34
27	.648766	4.23	.951980	1.05	.696787	5.27	.303213	33
28	.649020	4.23	.951917	1.05	.697103	5.28	.302897	32
29	.649274	4.22	.951854	1.05	.697420	5.27	.302580	31
30	9.649527	4.23	9.951791	1.05	9.697736	5.28	0.302264	30
31	.649781	4.22	.951728	1.05	.698053	5.27	.301947	29
32	.650034	4.22	.951665	1.05	.698369	5.27	.301631	28
33	.650287	4.20	.951602	1.05	.698685	5.27	.301315	27
34	.650539	4.22	.951539	1.05	.699001	5.25	.300999	26
35	9.650792	4.20	9.951476	1.07	9.699316	5.27	0.300684	25
36	.651044	4.22	.951412	1.05	.699632	5.25	.300368	24
37	.651297	4.20	.951349	1.05	.699947	5.27	.300053	23
38	.651549	4.18	.951286	1.07	.700263	5.25	.299737	22
39	.651800	4.20	.951222	1.05	.700578	5.25	.299422	21
40	9.652052	4.20	9.951159	1.05	9.700893	5.25	0.299107	20
41	.652304	4.18	.951096	1.07	.701208	5.25	.298792	19
42	.652555	4.18	.951032	1.07	.701523	5.23	.298477	18
43	.652806	4.18	.950968	1.05	.701837	5.25	.298163	17
44	.653057	4.18	.950905	1.07	.702152	5.23	.297848	16
45	9.653308	4.17	9.950841	1.05	9.702466	5.25	0.297534	15
46	.653558	4.17	.950778	1.07	.702781	5.23	.297219	14
47	.653808	4.18	.950714	1.07	.703095	5.23	.296905	13
48	.654059	4.17	.950650	1.07	.703409	5.22	.296591	12
49	.654309	4.15	.950586	1.07	.703722	5.23	.296278	11
50	9.654558	4.17	9.950522	1.07	9.704036	5.23	0.295964	10
51	.654808	4.17	.950458	1.07	.704350	5.22	.295650	9
52	.655058	4.15	.950394	1.07	.704663	5.22	.295337	8
53	.655307	4.15	.950330	1.07	.704976	5.23	.295024	7
54	.655556	4.15	.950. 6	1.07	.705290	5.22	.294710	6
55	9.655805	4.15	9.950202	1.07	9.705603	5.22	0.294397	5
56	.656054	4.13	.950138	1.07	.705916	5.20	.294084	4
57	.656302	4.15	.950074	1.07	.706228	5.22	.293772	3
58	.656551	4.13	.950010	1.08	.706541	5.22	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.20	.293146	1
60	9.657047		9.949881		9.707166		0.292834	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.657047	4.13	9.949881	1.08	9.707166	5.20	0.292834	60
1	.657295	4.12	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.13	.949752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5.20	.291586	56
5	9.658284	4.12	9.949558	1.07	9.708726	5.18	0.291274	55
6	.658531	4.12	.949494	1.08	.709037	5.20	.290963	54
7	.658778	4.12	.949429	1.08	.709349	5.18	.290651	53
8	.659025	4.10	.949364	1.07	.709660	5.18	.290340	52
9	.659271	4.10	.949300	1.08	.709971	5.18	.290029	51
10	9.659517	4.10	9.949235	1.08	9.710282	5.18	0.289718	50
11	.659763	4.10	.949170	1.08	.710593	5.18	.289407	49
12	.660009	4.10	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.10	.949040	1.08	.711215	5.17	.288785	47
14	.660501	4.08	.948975	1.08	.711525	5.18	.288475	46
15	9.660746	4.08	9.948910	1.08	9.711836	5.17	0.288164	45
16	.660991	4.08	.948845	1.08	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.08	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1.08	.712766	5.17	.287234	42
19	.661726	4.07	.948650	1.10	.713076	5.17	.286924	41
20	9.661970	4.07	9.948584	1.08	9.713386	5.17	0.286614	40
21	.662214	4.08	.948519	1.08	.713696	5.15	.286304	39
22	.662459	4.07	.948454	1.10	.714005	5.15	.285995	38
23	.662703	4.05	.948388	1.08	.714314	5.17	.285686	37
24	.662946	4.07	.948323	1.10	.714624	5.15	.285376	36
25	9.663190	4.05	9.948257	1.08	9.714933	5.15	0.285067	35
26	.663433	4.07	.948192	1.10	.715242	5.15	.284758	34
27	.663677	4.05	.948126	1.10	.715551	5.15	.284449	33
28	.663920	4.05	.948060	1.08	.715860	5.13	.284140	32
29	.664163	4.05	.947995	1.10	.716168	5.15	.283832	31
30	9.664406	4.03	9.947929	1.10	9.716477	5.13	0.283523	30
31	.664648	4.05	.947863	1.10	.716785	5.13	.283215	29
32	.664891	4.03	.947797	1.10	.717093	5.13	.282907	28
33	.665133	4.03	.947731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.947665	1.08	.717709	5.13	.282291	26
35	9.665617	4.03	9.947600	1.12	9.718017	5.13	0.281983	25
36	.665859	4.02	.947533	1.10	.718325	5.13	.281675	24
37	.666100	4.03	.947467	1.10	.718633	5.12	.281367	23
38	.666342	4.02	.947401	1.10	.718940	5.13	.281060	22
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	21
40	9.666824	4.02	9.947269	1.10	9.719555	5.12	0.280445	20
41	.667065	4.00	.947203	1.12	.719862	5.12	.280138	19
42	.667305	4.02	.947136	1.10	.720169	5.12	.279831	18
43	.667546	4.00	.947070	1.10	.720476	5.12	.279524	17
44	.667786	4.02	.947004	1.12	.720783	5.10	.279217	16
45	9.668027	4.00	9.946937	1.10	9.721089	5.12	0.278911	15
46	.668267	3.98	.946871	1.12	.721396	5.10	.278604	14
47	.668506	4.00	.946804	1.10	.721702	5.12	.278298	13
48	.668746	4.00	.946738	1.12	.722009	5.10	.277991	12
49	.668986	3.98	.946671	1.12	.722315	5.10	.277685	11
50	9.669225	3.98	9.946604	1.10	9.722621	5.10	0.277379	10
51	.669464	3.98	.946538	1.12	.722927	5.08	.277073	9
52	.669703	3.98	.946471	1.12	.723232	5.10	.276768	8
53	.669942	3.98	.946404	1.12	.723538	5.10	.276462	7
54	.670181	3.97	.946337	1.12	.723844	5.08	.276156	6
55	9.670419	3.98	9.946270	1.12	9.724149	5.08	0.275851	5
56	.670658	3.97	.946203	1.12	.724454	5.10	.275546	4
57	.670896	3.97	.946136	1.12	.724760	5.08	.275240	3
58	.671134	3.97	.946069	1.12	.725065	5.08	.274935	2
59	.671372	3.95	.946002	1.12	.725370	5.07	.274630	1
60	9.671609		9.945935		9.725674		0.274326	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.671609	3.97	9.945935	1.12	9.725674	5.08	0.274326	60
1	.671847	3.95	.945868	1.13	.725979	5.08	.274021	59
2	.672084	3.95	.945800	1.12	.726284	5.07	.273716	58
3	.672321	3.95	.945733	1.12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1.13	.726892	5.08	.273108	56
5	9.672795	3.95	9.945598	1.12	9.727197	5.07	0.272803	55
6	.673032	3.93	.945531	1.12	.727501	5.07	.272499	54
7	.673268	3.95	.945464	1.13	.727805	5.07	.272195	53
8	.673505	3.93	.945396	1.13	.728109	5.05	.271891	52
9	.673741	3.93	.945328	1.12	.728412	5.07	.271588	51
10	9.673977	3.93	9.945261	1.13	9.728716	5.07	0.271284	50
11	.674213	3.92	.945193	1.13	.729020	5.05	.270980	49
12	.674448	3.93	.945125	1.12	.729323	5.05	.270677	48
13	.674684	3.92	.945058	1.13	.729626	5.05	.270374	47
14	.674919	3.93	.944990	1.13	.729929	5.07	.270071	46
15	9.675155	3.92	9.944922	1.13	9.730233	5.03	0.269767	45
16	.675390	3.90	.944854	1.13	.730535	5.05	.269465	44
17	.675624	3.92	.944786	1.13	.730838	5.05	.269162	43
18	.675859	3.92	.944718	1.13	.731141	5.05	.268859	42
19	.676094	3.90	.944650	1.13	.731444	5.03	.268556	41
20	9.676328	3.90	9.944582	1.13	9.731746	5.03	0.268254	40
21	.676562	3.90	.944514	1.13	.732048	5.05	.267952	39
22	.676796	3.90	.944446	1.15	.732351	5.03	.267649	38
23	.677030	3.90	.944377	1.13	.732653	5.03	.267347	37
24	.677264	3.90	.944309	1.13	.732955	5.03	.267045	36
25	9.677498	3.88	9.944241	1.15	9.733257	5.02	0.266743	35
26	.677731	3.88	.944172	1.13	.733558	5.03	.266442	34
27	.677964	3.88	.944104	1.13	.733860	5.03	.266140	33
28	.678197	3.88	.944036	1.15	.734162	5.02	.265838	32
29	.678430	3.88	.943967	1.13	.734463	5.02	.265537	31
30	9.678663	3.87	9.943899	1.15	9.734764	5.03	0.265236	30
31	.678895	3.88	.943830	1.15	.735066	5.02	.264934	29
32	.679128	3.87	.943761	1.13	.735367	5.02	.264633	28
33	.679360	3.87	.943693	1.15	.735668	5.02	.264332	27
34	.679592	3.87	.943624	1.15	.735969	5.00	.264031	26
35	9.679824	3.87	9.943555	1.15	9.736269	5.02	0.263731	25
36	.680056	3.87	.943486	1.15	.736570	5.00	.263430	24
37	.680288	3.85	.943417	1.15	.736870	5.02	.263130	23
38	.680519	3.85	.943348	1.15	.737171	5.00	.262829	22
39	.680750	3.87	.943279	1.15	.737471	5.00	.262529	21
40	9.680982	3.85	9.943210	1.15	9.737771	5.00	0.262229	20
41	.681213	3.83	.943141	1.15	.738071	5.00	.261929	19
42	.681443	3.85	.943072	1.15	.738371	5.00	.261629	18
43	.681674	3.85	.943003	1.15	.738671	5.00	.261329	17
44	.681905	3.83	.942934	1.17	.738971	5.00	.261029	16
45	9.682135	3.83	9.942864	1.15	9.739271	4.98	0.260729	15
46	.682365	3.83	.942795	1.15	.739570	5.00	.260430	14
47	.682595	3.83	.942726	1.17	.739870	4.98	.260130	13
48	.682825	3.83	.942656	1.15	.740169	4.98	.259831	12
49	.683055	3.82	.942587	1.17	.740468	4.98	.259532	11
50	9.683284	3.83	9.942517	1.15	9.740767	4.98	0.259233	10
51	.683514	3.82	.942448	1.17	.741066	4.98	.258934	9
52	.683743	3.82	.942378	1.17	.741365	4.98	.258635	8
53	.683972	3.82	.942308	1.15	.741664	4.97	.258336	7
54	.684201	3.82	.942239	1.17	.741962	4.98	.258038	6
55	9.684430	3.80	9.942169	1.17	9.742261	4.97	0.257739	5
56	.684658	3.82	.942099	1.17	.742559	4.98	.257441	4
57	.684887	3.80	.942029	1.17	.742858	4.97	.257142	3
58	.685115	3.80	.941959	1.17	.743156	4.97	.256844	2
59	.685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60	9.685571		9.941819		9.743752		0.256248	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.685571	3.80	9.941819	1.17	9.743752	4.97	0.256248	60
1	.685799	3.80	.941749	1.17	.744050	4.97	.255950	59
2	.686027	3.78	.941679	1.17	.744348	4.95	.255652	58
3	.686254	3.80	.941609	1.17	.744645	4.97	.255355	57
4	.686482	3.78	.941539	1.17	.744943	4.95	.255057	56
5	9.686709	3.78	9.941469	1.18	9.745240	4.97	0.254760	55
6	.686936	3.78	.941398	1.17	.745538	4.95	.254462	54
7	.687163	3.77	.941328	1.17	.745835	4.95	.254165	53
8	.687389	3.78	.941258	1.18	.746132	4.95	.253868	52
9	.687616	3.78	.941187	1.17	.746429	4.95	.253571	51
10	9.687843	3.77	9.941117	1.18	9.746726	4.95	0.253274	50
11	.688069	3.77	.941046	1.18	.747023	4.93	.252977	49
12	.688295	3.77	.940975	1.17	.747319	4.95	.252681	48
13	.688521	3.77	.940905	1.18	.747616	4.95	.252384	47
14	.688747	3.75	.940834	1.18	.747913	4.93	.252087	46
15	9.688972	3.77	9.940763	1.17	9.748209	4.93	0.251791	45
16	.689198	3.75	.940693	1.18	.748505	4.93	.251495	44
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	9.690098	3.75	9.940409	1.18	9.749689	4.93	0.250311	40
21	.690323	3.75	.940338	1.18	.749985	4.93	.250015	39
22	.690548	3.73	.940267	1.18	.750281	4.92	.249719	38
23	.690772	3.73	.940196	1.18	.750576	4.93	.249424	37
24	.690996	3.73	.940125	1.18	.750872	4.92	.249128	36
25	9.691220	3.73	9.940054	1.20	9.751167	4.92	0.248833	35
26	.691444	3.73	.939982	1.18	.751462	4.92	.248538	34
27	.691668	3.73	.939911	1.18	.751757	4.92	.248243	33
28	.691892	3.72	.939840	1.20	.752052	4.92	.247948	32
29	.692115	3.73	.939768	1.18	.752347	4.92	.247653	31
30	9.692339	3.72	9.939697	1.20	9.752642	4.92	0.247358	30
31	.692562	3.72	.939625	1.18	.752937	4.90	.247063	29
32	.692785	3.72	.939554	1.20	.753231	4.92	.246769	28
33	.693008	3.72	.939482	1.20	.753526	4.90	.246474	27
34	.693231	3.70	.939410	1.18	.753820	4.92	.246180	26
35	9.693453	3.72	9.939339	1.20	9.754115	4.90	0.245885	25
36	.693676	3.70	.939267	1.20	.754409	4.90	.245591	24
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297	23
38	.694120	3.70	.939123	1.18	.754997	4.90	.245003	22
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709	21
40	9.694564	3.70	9.938980	1.20	9.755585	4.88	0.244415	20
41	.694786	3.68	.938908	1.20	.755878	4.90	.244122	19
42	.695007	3.70	.938836	1.22	.756172	4.88	.243828	18
43	.695229	3.68	.938763	1.20	.756465	4.90	.243535	17
44	.695450	3.68	.938691	1.20	.756759	4.88	.243241	16
45	9.695671	3.68	9.938619	1.20	9.757052	4.88	0.242948	15
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655	14
47	.696113	3.68	.938475	1.22	.757638	4.88	.242362	13
48	.696334	3.67	.938402	1.20	.757931	4.88	.242069	12
49	.696554	3.68	.938330	1.20	.758224	4.88	.241776	11
50	9.696775	3.67	9.938258	1.22	9.758517	4.88	0.241483	10
51	.696995	3.67	.938185	1.20	.758810	4.87	.241190	9
52	.697215	3.67	.938113	1.22	.759102	4.88	.240898	8
53	.697435	3.65	.938040	1.22	.759395	4.87	.240605	7
54	.697654	3.67	.937967	1.20	.759687	4.87	.240313	6
55	9.697874	3.67	9.937895	1.22	9.759979	4.88	0.240021	5
56	.698094	3.65	.937822	1.22	.760272	4.87	.239728	4
57	.698313	3.65	.937749	1.22	.760564	4.87	.239436	3
58	.698532	3.65	.937676	1.20	.760856	4.87	.239144	2
59	.698751	3.65	.937604	1.22	.761148	4.85	.238852	1
60	9.698970		9.937531		9.761439		0.238561	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.698970	3.65	9.937531	1.22	9.761439	4.87	0.238561	60
1	.699189	3.63	.937458	1.22	.761731	4.87	.238269	59
2	.699407	3.65	.937385	1.22	.762023	4.85	.237977	58
3	.699626	3.63	.937312	1.23	.762314	4.87	.237686	57
4	.699844	3.63	.937238	1.22	.762606	4.85	.237394	56
5	9.700062	3.63	9.937165	1.22	9.762897	4.85	0.237103	55
6	.700280	3.63	.937092	1.22	.763188	4.85	.236812	54
7	.700498	3.63	.937019	1.22	.763479	4.85	.236521	53
8	.700716	3.62	.936946	1.23	.763770	4.85	.236230	52
9	.700933	3.63	.936872	1.22	.764061	4.85	.235939	51
10	9.701151	3.62	9.936799	1.23	9.764352	4.85	0.235648	50
11	.701368	3.62	.936725	1.22	.764643	4.83	.235357	49
12	.701585	3.62	.936652	1.23	.764933	4.85	.235067	48
13	.701802	3.62	.936578	1.22	.765224	4.83	.234776	47
14	.702019	3.62	.936505	1.23	.765514	4.85	.234486	46
15	9.702236	3.60	9.936431	1.23	9.765805	4.83	0.234195	45
16	.702452	3.62	.936357	1.22	.766095	4.83	.233905	44
17	.702669	3.60	.936284	1.23	.766385	4.83	.233615	43
18	.702885	3.60	.936210	1.23	.766675	4.83	.233325	42
19	.703101	3.60	.936136	1.23	.766965	4.83	.233035	41
20	9.703317	3.60	9.936062	1.23	9.767255	4.83	0.232745	40
21	.703533	3.60	.935988	1.23	.767545	4.82	.232455	39
22	.703749	3.58	.935914	1.23	.767834	4.83	.232166	38
23	.703964	3.58	.935840	1.23	.768124	4.83	.231876	37
24	.704179	3.60	.935766	1.23	.768414	4.82	.231586	36
25	9.704395	3.58	9.935692	1.23	9.768703	4.82	0.231297	35
26	.704610	3.58	.935618	1.25	.768992	4.82	.231008	34
27	.704825	3.58	.935543	1.23	.769281	4.83	.230719	33
28	.705040	3.57	.935469	1.23	.769571	4.82	.230429	32
29	.705254	3.58	.935395	1.25	.769860	4.80	.230140	31
30	9.705469	3.57	9.935320	1.23	9.770148	4.82	0.229852	30
31	.705683	3.58	.935246	1.25	.770437	4.82	.229563	29
32	.705898	3.57	.935171	1.23	.770726	4.82	.229274	28
33	.706112	3.57	.935097	1.25	.771015	4.80	.228985	27
34	.706326	3.55	.935022	1.23	.771303	4.82	.228697	26
35	9.706539	3.57	9.934948	1.25	9.771592	4.80	0.228408	25
36	.706753	3.57	.934873	1.25	.771880	4.80	.228120	24
37	.706967	3.55	.934798	1.25	.772168	4.82	.227832	23
38	.707180	3.55	.934723	1.23	.772457	4.80	.227543	22
39	.707393	3.55	.934649	1.25	.772745	4.80	.227255	21
40	9.707606	3.55	9.934574	1.25	9.773033	4.80	0.226967	20
41	.707819	3.55	.934499	1.25	.773321	4.78	.226679	19
42	.708032	3.55	.934424	1.25	.773608	4.80	.226392	18
43	.708245	3.55	.934349	1.25	.773896	4.80	.226104	17
44	.708458	3.53	.934274	1.25	.774184	4.78	.225816	16
45	9.708670	3.53	9.934199	1.27	9.774471	4.80	0.225529	15
46	.708882	3.53	.934123	1.25	.774759	4.78	.225241	14
47	.709094	3.53	.934048	1.25	.775046	4.78	.224954	13
48	.709306	3.53	.933973	1.25	.775333	4.80	.224667	12
49	.709518	3.53	.933898	1.27	.775621	4.78	.224379	11
50	9.709730	3.52	9.933822	1.25	9.775908	4.78	0.224092	10
51	.709941	3.53	.933747	1.27	.776195	4.78	.223805	9
52	.710153	3.52	.933671	1.25	.776482	4.77	.223518	8
53	.710364	3.52	.933596	1.27	.776768	4.78	.223232	7
54	.710575	3.52	.933520	1.25	.777055	4.78	.222945	6
55	9.710786	3.52	9.933445	1.27	9.777342	4.77	0.222658	5
56	.710997	3.52	.933369	1.27	.777628	4.78	.222372	4
57	.711208	3.52	.933293	1.27	.777915	4.77	.222085	3
58	.711419	3.50	.933217	1.27	.778201	4.78	.221799	2
59	.711629	3.50	.933141	1.25	.778488	4.77	.221512	1
60	9.711839		9.933066		9.778774		0.221226	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.711839	3.52	9.933066	1.27	9.778774	4.77	0.221226	60
1	.712050	3.50	.932990	1.27	.779060	4.77	.220940	59
2	.712260	3.48	.932914	1.27	.779346	4.77	.220654	58
3	.712469	3.50	.932838	1.27	.779632	4.77	.220368	57
4	.712679	3.50	.932762	1.28	.779918	4.75	.220082	56
5	9.712889	3.48	9.932685	1.27	9.780203	4.77	0.219797	55
6	.713098	3.50	.932609	1.27	.780489	4.77	.219511	54
7	.713308	3.48	.932533	1.27	.780775	4.75	.219225	53
8	.713517	3.48	.932457	1.28	.781060	4.77	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.75	.218654	51
10	9.713935	3.48	9.932304	1.27	9.781631	4.75	0.218369	50
11	.714144	3.47	.932228	1.28	.781916	4.75	.218084	49
12	.714352	3.48	.932151	1.27	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.48	.931998	1.28	.782771	4.75	.217229	46
15	9.714978	3.47	9.931921	1.27	9.783056	4.75	0.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.47	.931768	1.28	.783626	4.73	.216374	43
18	.715602	3.45	.931691	1.28	.783910	4.75	.216090	42
19	.715809	3.47	.931614	1.28	.784195	4.73	.215805	41
20	9.716017	3.45	9.931537	1.28	9.784479	4.75	0.215521	40
21	.716224	3.47	.931460	1.28	.784764	4.73	.215236	39
22	.716432	3.45	.931383	1.28	.785048	4.73	.214952	38
23	.716639	3.45	.931306	1.28	.785332	4.73	.214668	37
24	.716846	3.45	.931229	1.28	.785616	4.73	.214384	36
25	9.717053	3.43	9.931152	1.28	9.785900	4.73	0.214100	35
26	.717259	3.45	.931075	1.28	.786184	4.73	.213816	34
27	.717466	3.45	.930998	1.28	.786468	4.73	.213532	33
28	.717673	3.43	.930921	1.30	.786752	4.73	.213248	32
29	.717879	3.43	.930843	1.28	.787036	4.72	.212964	31
30	9.718085	3.43	9.930766	1.30	9.787319	4.73	0.212681	30
31	.718291	3.43	.930688	1.28	.787603	4.72	.212397	29
32	.718497	3.43	.930611	1.30	.787886	4.73	.212114	28
33	.718703	3.43	.930533	1.28	.788170	4.72	.211830	27
34	.718909	3.42	.930456	1.30	.788453	4.72	.211547	26
35	9.719114	3.43	9.930378	1.30	9.788736	4.72	0.211264	25
36	.719320	3.42	.930300	1.28	.789019	4.72	.210981	24
37	.719525	3.42	.930223	1.30	.789302	4.72	.210698	23
38	.719730	3.42	.930145	1.30	.789585	4.72	.210415	22
39	.719935	3.42	.930067	1.30	.789868	4.72	.210132	21
40	9.720140	3.42	9.929989	1.30	9.790151	4.72	0.209849	20
41	.720345	3.40	.929911	1.30	.790434	4.70	.209566	19
42	.720549	3.42	.929833	1.30	.790716	4.72	.209284	18
43	.720754	3.40	.929755	1.30	.790999	4.70	.209001	17
44	.720958	3.40	.929677	1.30	.791281	4.70	.208719	16
45	9.721162	3.40	9.929599	1.30	9.791563	4.72	0.208437	15
46	.721366	3.40	.929521	1.32	.791846	4.70	.208154	14
47	.721570	3.40	.929442	1.30	.792128	4.70	.207872	13
48	.721774	3.40	.929364	1.30	.792410	4.70	.207590	12
49	.721978	3.38	.929286	1.32	.792692	4.70	.207308	11
50	9.722181	3.40	9.929207	1.30	9.792974	4.70	0.207026	10
51	.722385	3.38	.929129	1.32	.793256	4.70	.206744	9
52	.722588	3.38	.929050	1.30	.793538	4.68	.206462	8
53	.722791	3.38	.928972	1.32	.793819	4.70	.206181	7
54	.722994	3.38	.928893	1.30	.794101	4.70	.205899	6
55	9.723197	3.38	9.928815	1.32	9.794383	4.68	0.205617	5
56	.723400	3.38	.928736	1.32	.794664	4.70	.205336	4
57	.723603	3.37	.928657	1.32	.794946	4.68	.205054	3
58	.723805	3.37	.928578	1.32	.795227	4.68	.204773	2
59	.724007	3.38	.928499	1.32	.795508	4.68	.204492	1
60	9.724210		9.928420		9.795789		0.204211	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.724210	3.37	9.928420	1.30	9.795789	4.68	0.204211	60
1	.724412	3.37	.928342	1.32	.796070	4.68	.203930	59
2	.724614	3.37	.928263	1.33	.796351	4.68	.203649	58
3	.724816	3.35	.928183	1.32	.796632	4.68	.203368	57
4	.725017	3.37	.928104	1.32	.796913	4.68	.203087	56
5	9.725219	3.35	9.928025	1.32	9.797194	4.67	0.202806	55
6	.725420	3.37	.927946	1.32	.797474	4.68	.202526	54
7	.725622	3.35	.927867	1.33	.797755	4.68	.202245	53
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	9.726225	3.35	9.927629	1.33	9.798596	4.68	0.201404	50
11	.726426	3.33	.927549	1.32	.798877	4.67	.201123	49
12	.726626	3.35	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.33	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.35	.927310	1.32	.799717	4.67	.200283	46
15	9.727228	3.33	9.927231	1.33	9.799997	4.67	0.200003	45
16	.727428	3.33	.927151	1.33	.800277	4.67	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.65	.199443	43
18	.727828	3.32	.926991	1.33	.800836	4.67	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.67	.198884	41
20	9.728227	3.33	9.926831	1.33	9.801396	4.65	0.198604	40
21	.728427	3.32	.926751	1.33	.801675	4.67	.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.65	.198045	38
23	.728825	3.32	.926591	1.33	.802234	4.65	.197766	37
24	.729024	3.32	.926511	1.33	.802513	4.65	.197487	36
25	9.729223	3.32	9.926431	1.33	9.802792	4.67	0.197208	35
26	.729422	3.32	.926351	1.35	.803072	4.65	.196928	34
27	.729621	3.32	.926270	1.33	.803351	4.65	.196649	33
28	.729820	3.30	.926190	1.33	.803630	4.65	.196370	32
29	.730018	3.32	.926110	1.35	.803909	4.63	.196091	31
30	9.730217	3.30	9.926029	1.33	9.804187	4.65	0.195813	30
31	.730415	3.30	.925949	1.35	.804466	4.65	.195534	29
32	.730613	3.30	.925868	1.33	.804745	4.63	.195255	28
33	.730811	3.30	.925788	1.35	.805023	4.65	.194977	27
34	.731009	3.28	.925707	1.35	.805302	4.63	.194698	26
35	9.731206	3.30	9.925626	1.35	9.805580	4.65	0.194420	25
36	.731404	3.30	.925545	1.33	.805859	4.63	.194141	24
37	.731602	3.28	.925465	1.35	.806137	4.63	.193863	23
38	.731799	3.28	.925384	1.35	.806415	4.63	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	9.732193	3.28	9.925222	1.35	9.806971	4.63	0.193029	20
41	.732390	3.28	.925141	1.35	.807249	4.63	.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.27	.924979	1.37	.807805	4.63	.192195	17
44	.732980	3.28	.924897	1.35	.808083	4.63	.191917	16
45	9.733177	3.27	9.924816	1.35	9.808361	4.62	0.191639	15
46	.733373	3.27	.924735	1.35	.808638	4.63	.191362	14
47	.733569	3.27	.924654	1.37	.808916	4.62	.191084	13
48	.733765	3.27	.924572	1.35	.809193	4.63	.190807	12
49	.733961	3.27	.924491	1.37	.809471	4.62	.190529	11
50	9.734157	3.27	9.924409	1.35	9.809748	4.62	0.190252	10
51	.734353	3.27	.924328	1.37	.810025	4.62	.189975	9
52	.734549	3.25	.924246	1.37	.810302	4.63	.189698	8
53	.734744	3.25	.924164	1.35	.810580	4.62	.189420	7
54	.734939	3.27	.924083	1.37	.810857	4.62	.189143	6
55	9.735135	3.25	9.924001	1.37	9.811134	4.60	0.188866	5
56	.735330	3.25	.923919	1.37	.811410	4.62	.188590	4
57	.735525	3.23	.923837	1.37	.811687	4.62	.188313	3
58	.735719	3.25	.923755	1.37	.811964	4.62	.188036	2
59	.735914	3.25	.923673	1.37	.812241	4.60	.187759	1
60	9.736109		9.923591		9.812517		0.187483	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.736109	3.23	9.923591	1.37	9.812517	4.62	0.187483	60
1	.736303	3.25	.923509	1.37	.812794	4.60	.187206	59
2	.736498	3.23	.923427	1.37	.813070	4.62	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.60	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	9.737080	3.23	9.923181	1.38	9.813899	4.62	0.186101	55
6	.737274	3.22	.923098	1.37	.814176	4.60	.185824	54
7	.737467	3.23	.923016	1.38	.814452	4.60	.185548	53
8	.737661	3.23	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.38	.815004	4.60	.184996	51
10	9.738048	3.22	9.922768	1.37	9.815280	4.58	0.184720	50
11	.738241	3.22	.922686	1.38	.815555	4.60	.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.60	.184169	48
13	.738627	3.22	.922520	1.37	.816107	4.58	.183893	47
14	.738820	3.22	.922438	1.38	.816382	4.60	.183618	46
15	9.739013	3.22	9.922355	1.38	9.816658	4.58	0.183342	45
16	.739206	3.20	.922272	1.38	.816933	4.60	.183067	44
17	.739398	3.20	.922189	1.38	.817209	4.58	.182791	43
18	.739590	3.22	.922106	1.38	.817484	4.58	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.60	.182241	41
20	9.739975	3.20	9.921940	1.38	9.818035	4.58	0.181965	40
21	.740167	3.20	.921857	1.38	.818310	4.58	.181690	39
22	.740359	3.18	.921774	1.38	.818585	4.58	.181415	38
23	.740550	3.20	.921691	1.40	.818860	4.58	.181140	37
24	.740742	3.20	.921607	1.38	.819135	4.58	.180865	36
25	9.740934	3.18	9.921524	1.38	9.819410	4.57	0.180590	35
26	.741125	3.18	.921441	1.40	.819684	4.58	.180316	34
27	.741316	3.20	.921357	1.38	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.40	.820234	4.57	.179766	32
29	.741699	3.17	.921190	1.38	.820508	4.58	.179492	31
30	9.741889	3.18	9.921107	1.40	9.820783	4.57	0.179217	30
31	.742080	3.18	.921023	1.40	.821057	4.58	.178943	29
32	.742271	3.18	.920939	1.38	.821332	4.57	.178668	28
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	27
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	26
35	9.742842	3.18	9.920688	1.40	9.822154	4.58	0.177846	25
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	24
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	23
38	.743413	3.15	.920436	1.40	.822977	4.57	.177023	22
39	.743602	3.17	.920352	1.40	.823251	4.55	.176749	21
40	9.743792	3.17	9.920268	1.40	9.823524	4.57	0.176476	20
41	.743982	3.15	.920184	1.42	.823798	4.57	.176202	19
42	.744171	3.17	.920099	1.40	.824072	4.55	.175928	18
43	.744361	3.15	.920015	1.40	.824345	4.57	.175655	17
44	.744550	3.15	.919931	1.42	.824619	4.57	.175381	16
45	9.744739	3.15	9.919846	1.40	9.824893	4.55	0.175107	15
46	.744928	3.15	.919762	1.42	.825166	4.55	.174834	14
47	.745117	3.15	.919677	1.40	.825439	4.57	.174561	13
48	.745306	3.13	.919593	1.42	.825713	4.55	.174287	12
49	.745494	3.15	.919508	1.40	.825986	4.55	.174014	11
50	9.745683	3.13	9.919424	1.42	9.826259	4.55	0.173741	10
51	.745871	3.15	.919339	1.42	.826532	4.55	.173468	9
52	.746060	3.13	.919254	1.42	.826805	4.55	.173195	8
53	.746248	3.13	.919169	1.40	.827078	4.55	.172922	7
54	.746436	3.13	.919085	1.42	.827351	4.55	.172649	6
55	9.746624	3.13	9.919000	1.42	9.827624	4.55	0.172376	5
56	.746812	3.12	.918915	1.42	.827897	4.55	.172103	4
57	.746999	3.13	.918830	1.42	.828170	4.53	.171830	3
58	.747187	3.12	.918745	1.43	.828442	4.55	.171558	2
59	.747374	3.13	.918659	1.42	.828715	4.53	.171285	1
60	9.747562		9.918574		9.828987		0.171013	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.747562	3.12	9.918574	1.42	9.828987	4.55	0.171013	60
1	.747749	3.12	.918489	1.42	.829260	4.53	.170740	59
2	.747936	3.12	.918404	1.43	.829532	4.55	.170468	58
3	.748123	3.12	.918318	1.42	.829805	4.53	.170195	57
4	.748310	3.12	.918233	1.43	.830077	4.53	.169923	56
5	9.748497	3.10	9.918147	1.42	9.830349	4.53	0.169651	55
6	.748683	3.12	.918062	1.43	.830621	4.53	.169379	54
7	.748870	3.10	.917976	1.42	.830893	4.53	.169107	53
8	.749056	3.12	.917891	1.43	.831165	4.53	.168835	52
9	.749243	3.10	.917805	1.43	.831437	4.53	.168563	51
10	9.749429	3.10	9.917719	1.42	9.831709	4.53	0.168291	50
11	.749615	3.10	.917634	1.43	.831981	4.53	.168019	49
12	.749801	3.10	.917548	1.43	.832253	4.53	.167747	48
13	.749987	3.08	.917462	1.43	.832525	4.52	.167475	47
14	.750172	3.10	.917376	1.43	.832796	4.53	.167204	46
15	9.750358	3.08	9.917290	1.43	9.833068	4.52	0.166932	45
16	.750543	3.10	.917204	1.43	.833339	4.53	.166661	44
17	.750729	3.08	.917118	1.43	.833611	4.52	.166389	43
18	.750914	3.08	.917032	1.43	.833882	4.53	.166118	42
19	.751099	3.08	.916946	1.45	.834154	4.52	.165846	41
20	9.751284	3.08	9.916859	1.43	9.834425	4.52	0.165575	40
21	.751469	3.08	.916773	1.43	.834696	4.52	.165304	39
22	.751654	3.08	.916687	1.45	.834967	4.52	.165033	38
23	.751839	3.07	.916600	1.43	.835238	4.52	.164762	37
24	.752023	3.08	.916514	1.45	.835509	4.52	.164491	36
25	9.752208	3.07	9.916427	1.43	9.835780	4.52	0.164220	35
26	.752392	3.07	.916341	1.45	.836051	4.52	.163949	34
27	.752576	3.07	.916254	1.45	.836322	4.52	.163678	33
28	.752760	3.07	.916167	1.43	.836593	4.52	.163407	32
29	.752944	3.07	.916081	1.45	.836864	4.50	.163136	31
30	9.753128	3.07	9.915994	1.45	9.837134	4.52	0.162866	30
31	.753312	3.05	.915907	1.45	.837405	4.50	.162595	29
32	.753495	3.07	.915820	1.45	.837675	4.52	.162325	28
33	.753679	3.05	.915733	1.45	.837946	4.50	.162054	27
34	.753862	3.07	.915646	1.45	.838216	4.52	.161784	26
35	9.754046	3.05	9.915559	1.45	9.838487	4.50	0.161513	25
36	.754229	3.05	.915472	1.45	.838757	4.50	.161243	24
37	.754412	3.05	.915385	1.47	.839027	4.50	.160973	23
38	.754595	3.05	.915297	1.45	.839297	4.52	.160703	22
39	.754778	3.03	.915210	1.45	.839568	4.50	.160432	21
40	9.754960	3.05	9.915123	1.47	9.839838	4.50	0.160162	20
41	.755143	3.05	.915035	1.45	.840108	4.50	.159892	19
42	.755326	3.03	.914948	1.47	.840378	4.50	.159622	18
43	.755508	3.03	.914860	1.45	.840648	4.48	.159352	17
44	.755690	3.03	.914773	1.47	.840917	4.50	.159083	16
45	9.755872	3.03	9.914685	1.45	9.841187	4.50	0.158813	15
46	.756054	3.03	.914598	1.47	.841457	4.50	.158543	14
47	.756236	3.03	.914510	1.47	.841727	4.48	.158273	13
48	.756418	3.03	.914422	1.47	.841996	4.50	.158004	12
49	.756600	3.03	.914334	1.47	.842266	4.48	.157734	11
50	9.756782	3.02	9.914246	1.47	9.842535	4.50	0.157465	10
51	.756963	3.02	.914158	1.47	.842805	4.48	.157195	9
52	.757144	3.03	.914070	1.47	.843074	4.48	.156926	8
53	.757326	3.02	.913982	1.47	.843343	4.48	.156657	7
54	.757507	3.02	.913894	1.47	.843612	4.50	.156388	6
55	9.757688	3.02	9.913806	1.47	9.843882	4.48	0.156118	5
56	.757869	3.02	.913718	1.47	.844151	4.48	.155849	4
57	.758050	3.00	.913630	1.48	.844420	4.48	.155580	3
58	.758230	3.02	.913541	1.47	.844689	4.48	.155311	2
59	.758411	3.00	.913453	1.47	.844958	4.48	.155042	1
60	9.758591		9.913365		9.845227		0.154773	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.758591	3.02	9.913365	1.48	9.845227	4.48	0.154773	60
1	.758772	3.00	.913276	1.48	.845496	4.47	.154504	59
2	.758952	3.00	.913187	1.47	.845764	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.846033	4.48	.153967	57
4	.759312	3.00	.913010	1.47	.846302	4.47	.153698	56
5	9.759492	3.00	9.912922	1.48	9.846570	4.48	0.153430	55
6	.759672	3.00	.912833	1.48	.846839	4.48	.153161	54
7	.759852	2.98	.912744	1.48	.847108	4.47	.152892	53
8	.760031	3.00	.912655	1.48	.847376	4.47	.152624	52
9	.760211	2.98	.912566	1.48	.847644	4.48	.152356	51
10	9.760390	2.98	9.912477	1.48	9.847913	4.47	0.152087	50
11	.760569	2.98	.912388	1.48	.848181	4.47	.151819	49
12	.760748	2.98	.912299	1.48	.848449	4.47	.151551	48
13	.760927	2.98	.912210	1.48	.848717	4.48	.151283	47
14	.761106	2.98	.912121	1.50	.848986	4.47	.151014	46
15	9.761285	2.98	9.912031	1.48	9.849254	4.47	0.150746	45
16	.761464	2.97	.911942	1.48	.849522	4.47	.150478	44
17	.761642	2.98	.911853	1.50	.849790	4.45	.150210	43
18	.761821	2.97	.911763	1.48	.850057	4.47	.149943	42
19	.761999	2.97	.911674	1.50	.850325	4.47	.149675	41
20	9.762177	2.98	9.911584	1.48	9.850593	4.47	0.149407	40
21	.762356	2.97	.911495	1.50	.850861	4.47	.149139	39
22	.762534	2.97	.911405	1.50	.851129	4.45	.148871	38
23	.762712	2.95	.911315	1.48	.851396	4.47	.148604	37
24	.762889	2.97	.911226	1.50	.851664	4.45	.148336	36
25	9.763067	2.97	9.911136	1.50	9.851931	4.47	0.148069	35
26	.763245	2.95	.911046	1.50	.852199	4.45	.147801	34
27	.763422	2.97	.910956	1.50	.852466	4.45	.147534	33
28	.763600	2.95	.910866	1.50	.852733	4.47	.147267	32
29	.763777	2.95	.910776	1.50	.853001	4.45	.146999	31
30	9.763954	2.95	9.910686	1.50	9.853268	4.45	0.146732	30
31	.764131	2.95	.910596	1.50	.853535	4.45	.146465	29
32	.764308	2.95	.910506	1.52	.853802	4.45	.146198	28
33	.764485	2.95	.910415	1.50	.854069	4.45	.145931	27
34	.764662	2.93	.910325	1.50	.854336	4.45	.145664	26
35	9.764838	2.95	9.910235	1.52	9.854603	4.45	0.145397	25
36	.765015	2.93	.910144	1.50	.854870	4.45	.145130	24
37	.765191	2.93	.910054	1.52	.855137	4.45	.144863	23
38	.765367	2.95	.909963	1.50	.855404	4.45	.144596	22
39	.765544	2.93	.909873	1.52	.855671	4.45	.144329	21
40	9.765720	2.93	9.909782	1.52	9.855938	4.43	0.144062	20
41	.765896	2.93	.909691	1.50	.856204	4.45	.143796	19
42	.766072	2.92	.909601	1.52	.856471	4.43	.143529	18
43	.766247	2.93	.909510	1.52	.856737	4.45	.143263	17
44	.766423	2.92	.909419	1.52	.857004	4.43	.142996	16
45	9.766598	2.93	9.909328	1.52	9.857270	4.45	0.142730	15
46	.766774	2.92	.909237	1.52	.857537	4.43	.142463	14
47	.766949	2.92	.909146	1.52	.857803	4.43	.142197	13
48	.767124	2.93	.909055	1.52	.858069	4.45	.141931	12
49	.767300	2.92	.908964	1.52	.858336	4.43	.141664	11
50	9.767475	2.90	9.908873	1.53	9.858602	4.43	0.141398	10
51	.767649	2.92	.908781	1.52	.858868	4.43	.141132	9
52	.767824	2.92	.908690	1.52	.859134	4.43	.140866	8
53	.767999	2.90	.908599	1.53	.859400	4.43	.140600	7
54	.768173	2.92	.908507	1.52	.859666	4.43	.140334	6
55	9.768348	2.90	9.908416	1.53	9.859932	4.43	0.140068	5
56	.768522	2.92	.908324	1.52	.860198	4.43	.139802	4
57	.768697	2.90	.908233	1.53	.860464	4.43	.139536	3
58	.768871	2.90	.908141	1.53	.860730	4.42	.139270	2
59	.769045	2.90	.908049	1.52	.860995	4.43	.139005	1
60	9.769219		9.907958		9.861261		0.138739	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.769219	2.90	9.907958	1.53	9.861261	4.43	0.138739	60
1	.769393	2.88	.907866	1.53	.861527	4.42	.138473	59
2	.769566	2.90	.907774	1.53	.861792	4.43	.138208	58
3	.769740	2.88	.907682	1.53	.862058	4.42	.137942	57
4	.769913	2.90	.907590	1.53	.862323	4.43	.137677	56
5	9.770087	2.88	9.907498	1.53	9.862589	4.42	0.137411	55
6	.770260	2.88	.907406	1.53	.862854	4.42	.137146	54
7	.770433	2.88	.907314	1.53	.863119	4.43	.136881	53
8	.770606	2.88	.907222	1.55	.863385	4.42	.136615	52
9	.770779	2.88	.907129	1.53	.863650	4.42	.136350	51
10	9.770952	2.88	9.907037	1.53	9.863915	4.42	0.136085	50
11	.771125	2.88	.906945	1.55	.864180	4.42	.135820	49
12	.771298	2.87	.906852	1.53	.864445	4.42	.135555	48
13	.771470	2.88	.906760	1.55	.864710	4.42	.135290	47
14	.771643	2.87	.906667	1.53	.864975	4.42	.135025	46
15	9.771815	2.87	9.906575	1.55	9.865240	4.42	0.134760	45
16	.771987	2.87	.906482	1.55	.865505	4.42	.134495	44
17	.772159	2.87	.906389	1.55	.865770	4.42	.134230	43
18	.772331	2.87	.906296	1.53	.866035	4.42	.133965	42
19	.772503	2.87	.906204	1.55	.866300	4.40	.133700	41
20	9.772675	2.87	9.906111	1.55	9.866564	4.42	0.133436	40
21	.772847	2.85	.906018	1.55	.866829	4.42	.133171	39
22	.773018	2.87	.905925	1.55	.867094	4.40	.132906	38
23	.773190	2.85	.905832	1.55	.867358	4.42	.132642	37
24	.773361	2.87	.905739	1.57	.867623	4.40	.132377	36
25	9.773533	2.85	9.905645	1.55	9.867887	4.42	0.132113	35
26	.773704	2.85	.905552	1.55	.868152	4.40	.131848	34
27	.773875	2.85	.905459	1.55	.868416	4.40	.131584	33
28	.774046	2.85	.905366	1.57	.868680	4.42	.131320	32
29	.774217	2.85	.905272	1.55	.868945	4.40	.131055	31
30	9.774388	2.83	9.905179	1.57	9.869209	4.40	0.130791	30
31	.774558	2.85	.905085	1.55	.869473	4.40	.130527	29
32	.774729	2.83	.904992	1.57	.869737	4.40	.130263	28
33	.774899	2.85	.904898	1.57	.870001	4.40	.129999	27
34	.775070	2.83	.904804	1.55	.870265	4.40	.129735	26
35	9.775240	2.83	9.904711	1.57	9.870529	4.40	0.129471	25
36	.775410	2.83	.904617	1.57	.870793	4.40	.129207	24
37	.775580	2.83	.904523	1.57	.871057	4.40	.128943	23
38	.775750	2.83	.904429	1.57	.871321	4.40	.128679	22
39	.775920	2.83	.904335	1.57	.871585	4.40	.128415	21
40	9.776090	2.82	9.904241	1.57	9.871849	4.38	0.128151	20
41	.776259	2.83	.904147	1.57	.872112	4.40	.127888	19
42	.776429	2.82	.904053	1.57	.872376	4.40	.127624	18
43	.776598	2.83	.903959	1.58	.872640	4.38	.127360	17
44	.776768	2.82	.903864	1.57	.872903	4.40	.127097	16
45	9.776937	2.82	9.903770	1.57	9.873167	4.38	0.126833	15
46	.777106	2.82	.903676	1.58	.873430	4.40	.126570	14
47	.777275	2.82	.903581	1.57	.873694	4.38	.126306	13
48	.777444	2.82	.903487	1.58	.873957	4.38	.126043	12
49	.777613	2.80	.903392	1.57	.874220	4.40	.125780	11
50	9.777781	2.82	9.903298	1.58	9.874484	4.38	0.125516	10
51	.777950	2.82	.903203	1.58	.874747	4.38	.125253	9
52	.778119	2.80	.903108	1.57	.875010	4.38	.124990	8
53	.778287	2.80	.903014	1.58	.875273	4.40	.124727	7
54	.778455	2.82	.902919	1.58	.875537	4.38	.124463	6
55	9.778624	2.80	9.902824	1.58	9.875800	4.38	0.124200	5
56	.778792	2.80	.902729	1.58	.876063	4.38	.123937	4
57	.778960	2.80	.902634	1.58	.876326	4.38	.123674	3
58	.779128	2.78	.902539	1.58	.876589	4.38	.123411	2
59	.779295	2.80	.902444	1.58	.876852	4.37	.123148	1
60	9.779463		9.902349		9.877114		0.122886	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.779463	2.80	9.902349	1.60	9.877114	4.38	0.122886	60
1	.779631	2.78	.902253	1.58	.877377	4.38	.122623	59
2	.779798	2.80	.902158	1.58	.877640	4.38	.122360	58
3	.779966	2.78	.902063	1.60	.877903	4.37	.122097	57
4	.780133	2.78	.901967	1.58	.878165	4.38	.121835	56
5	9.780300	2.78	9.901872	1.60	9.878428	4.38	0.121572	55
6	.780467	2.78	.901776	1.58	.878691	4.37	.121309	54
7	.780634	2.78	.901681	1.60	.878953	4.38	.121047	53
8	.780801	2.78	.901585	1.58	.879216	4.37	.120784	52
9	.780968	2.77	.901490	1.60	.879478	4.38	.120522	51
10	9.781134	2.78	9.901394	1.60	9.879741	4.37	0.120259	50
11	.781301	2.78	.901298	1.60	.880003	4.37	.119997	49
12	.781468	2.77	.901202	1.60	.880265	4.38	.119735	48
13	.781634	2.77	.901106	1.60	.880528	4.37	.119472	47
14	.781800	2.77	.901010	1.60	.880790	4.37	.119210	46
15	9.781966	2.77	9.900914	1.60	9.881052	4.37	0.118948	45
16	.782132	2.77	.900818	1.60	.881314	4.38	.118686	44
17	.782298	2.77	.900722	1.60	.881577	4.37	.118423	43
18	.782464	2.77	.900626	1.62	.881839	4.37	.118161	42
19	.782630	2.77	.900529	1.60	.882101	4.37	.117899	41
20	9.782796	2.75	9.900433	1.60	9.882363	4.37	0.117637	40
21	.782961	2.77	.900337	1.62	.882625	4.37	.117375	39
22	.783127	2.75	.900240	1.60	.882887	4.35	.117113	38
23	.783292	2.77	.900144	1.62	.883148	4.37	.116852	37
24	.783458	2.75	.900047	1.60	.883410	4.37	.116590	36
25	9.783623	2.75	9.899951	1.62	9.883672	4.37	0.116328	35
26	.783788	2.75	.899854	1.62	.883934	4.37	.116066	34
27	.783953	2.75	.899757	1.62	.884196	4.35	.115804	33
28	.784118	2.73	.899660	1.60	.884457	4.37	.115543	32
29	.784282	2.75	.899564	1.62	.884719	4.35	.115281	31
30	9.784447	2.75	9.899467	1.62	9.884980	4.37	0.115020	30
31	.784612	2.73	.899370	1.62	.885242	4.37	.114758	29
32	.784776	2.75	.899273	1.62	.885504	4.35	.114496	28
33	.784941	2.73	.899176	1.63	.885765	4.35	.114235	27
34	.785105	2.73	.899078	1.62	.886026	4.37	.113974	26
35	9.785269	2.73	9.898981	1.62	9.886288	4.35	0.113712	25
36	.785433	2.73	.898884	1.62	.886549	4.37	.113451	24
37	.785597	2.73	.898787	1.63	.886811	4.35	.113189	23
38	.785761	2.73	.898689	1.62	.887072	4.35	.112928	22
39	.785925	2.73	.898592	1.63	.887333	4.35	.112667	21
40	9.786089	2.72	9.898494	1.62	9.887594	4.35	0.112406	20
41	.786252	2.73	.898397	1.63	.887855	4.35	.112145	19
42	.786416	2.72	.898299	1.62	.888116	4.37	.111884	18
43	.786579	2.72	.898202	1.63	.888378	4.35	.111622	17
44	.786742	2.73	.898104	1.63	.888639	4.35	.111361	16
45	9.786906	2.72	9.898006	1.63	9.888900	4.35	0.111100	15
46	.787069	2.72	.897908	1.63	.889161	4.33	.110839	14
47	.787232	2.72	.897810	1.63	.889421	4.35	.110579	13
48	.787395	2.70	.897712	1.63	.889682	4.35	.110318	12
49	.787557	2.72	.897614	1.63	.889943	4.35	.110057	11
50	9.787720	2.72	9.897516	1.63	9.890204	4.35	0.109796	10
51	.787883	2.70	.897418	1.63	.890465	4.33	.109535	9
52	.788045	2.72	.897320	1.63	.890725	4.35	.109275	8
53	.788208	2.70	.897222	1.65	.890986	4.35	.109014	7
54	.788370	2.70	.897123	1.63	.891247	4.33	.108753	6
55	9.788532	2.70	9.897025	1.65	9.891507	4.35	0.108493	5
56	.788694	2.70	.896926	1.63	.891768	4.33	.108232	4
57	.788856	2.70	.896828	1.65	.892028	4.35	.107972	3
58	.789018	2.70	.896729	1.63	.892289	4.33	.107711	2
59	.789180	2.70	.896631	1.65	.892549	4.35	.107451	1
60	9.789342		9.896532		9.892810		0.107190	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.789342	2.70	9.896532	1.65	9.892810	4.33	0.107190	60
1	.789504	2.68	.896433	1.63	.893070	4.35	.106930	59
2	.789665	2.70	.896335	1.65	.893331	4.33	.106669	58
3	.789827	2.68	.896236	1.65	.893591	4.33	.106409	57
4	.789988	2.68	.896137	1.65	.893851	4.33	.106149	56
5	9.790149	2.68	9.896038	1.65	9.894111	4.35	0.105889	55
6	.790310	2.68	.895939	1.65	.894372	4.33	.105628	54
7	.790471	2.68	.895840	1.65	.894632	4.33	.105368	53
8	.790632	2.68	.895741	1.67	.894892	4.33	.105108	52
9	.790793	2.68	.895641	1.65	.895152	4.33	.104848	51
10	9.790954	2.68	9.895542	1.65	9.895412	4.33	0.104588	50
11	.791115	2.67	.895443	1.67	.895672	4.33	.104328	49
12	.791275	2.68	.895343	1.65	.895932	4.33	.104068	48
13	.791436	2.67	.895244	1.65	.896192	4.33	.103808	47
14	.791596	2.68	.895145	1.67	.896452	4.33	.103548	46
15	9.791757	2.67	9.895045	1.67	9.896712	4.32	0.103288	45
16	.791917	2.67	.894945	1.65	.896971	4.33	.103029	44
17	.792077	2.67	.894846	1.67	.897231	4.33	.102769	43
18	.792237	2.67	.894746	1.67	.897491	4.33	.102509	42
19	.792397	2.67	.894646	1.67	.897751	4.32	.102249	41
20	9.792557	2.65	9.894546	1.67	9.898010	4.33	0.101990	40
21	.792716	2.67	.894446	1.67	.898270	4.33	.101730	39
22	.792876	2.65	.894346	1.67	.898530	4.32	.101470	38
23	.793035	2.67	.894246	1.67	.898789	4.33	.101211	37
24	.793195	2.65	.894146	1.67	.899049	4.32	.100951	36
25	9.793354	2.67	9.894046	1.67	9.899308	4.33	0.100692	35
26	.793514	2.65	.893946	1.67	.899568	4.32	.100432	34
27	.793673	2.65	.893846	1.68	.899827	4.33	.100173	33
28	.793832	2.65	.893745	1.67	.900087	4.32	.099913	32
29	.793991	2.65	.893645	1.68	.900346	4.32	.099654	31
30	9.794150	2.63	9.893544	1.67	9.900605	4.32	0.099395	30
31	.794308	2.65	.893444	1.68	.900864	4.33	.099136	29
32	.794467	2.65	.893343	1.67	.901124	4.32	.098876	28
33	.794626	2.63	.893243	1.68	.901383	4.32	.098617	27
34	.794784	2.63	.893142	1.68	.901642	4.32	.098358	26
35	9.794942	2.65	9.893041	1.68	9.901901	4.32	0.098099	25
36	.795101	2.63	.892940	1.68	.902160	4.33	.097840	24
37	.795259	2.63	.892839	1.67	.902420	4.32	.097580	23
38	.795417	2.63	.892739	1.68	.902679	4.32	.097321	22
39	.795575	2.63	.892638	1.70	.902938	4.32	.097062	21
40	9.795733	2.63	9.892536	1.68	9.903197	4.32	0.096803	20
41	.795891	2.63	.892435	1.68	.903456	4.30	.096544	19
42	.796049	2.62	.892334	1.68	.903714	4.32	.096286	18
43	.796206	2.63	.892233	1.68	.903973	4.32	.096027	17
44	.796364	2.62	.892132	1.70	.904232	4.32	.095768	16
45	9.796521	2.63	9.892030	1.68	9.904491	4.32	0.095509	15
46	.796679	2.62	.891929	1.70	.904750	4.30	.095250	14
47	.796836	2.62	.891827	1.68	.905008	4.32	.094992	13
48	.796993	2.62	.891726	1.70	.905267	4.32	.094733	12
49	.797150	2.62	.891624	1.68	.905526	4.32	.094474	11
50	9.797307	2.62	9.891523	1.70	9.905785	4.30	0.094215	10
51	.797464	2.62	.891421	1.70	.906043	4.32	.093957	9
52	.797621	2.60	.891319	1.70	.906302	4.30	.093698	8
53	.797777	2.62	.891217	1.70	.906560	4.32	.093440	7
54	.797934	2.62	.891115	1.70	.906819	4.30	.093181	6
55	9.798091	2.60	9.891013	1.70	9.907077	4.32	0.092923	5
56	.798247	2.60	.890911	1.70	.907336	4.30	.092664	4
57	.798403	2.62	.890809	1.70	.907594	4.32	.092406	3
58	.798560	2.60	.890707	1.70	.907853	4.30	.092147	2
59	.798716	2.60	.890605	1.70	.908111	4.30	.091889	1
60	9.798872		9.890503		9.908369		0.091631	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.798872	2.60	9.890503	1.72	9.908369	4.32	0.091631	60
1	.799028	2.60	.890400	1.70	.908628	4.30	.091372	59
2	.799184	2.58	.890298	1.72	.908886	4.30	.091114	58
3	.799339	2.60	.890195	1.70	.909144	4.30	.090856	57
4	.799495	2.60	.890093	1.72	.909402	4.30	.090598	56
5	9.799651	2.58	9.889990	1.70	9.909660	4.30	0.090340	55
6	.799806	2.60	.889888	1.72	.909918	4.32	.090082	54
7	.799962	2.58	.889785	1.72	.910177	4.30	.089823	53
8	.800117	2.58	.889682	1.72	.910435	4.30	.089565	52
9	.800272	2.58	.889579	1.70	.910693	4.30	.089307	51
10	9.800427	2.58	9.889477	1.72	9.910951	4.30	0.089049	50
11	.800582	2.58	.889374	1.72	.911209	4.30	.088791	49
12	.800737	2.58	.889271	1.72	.911467	4.30	.088533	48
13	.800892	2.58	.889168	1.73	.911725	4.28	.088275	47
14	.801047	2.57	.889064	1.72	.911982	4.30	.088018	46
15	9.801201	2.58	9.888961	1.72	9.912240	4.30	0.087760	45
16	.801356	2.58	.888858	1.72	.912498	4.30	.087502	44
17	.801511	2.57	.888755	1.73	.912756	4.30	.087244	43
18	.801665	2.57	.888651	1.72	.913014	4.28	.086986	42
19	.801819	2.57	.888548	1.73	.913271	4.30	.086729	41
20	9.801973	2.58	9.888444	1.72	9.913529	4.30	0.086471	40
21	.802128	2.57	.888341	1.73	.913787	4.28	.086213	39
22	.802282	2.57	.888237	1.72	.914044	4.30	.085956	38
23	.802436	2.55	.888134	1.73	.914302	4.30	.085698	37
24	.802589	2.57	.888030	1.73	.914560	4.28	.085440	36
25	9.802743	2.57	9.887926	1.73	9.914817	4.30	0.085183	35
26	.802897	2.55	.887822	1.73	.915075	4.28	.084925	34
27	.803050	2.57	.887718	1.73	.915332	4.30	.084668	33
28	.803204	2.55	.887614	1.73	.915590	4.28	.084410	32
29	.803357	2.57	.887510	1.73	.915847	4.28	.084153	31
30	9.803511	2.55	9.887406	1.73	9.916104	4.30	0.083896	30
31	.803664	2.55	.887302	1.73	.916362	4.28	.083638	29
32	.803817	2.55	.887198	1.75	.916619	4.30	.083381	28
33	.803970	2.55	.887093	1.73	.916877	4.28	.083123	27
34	.804123	2.55	.886989	1.73	.917134	4.28	.082866	26
35	9.804276	2.53	9.886885	1.75	9.917391	4.28	0.082609	25
36	.804428	2.55	.886780	1.73	.917648	4.30	.082352	24
37	.804581	2.55	.886676	1.75	.917906	4.28	.082094	23
38	.804734	2.53	.886571	1.75	.918163	4.28	.081837	22
39	.804886	2.55	.886466	1.73	.918420	4.28	.081580	21
40	9.805039	2.53	9.886362	1.75	9.918677	4.28	0.081323	20
41	.805191	2.53	.886257	1.75	.918934	4.28	.081066	19
42	.805343	2.53	.886152	1.75	.919191	4.28	.080809	18
43	.805495	2.53	.886047	1.75	.919448	4.28	.080552	17
44	.805647	2.53	.885942	1.75	.919705	4.28	.080295	16
45	9.805799	2.53	9.885837	1.75	9.919962	4.28	0.080038	15
46	.805951	2.53	.885732	1.75	.920219	4.28	.079781	14
47	.806103	2.52	.885627	1.75	.920476	4.28	.079524	13
48	.806254	2.53	.885522	1.77	.920733	4.28	.079267	12
49	.806406	2.52	.885416	1.75	.920990	4.28	.079010	11
50	9.806557	2.53	9.885311	1.77	9.921247	4.27	0.078753	10
51	.806709	2.52	.885205	1.75	.921503	4.28	.078497	9
52	.806860	2.52	.885100	1.77	.921760	4.28	.078240	8
53	.807011	2.53	.884994	1.75	.922017	4.28	.077983	7
54	.807163	2.52	.884889	1.77	.922274	4.27	.077726	6
55	9.807314	2.52	9.884783	1.77	9.922530	4.28	0.077470	5
56	.807465	2.50	.884677	1.75	.922787	4.28	.077213	4
57	.807615	2.52	.884572	1.77	.923044	4.27	.076956	3
58	.807766	2.52	.884466	1.77	.923300	4.28	.076700	2
59	.807917	2.50	.884360	1.77	.923557	4.28	.076443	1
60	9.808067		9.884254		9.923814		0.076186	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.808067	2.52	9.884254	1.77	9.923814	4.27	0.076186	60
1	.808218	2.50	.884148	1.77	.924070	4.28	.075930	59
2	.808368	2.52	.884042	1.77	.924327	4.27	.075673	58
3	.808519	2.50	.883936	1.78	.924583	4.28	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	9.808819	2.50	9.883723	1.77	9.925096	4.27	0.074904	55
6	.808969	2.50	.883617	1.78	.925352	4.28	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.78	.925865	4.28	.074135	52
9	.809419	2.50	.883297	1.77	.926122	4.27	.073878	51
10	9.809569	2.48	9.883191	1.78	9.926378	4.27	0.073622	50
11	.809718	2.50	.883084	1.78	.926634	4.27	.073366	49
12	.809868	2.48	.882977	1.77	.926890	4.28	.073110	48
13	.810017	2.50	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.48	.882764	1.78	.927403	4.27	.072597	46
15	9.810316	2.48	9.882657	1.78	9.927659	4.27	0.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.78	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.78	.928427	4.28	.071573	42
19	.810912	2.48	.882229	1.80	.928684	4.27	.071316	41
20	9.811061	2.48	9.882121	1.78	9.928940	4.27	0.071060	40
21	.811210	2.47	.882014	1.78	.929196	4.27	.070804	39
22	.811358	2.48	.881907	1.80	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.78	.929708	4.27	.070292	37
24	.811655	2.48	.881692	1.80	.929964	4.27	.070036	36
25	9.811804	2.47	9.881584	1.78	9.930220	4.25	0.069780	35
26	.811952	2.47	.881477	1.80	.930475	4.27	.069525	34
27	.812100	2.47	.881369	1.80	.930731	4.27	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.27	.069013	32
29	.812396	2.47	.881153	1.78	.931243	4.27	.068757	31
30	9.812544	2.47	9.881046	1.80	9.931499	4.27	0.068501	30
31	.812692	2.47	.880938	1.80	.931755	4.25	.068245	29
32	.812840	2.47	.880830	1.80	.932010	4.27	.067990	28
33	.812988	2.45	.880722	1.82	.932266	4.27	.067734	27
34	.813135	2.47	.880613	1.80	.932522	4.27	.067478	26
35	9.813283	2.45	9.880505	1.80	9.932778	4.25	0.067222	25
36	.813430	2.47	.880397	1.80	.933033	4.27	.066967	24
37	.813578	2.45	.880289	1.82	.933289	4.27	.066711	23
38	.813725	2.45	.880180	1.80	.933545	4.25	.066455	22
39	.813872	2.45	.880072	1.82	.933800	4.27	.066200	21
40	9.814019	2.45	9.879963	1.80	9.934056	4.25	0.065944	20
41	.814166	2.45	.879855	1.82	.934311	4.27	.065689	19
42	.814313	2.45	.879746	1.82	.934567	4.25	.065433	18
43	.814460	2.45	.879637	1.80	.934822	4.27	.065178	17
44	.814607	2.43	.879529	1.82	.935078	4.25	.064922	16
45	9.814753	2.45	9.879420	1.82	9.935333	4.27	0.064667	15
46	.814900	2.43	.879311	1.82	.935589	4.25	.064411	14
47	.815046	2.45	.879202	1.82	.935844	4.27	.064156	13
48	.815193	2.43	.879093	1.82	.936100	4.25	.063900	12
49	.815339	2.43	.878984	1.82	.936355	4.27	.063645	11
50	9.815485	2.45	9.878875	1.82	9.936611	4.25	0.063389	10
51	.815632	2.43	.878766	1.83	.936866	4.25	.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.27	.062879	8
53	.815924	2.42	.878547	1.82	.937377	4.25	.062623	7
54	.816069	2.43	.878438	1.83	.937632	4.25	.062368	6
55	9.816215	2.43	9.878328	1.82	9.937887	4.25	0.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.27	.061858	4
57	.816507	2.42	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.43	.877999	1.82	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60	9.816943		9.877780		9.939163		0.060837	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	0.060837
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582
2	.817233	2.43	.877560	1.83	.939673	4.25	.060327
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072
4	.817524	2.40	.877340	1.83	.940183	4.27	.059817
5	9.817668	2.42	9.877230	1.83	9.940439	4.25	0.059561
6	.817813	2.42	.877120	1.83	.940694	4.25	.059306
7	.817958	2.42	.877010	1.85	.940949	4.25	.059051
8	.818103	2.40	.876899	1.83	.941204	4.25	.058796
9	.818247	2.42	.876789	1.85	.941459	4.23	.058541
10	9.818392	2.40	9.876678	1.83	9.941713	4.25	0.058287
11	.818536	2.42	.876568	1.85	.941968	4.25	.058032
12	.818681	2.40	.876457	1.83	.942223	4.25	.057777
13	.818825	2.40	.876347	1.85	.942478	4.25	.057522
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267
15	9.819113	2.40	9.876125	1.85	9.942988	4.25	0.057012
16	.819257	2.40	.876014	1.83	.943243	4.25	.056757
17	.819401	2.40	.875904	1.85	.943498	4.23	.056502
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248
19	.819689	2.38	.875682	1.85	.944007	4.25	.055993
20	9.819832	2.40	9.875571	1.87	9.944262	4.25	0.055738
21	.819976	2.40	.875459	1.85	.944517	4.23	.055483
22	.820120	2.38	.875348	1.85	.944771	4.25	.055229
23	.820263	2.38	.875237	1.85	.945026	4.25	.054974
24	.820406	2.40	.875126	1.87	.945281	4.23	.054719
25	9.820550	2.38	9.875014	1.85	9.945535	4.25	0.054465
26	.820693	2.38	.874903	1.87	.945790	4.25	.054210
27	.820836	2.38	.874791	1.85	.946045	4.23	.053955
28	.820979	2.38	.874680	1.87	.946299	4.25	.053701
29	.821122	2.38	.874568	1.87	.946554	4.23	.063446
30	9.821265	2.37	9.874456	1.87	9.946808	4.25	0.053192
31	.821407	2.38	.874344	1.87	.947063	4.25	.052937
32	.821550	2.38	.874232	1.85	.947318	4.23	.052682
33	.821693	2.37	.874121	1.87	.947572	4.25	.052428
34	.821835	2.37	.874009	1.88	.947827	4.23	.052173
35	9.821977	2.38	9.873896	1.87	9.948081	4.23	0.051919
36	.822120	2.37	.873784	1.87	.948335	4.25	.051665
37	.822262	2.37	.873672	1.87	.948590	4.23	.051410
38	.822404	2.37	.873560	1.87	.948844	4.25	.051156
39	.822546	2.37	.873448	1.88	.949099	4.23	.050901
40	9.822688	2.37	9.873335	1.87	9.949353	4.25	0.050647
41	.822830	2.37	.873223	1.88	.949608	4.23	.050392
42	.822972	2.37	.873110	1.87	.949862	4.23	.050138
43	.823114	2.35	.872998	1.88	.950116	4.25	.049884
44	.823255	2.37	.872885	1.88	.950371	4.23	.049629
45	9.823397	2.37	9.872772	1.88	9.950625	4.23	0.049375
46	.823539	2.35	.872659	1.87	.950879	4.23	.049121
47	.823680	2.35	.872547	1.88	.951133	4.25	.048867
48	.823821	2.37	.872434	1.88	.951388	4.23	.048612
49	.823963	2.35	.872321	1.88	.951642	4.23	.048358
50	9.824104	2.35	9.872208	1.88	9.951896	4.23	0.048104
51	.824245	2.35	.872095	1.90	.952150	4.25	.047850
52	.824386	2.35	.871981	1.88	.952405	4.23	.047595
53	.824527	2.35	.871868	1.88	.952659	4.23	.047341
54	.824668	2.33	.871755	1.90	.952913	4.23	.047087
55	9.824808	2.35	9.871641	1.88	9.953167	4.23	0.046833
56	.824949	2.35	.871528	1.90	.953421	4.23	.046579
57	.825090	2.33	.871414	1.88	.953675	4.23	.046325
58	.825230	2.35	.871301	1.90	.953929	4.23	.046071
59	.825371	2.33	.871187	1.90	.954183	4.23	.045817
60	9.825511		9.871073		9.954437		0.045563
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.

TABLE XII.—*Continued*

Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
9.825511	2.33	9.871073	1.88	9.954437	4.23	0.045563	60
.825651	2.33	.870960	1.90	.954691	4.25	.045309	59
.825791	2.33	.870846	1.90	.954946	4.23	.045054	58
.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
9.826211	2.33	9.870504	1.90	9.955708	4.22	0.044292	55
.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
.826491	2.33	.870276	1.92	.956215	4.23	.043785	53
.826631	2.32	.870161	1.90	.956469	4.23	.043531	52
.826770	2.33	.870047	1.90	.956723	4.23	.043277	51
9.826910	2.32	9.869933	1.92	9.956977	4.23	0.043023	50
.827049	2.33	.869818	1.90	.957231	4.23	.042769	49
.827189	2.32	.869704	1.92	.957485	4.23	.042515	48
.827328	2.32	.869589	1.92	.957739	4.23	.042261	47
.827467	2.32	.869474	1.90	.957993	4.23	.042007	46
9.827606	2.32	9.869360	1.92	9.958247	4.22	0.041753	45
.827745	2.32	.869245	1.92	.958500	4.23	.041500	44
.827884	2.32	.869130	1.92	.958754	4.23	.041246	43
.828023	2.32	.869015	1.92	.959008	4.23	.040992	42
.828162	2.32	.868900	1.92	.959262	4.23	.040738	41
9.828301	2.30	9.868785	1.92	9.959516	4.22	0.040484	40
.828439	2.32	.868670	1.92	.959769	4.23	.040231	39
.828578	2.30	.868555	1.92	.960023	4.23	.039977	38
.828716	2.32	.868440	1.93	.960277	4.22	.039723	37
.828855	2.30	.868324	1.92	.960530	4.23	.039470	36
9.828993	2.30	9.868209	1.93	9.960784	4.23	0.039216	35
.829131	2.30	.868093	1.92	.961038	4.23	.038962	34
.829269	2.30	.867978	1.93	.961292	4.22	.038708	33
.829407	2.30	.867862	1.92	.961545	4.23	.038455	32
.829545	2.30	.867747	1.93	.961799	4.22	.038201	31
9.829683	2.30	9.867631	1.93	9.962052	4.23	0.037948	30
.829821	2.30	.867515	1.93	.962306	4.23	.037694	29
.829959	2.30	.867399	1.93	.962560	4.22	.037440	28
.830097	2.28	.867283	1.93	.962813	4.23	.037187	27
.830234	2.30	.867167	1.93	.963067	4.22	.036933	26
9.830372	2.28	9.867051	1.93	9.963320	4.23	0.036680	25
.830509	2.28	.866935	1.93	.963574	4.23	.036426	24
.830646	2.30	.866819	1.93	.963828	4.22	.036172	23
.830784	2.28	.866703	1.95	.964081	4.23	.035919	22
.830921	2.28	.866586	1.93	.964335	4.22	.035665	21
9.831058	2.28	9.866470	1.95	9.964588	4.23	0.035412	20
.831195	2.28	.866353	1.93	.964842	4.22	.035158	19
.831332	2.28	.866237	1.95	.965095	4.23	.034905	18
.831469	2.28	.866120	1.93	.965349	4.22	.034651	17
.831606	2.27	.866004	1.95	.965602	4.22	.034398	16
9.831742	2.28	9.865887	1.95	9.965855	4.23	0.034145	15
.831879	2.27	.865770	1.95	.966109	4.22	.033891	14
.832015	2.28	.865653	1.95	.966362	4.23	.033638	13
.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
.832288	2.28	.865419	1.95	.966869	4.23	.033131	11
9.832425	2.27	9.865302	1.95	9.967123	4.22	0.032877	10
.832561	2.27	.865185	1.95	.967376	4.22	.032624	9
.832697	2.27	.865068	1.97	.967629	4.23	.032371	8
.832833	2.27	.864950	1.95	.967883	4.22	.032117	7
.832969	2.27	.864833	1.95	.968136	4.22	.031864	6
9.833105	2.27	9.864716	1.97	9.968389	4.23	0.031611	5
.833241	2.27	.864598	1.95	.968643	4.22	.031357	4
.833377	2.25	.864481	1.97	.968896	4.22	.031104	3
.833512	2.27	.864363	1.97	.969149	4.23	.030851	2
.833648	2.25	.864245	1.97	.969403	4.22	.030597	1
9.833783		9.864127		9.969656		0.030344	0
Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Continued*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.833783	2.27	9.864127	1.95	9.969656	4.22	0.030344	60
1	.833919	2.25	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.23	.029838	58
3	.834189	2.27	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	9.834460	2.25	9.863538	1.98	9.970922	4.22	0.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.23	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.23	.863183	1.98	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	9.835134	2.25	9.862946	1.98	9.972188	4.22	0.027812	50
11	.835269	2.23	.862827	1.97	.972441	4.23	.027559	49
12	.835403	2.25	.862709	1.98	.972695	4.22	.027305	48
13	.835538	2.23	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.25	.862471	1.97	.973201	4.22	.026799	46
15	9.835807	2.23	9.862353	1.98	9.973454	4.22	0.026546	45
16	.835941	2.23	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.98	.974466	4.23	.025534	41
20	9.836477	2.23	9.861758	2.00	9.974720	4.22	0.025280	40
21	.836611	2.23	.861638	1.98	.974973	4.22	.025027	39
22	.836745	2.22	.861519	1.98	.975226	4.22	.024774	38
23	.836878	2.23	.861400	2.00	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.98	.975732	4.22	.024268	36
25	9.837146	2.22	9.861161	2.00	9.975985	4.22	0.024015	35
26	.837279	2.22	.861041	1.98	.976238	4.22	.023762	34
27	.837412	2.23	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	9.837812	2.22	9.860562	2.00	9.977250	4.22	0.022750	30
31	.837945	2.22	.860442	2.00	.977503	4.22	.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.22	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.22	.860082	2.00	.978262	4.22	.021738	26
35	9.838477	2.22	9.859962	2.00	9.978515	4.22	0.021485	25
36	.838610	2.20	.859842	2.02	.978768	4.22	.021232	24
37	.838742	2.22	.859721	2.00	.979021	4.22	.020979	23
38	.838875	2.20	.859601	2.02	.979274	4.22	.020726	22
39	.839007	2.22	.859480	2.00	.979527	4.22	.020473	21
40	9.839140	2.20	9.859360	2.02	9.979780	4.22	0.020220	20
41	.839272	2.20	.859239	2.00	.980033	4.22	.019967	19
42	.839404	2.20	.859119	2.02	.980286	4.20	.019714	18
43	.839536	2.20	.858998	2.02	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	9.839800	2.20	9.858756	2.02	9.981044	4.22	0.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.22	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.22	.018450	13
48	.840196	2.20	.858393	2.02	.981803	4.22	.018197	12
49	.840328	2.18	.858272	2.02	.982056	4.22	.017944	11
50	9.840459	2.20	9.858151	2.03	9.982309	4.22	0.017691	10
51	.840591	2.18	.858029	2.02	.982562	4.20	.017438	9
52	.840722	2.20	.857908	2.03	.982814	4.22	.017186	8
53	.840854	2.18	.857786	2.02	.983067	4.22	.016933	7
54	.840985	2.18	.857665	2.03	.983320	4.22	.016680	6
55	9.841116	2.18	9.857543	2.02	9.983573	4.22	0.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.22	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.22	.015921	3
58	.841509	2.18	.857178	2.03	.984332	4.20	.015668	2
59	.841640	2.18	.857056	2.03	.984584	4.22	.015416	1
60	9.841771		9.856934		9.984837		0.015163	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

TABLE XII.—*Concluded*

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.841771	2.18	9.856934	2.03	9.984837	4.22	0.015163	60
1	.841902	2.18	.856812	2.03	.985090	4.22	.014910	59
2	.842033	2.17	.856690	2.03	.985343	4.22	.014657	58
3	.842163	2.18	.856568	2.03	.985596	4.20	.014404	57
4	.842294	2.17	.856446	2.05	.985848	4.22	.014152	56
5	9.842424	2.18	9.856323	2.03	9.986101	4.22	0.013899	55
6	.842555	2.17	.856201	2.05	.986354	4.22	.013646	54
7	.842685	2.17	.856078	2.03	.986607	4.22	.013393	53
8	.842815	2.18	.855956	2.05	.986860	4.20	.013140	52
9	.842946	2.17	.855833	2.03	.987112	4.22	.012888	51
10	9.843076	2.17	9.855711	2.05	9.987365	4.22	0.012635	50
11	.843206	2.17	.855588	2.05	.987618	4.22	.012382	49
12	.843336	2.17	.855465	2.05	.987871	4.20	.012129	48
13	.843466	2.15	.855342	2.05	.988123	4.22	.011877	47
14	.843595	2.17	.855219	2.05	.988376	4.22	.011624	46
15	9.843725	2.17	9.855096	2.05	9.988629	4.22	0.011371	45
16	.843855	2.15	.854973	2.05	.988882	4.20	.011118	44
17	.843984	2.17	.854850	2.05	.989134	4.22	.010866	43
18	.844114	2.15	.854727	2.07	.989387	4.22	.010613	42
19	.844243	2.15	.854603	2.05	.989640	4.22	.010360	41
20	9.844372	2.17	9.854480	2.07	9.989893	4.20	0.010107	40
21	.844502	2.15	.854356	2.05	.990145	4.22	.009855	39
22	.844631	2.15	.854233	2.07	.990398	4.22	.009602	38
23	.844760	2.15	.854109	2.05	.990651	4.20	.009349	37
24	.844889	2.15	.853986	2.07	.990903	4.22	.009097	36
25	9.845018	2.15	9.853862	2.07	9.991156	4.22	0.008844	35
26	.845147	2.15	.853738	2.07	.991409	4.22	.008591	34
27	.845276	2.15	.853614	2.07	.991662	4.20	.008338	33
28	.845405	2.13	.853490	2.07	.991914	4.22	.008086	32
29	.845533	2.15	.853366	2.07	.992167	4.22	.007833	31
30	9.845662	2.13	9.853242	2.07	9.992420	4.20	0.007580	30
31	.845790	2.15	.853118	2.07	.992672	4.22	.007328	29
32	.845919	2.13	.852994	2.08	.992925	4.22	.007075	28
33	.846047	2.13	.852869	2.07	.993178	4.22	.006822	27
34	.846175	2.15	.852745	2.08	.993431	4.20	.006569	26
35	9.846304	2.13	9.852620	2.07	9.993683	4.22	0.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.22	.006064	24
37	.846560	2.13	.852371	2.07	.994189	4.20	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.22	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.22	.005306	21
40	9.846944	2.12	9.851997	2.08	9.994947	4.20	0.005053	20
41	.847071	2.13	.851872	2.08	.995199	4.22	.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.22	.004548	18
43	.847327	2.12	.851622	2.08	.995705	4.20	.004295	17
44	.847454	2.13	.851497	2.08	.995957	4.22	.004043	16
45	9.847582	2.12	9.851372	2.10	9.996210	4.22	0.003790	15
46	.847709	2.12	.851246	2.08	.996463	4.20	.003537	14
47	.847836	2.13	.851121	2.08	.996715	4.22	.003285	13
48	.847964	2.12	.850996	2.10	.996968	4.22	.003032	12
49	.848091	2.12	.850870	2.08	.997221	4.20	.002779	11
50	9.848218	2.12	9.850745	2.10	9.997473	4.22	0.002527	10
51	.848345	2.12	.850619	2.10	.997726	4.22	.002274	9
52	.848472	2.12	.850493	2.08	.997979	4.20	.002021	8
53	.848599	2.12	.850368	2.10	.998231	4.22	.001769	7
54	.848726	2.10	.850242	2.10	.998484	4.22	.001516	6
55	9.848852	2.12	9.850116	2.10	9.998737	4.20	0.001263	5
56	.848979	2.12	.849990	2.10	.998989	4.22	.001011	4
57	.849106	2.10	.849864	2.10	.999242	4.22	.000758	3
58	.849232	2.12	.849738	2.12	.999495	4.20	.000505	2
59	.849359	2.10	.849611	2.10	.999747	4.22	.000253	1
60	9.849485		9.849485		0.000000		0.000000	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

Table XIII

	0°				0°				0°		
′	Sine	Cosine	′	′	Sine	Cosine	′	′	Sine	Cosine	′
0	.00000	1	60	21	.00611	.99998	39	41	.01193	.99993	19
1	.00029	1	59	22	.00640	.99998	38	42	.01222	.99993	18
2	.00058	1	58	23	.00669	.99998	37	43	.01251	.99992	17
3	.00087	1	57	24	.00698	.99998	36	44	.01280	.99992	16
4	.00116	1	56	25	.00727	.99997	35	45	.01309	.99991	15
5	.00145	1	55	26	.00756	.99997	34	46	.01338	.99991	14
6	.00175	1	54	27	.00785	.99997	33	47	.01367	.99991	13
7	.00204	1	53	28	.00814	.99997	32	48	.01396	.99990	12
8	.00233	1	52	29	.00844	.99996	31	49	.01425	.99990	11
9	.00262	1	51	30	.00873	.99996	30	50	.01454	.99989	10
10	.00291	1	50	31	.00902	.99996	29	51	.01483	.99989	9
11	.00320	.99999	49	32	.00931	.99996	28	52	.01513	.99989	8
12	.00349	.99999	48	33	.00960	.99995	27	53	.01542	.99988	7
13	.00378	.99999	47	34	.00989	.99995	26	54	.01571	.99988	6
14	.00407	.99999	46	35	.01018	.99995	25	55	.01600	.99987	5
15	.00436	.99999	45	36	.01047	.99995	24	56	.01629	.99987	4
16	.00465	.99999	44	37	.01076	.99994	23	57	.01658	.99986	3
17	.00495	.99999	43	38	.01105	.99994	22	58	.01687	.99986	2
18	.00524	.99999	42	39	.01134	.99994	21	59	.01716	.99985	1
19	.00553	.99998	41	40	.01164	.99993	20	60	.01745	.99985	0
20	.00582	.99998	40								
′	Cosine	Sine	′	′	Cosine	Sine	′	′	Cosine	Sine	′
	89°				89°				89°		

TABLE XIII.—*Continued*

′	1° Sine	1° Cosine	2° Sine	2° Cosine	3° Sine	3° Cosine	4° Sine	4° Cosine	′
0	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	88° Cosine	88° Sine	87° Cosine	87° Sine	86° Cosine	86° Sine	85° Cosine	85° Sine	′

TABLE XIII.—*Continued*

′	5° Sine	5° Cosine	6° Sine	6° Cosine	7° Sine	7° Cosine	8° Sine	8° Cosine	′
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		

TABLE XIII.—*Continued*

′	9°		10°		11°		12°		′
	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	
0	.15643	.98769	.17365	.98481	.19081	.98163	.20791	.97815	60
1	.15672	.98764	.17393	.98476	.19109	.98157	.20820	.97809	59
2	.15701	.98760	.17422	.98471	.19138	.98152	.20848	.97803	58
3	.15730	.98755	.17451	.98466	.19167	.98146	.20877	.97797	57
4	.15758	.98751	.17479	.98461	.19195	.98140	.20905	.97791	56
5	.15787	.98746	.17508	.98455	.19224	.98135	.20933	.97784	55
6	.15816	.98741	.17537	.98450	.19252	.98129	.20962	.97778	54
7	.15845	.98737	.17565	.98445	.19281	.98124	.20990	.97772	53
8	.15873	.98732	.17594	.98440	.19309	.98118	.21019	.97766	52
9	.15902	.98728	.17623	.98435	.19338	.98112	.21047	.97760	51
10	.15931	.98723	.17651	.98430	.19366	.98107	.21076	.97754	50
11	.15959	.98718	.17680	.98425	.19395	.98101	.21104	.97748	49
12	.15988	.98714	.17708	.98420	.19423	.98096	.21132	.97742	48
13	.16017	.98709	.17737	.98414	.19452	.98090	.21161	.97735	47
14	.16046	.98704	.17766	.98409	.19481	.98084	.21189	.97729	46
15	.16074	.98700	.17794	.98404	.19509	.98079	.21218	.97723	45
16	.16103	.98695	.17823	.98399	.19538	.98073	.21246	.97717	44
17	.16132	.98690	.17852	.98394	.19566	.98067	.21275	.97711	43
18	.16160	.98689	.17880	.98389	.19595	.98061	.21303	.97705	42
19	.16189	.98681	.17909	.98383	.19623	.98056	.21331	.97698	41
20	.16218	.98676	.17937	.98378	.19652	.98050	.21360	.97692	40
21	.16246	.98671	.17966	.98373	.19680	.98044	.21388	.97686	39
22	.16275	.98667	.17995	.98368	.19709	.98039	.21417	.97680	38
23	.16304	.98662	.18023	.98362	.19737	.98033	.21445	.97673	37
24	.16333	.98657	.18052	.98357	.19766	.98027	.21474	.97667	36
25	.16361	.98652	.18081	.98352	.19794	.98021	.21502	.97661	35
26	.16390	.98648	.18109	.98347	.19823	.98016	.21530	.97655	34
27	.16419	.98643	.18138	.98341	.19851	.98010	.21559	.97648	33
28	.16447	.98638	.18166	.98336	.19880	.98004	.21587	.97642	32
29	.16476	.98633	.18195	.98331	.19908	.97987	.21616	.97636	31
30	.16505	.98629	.18224	.98325	.19937	.97992	.21644	.97630	30
31	.16533	.98624	.18252	.98320	.19965	.97987	.21672	.97623	29
32	.16562	.98619	.18281	.98315	.19994	.97981	.21701	.97617	28
33	.16591	.98614	.18309	.98310	.20022	.97975	.21729	.97611	27
34	.16620	.98609	.18338	.98304	.20031	.97969	.21758	.97604	26
35	.16648	.98604	.18367	.98299	.20079	.97963	.21786	.97598	25
36	.16677	.98600	.18395	.98294	.20108	.97958	.21814	.97592	24
37	.16706	.98595	.18424	.98288	.20136	.97952	.21843	.97585	23
38	.16734	.98590	.18452	.98283	.20165	.97946	.21871	.97579	22
39	.16763	.98585	.18481	.98277	.20193	.97940	.21899	.97573	21
40	.16792	.98580	.18509	.98272	.20222	.97934	.21928	.97566	20
41	.16820	.98575	.18538	.98267	.20250	.97928	.21956	.97560	19
42	.16849	.98570	.18567	.98261	.20279	.97922	.21985	.97553	18
43	.16878	.98565	.18595	.98256	.20307	.97916	.22013	.97547	17
44	.16906	.98561	.18624	.98250	.20336	.97910	.22041	.97541	16
45	.16935	.98556	.18652	.98245	.20364	.97905	.22070	.97534	15
46	.16964	.98551	.18681	.98240	.20393	.97899	.22098	.97528	14
47	.16992	.98546	.18710	.98234	.20421	.97893	.22126	.97521	13
48	.17021	.98541	.18738	.98229	.20450	.97887	.22155	.97515	12
49	.17050	.98536	.18767	.98223	.20478	.97881	.22183	.97508	11
50	.17078	.98531	.18795	.98218	.20507	.97875	.22212	.97502	10
51	.17107	.98526	.18824	.98212	.20535	.97869	.22240	.97496	9
52	.17136	.98521	.18852	.98207	.20563	.97863	.22268	.97489	8
53	.17164	.98516	.18881	.98201	.20592	.97857	.22297	.97483	7
54	.17193	.98511	.18910	.98196	.20620	.97851	.22325	.97476	6
55	.17222	.98506	.18938	.98190	.20649	.97845	.22353	.97470	5
56	.17250	.98501	.18967	.98185	.20677	.97839	.22382	.97463	4
57	.17279	.98496	.18995	.98179	.20706	.97833	.22410	.97457	3
58	.17308	.98491	.19024	.98174	.20734	.97827	.22438	.97450	2
59	.17336	.98486	.19052	.98168	.20763	.97821	.22467	.97444	1
60	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	0
′	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	′
	80°		79°		78°		77°		

TABLE XIII.—*Continued*

′	13°		14°		15°		16°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.22495	.97437	.24192	.97030	.25882	.96593	.27564	.96126	60
1	.22523	.97430	.24220	.97023	.25910	.96585	.27592	.96118	59
2	.22552	.97424	.24249	.97015	.25938	.96578	.27620	.96110	58
3	.22580	.97417	.24277	.97008	.25966	.96570	.27648	.96102	57
4	.22608	.97411	.24305	.97001	.25994	.96562	.27676	.96094	56
5	.22637	.97404	.24333	.96994	.26022	.96555	.27704	.96086	55
6	.22665	.97398	.24362	.96987	.26050	.96547	.27731	.96078	54
7	.22693	.97391	.24390	.96980	.26079	.96540	.27759	.96070	53
8	.22722	.97384	.24418	.96973	.26107	.96532	.27787	.96062	52
9	.22750	.97378	.24446	.96966	.26135	.96524	.27815	.96054	51
10	.22778	.97371	.24474	.96959	.26163	.96517	.27843	.96046	50
11	.22807	.97365	.24503	.96952	.26191	.96509	.27871	.96037	49
12	.22835	.97358	.24531	.96945	.26219	.96502	.27899	.96029	48
13	.22863	.97351	.24559	.96937	.26247	.96494	.27927	.96021	47
14	.22892	.97345	.24587	.96930	.26275	.96486	.27955	.96013	46
15	.22920	.97338	.24615	.96923	.26303	.96479	.27983	.96005	45
16	.22948	.97331	.24644	.96916	.26331	.96471	.28011	.95997	44
17	.22977	.97325	.24672	.96909	.26359	.96463	.28039	.95989	43
18	.23005	.97318	.24700	.96902	.26387	.96456	.28067	.95981	42
19	.23033	.97311	.24728	.96894	.26415	.96448	.28095	.95972	41
20	.23062	.97304	.24756	.96887	.26443	.96440	.28123	.95964	40
21	.23090	.97298	.24784	.96880	.26471	.96433	.28150	.95956	39
22	.23118	.97291	.24813	.96873	.26500	.96425	.28178	.95948	38
23	.23146	.97284	.24841	.96866	.26528	.96417	.28206	.95940	37
24	.23175	.97278	.24869	.96858	.26556	.96410	.28234	.95931	36
25	.23203	.97271	.24897	.96851	.26584	.96402	.28262	.95923	35
26	.23231	.97264	.24925	.96844	.26612	.96394	.28290	.95915	34
27	.23260	.97257	.24954	.96837	.26640	.96386	.28318	.95907	33
28	.23288	.97251	.24982	.96829	.26668	.96379	.28346	.95898	32
29	.23316	.97244	.25010	.96822	.26696	.96371	.28374	.95890	31
30	.23345	.97237	.25038	.96815	.26724	.96363	.28402	.95882	30
31	.23373	.97230	.25066	.96807	.26752	.96355	.28429	.95874	29
32	.23401	.97223	.25094	.96800	.26780	.96347	.28457	.95865	28
33	.23429	.97217	.25122	.96793	.26808	.96340	.28485	.95857	27
34	.23458	.97210	.25151	.96786	.26836	.96332	.28513	.95849	26
35	.23486	.97203	.25179	.96778	.26864	.96324	.28541	.95841	25
36	.23514	.97196	.25207	.96771	.26892	.96316	.28569	.95832	24
37	.23542	.97189	.25235	.96764	.26920	.96308	.28597	.95824	23
38	.23571	.97182	.25263	.96756	.26948	.96301	.28625	.95816	22
39	.23599	.97176	.25291	.96749	.26976	.96293	.28652	.95807	21
40	.23627	.97169	.25320	.96742	.27004	.96285	.28680	.95799	20
41	.23656	.97162	.25348	.96734	.27032	.96277	.28708	.95791	19
42	.23684	.97155	.25376	.96727	.27060	.96269	.28736	.95782	18
43	.23712	.97148	.25404	.96719	.27088	.96261	.28764	.95774	17
44	.23740	.97141	.25432	.96712	.27116	.96253	.28792	.95766	16
45	.23769	.97134	.25460	.96705	.27144	.96246	.28820	.95757	15
46	.23797	.97127	.25488	.96697	.27172	.96238	.28847	.95749	14
47	.23825	.97120	.25516	.96690	.27200	.96230	.28875	.95740	13
48	.23853	.97113	.25545	.96682	.27228	.96222	.28903	.95732	12
49	.23882	.97106	.25573	.96675	.27256	.96214	.28931	.95724	11
50	.23910	.97100	.25601	.96667	.27284	.96206	.28959	.95715	10
51	.23938	.97093	.25629	.96660	.27312	.96198	.28987	.95707	9
52	.23966	.97086	.25657	.96653	.27340	.96190	.29015	.95698	8
53	.23995	.97079	.25685	.96645	.27368	.96182	.29042	.95690	7
54	.24023	.97072	.25713	.96638	.27396	.96174	.29070	.95681	6
55	.24051	.97065	.25741	.96630	.27424	.96166	.29098	.95673	5
56	.24079	.97058	.25769	.96623	.27452	.96158	.29126	.95664	4
57	.24108	.97051	.25798	.96615	.27480	.96150	.29154	.95656	3
58	.24136	.97044	.25826	.96608	.27508	.96142	.29182	.95647	2
59	.24164	.97037	.25854	.96600	.27536	.96134	.29209	.95639	1
60	.24192	.97030	.25882	.96593	.27564	.96126	.29237	.95630	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	76°		75°		74°		73°		

TABLE XIII.—*Continued*

′	17° Sine	17° Cosine	18° Sine	18° Cosine	19° Sine	19° Cosine	20° Sine	20° Cosine	′
0	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	60
1	.29265	.95622	.30929	.95097	.32584	.94542	.34229	.93959	59
2	.29293	.95613	.30957	.95088	.32612	.94533	.34257	.93949	58
3	.29321	.95605	.30985	.95079	.32639	.94523	.34284	.93939	57
4	.29348	.95596	.31012	.95070	.32667	.94514	.34311	.93929	56
5	.29376	.95588	.31040	.95061	.32694	.94504	.34339	.93919	55
6	.29404	.95579	.31068	.95052	.32722	.94495	.34366	.93909	54
7	.29432	.95571	.31095	.95043	.32749	.94485	.34393	.93899	53
8	.29460	.95562	.31123	.95033	.32777	.94476	.34421	.93889	52
9	.29487	.95554	.31151	.95024	.32804	.94466	.34448	.93879	51
10	.29515	.95545	.31178	.95015	.32832	.94457	.34475	.93869	50
11	.29543	.95536	.31206	.95006	.32859	.94447	.34503	.93859	49
12	.29571	.95528	.31233	.94997	.32887	.94438	.34530	.93849	48
13	.29599	.95519	.31261	.94988	.32914	.94428	.34557	.93839	47
14	.29626	.95511	.31289	.94979	.32942	.94418	.34584	.93829	46
15	.29654	.95502	.31316	.94970	.32969	.94409	.34612	.93819	45
16	.29682	.95493	.31344	.94961	.32997	.94399	.34639	.93809	44
17	.29710	.95485	.31372	.94952	.33024	.94390	.34666	.93799	43
18	.29737	.95476	.31399	.94943	.33051	.94380	.34694	.93789	42
19	.29765	.95467	.31427	.94933	.33079	.94370	.34721	.93779	41
20	.29793	.95459	.31454	.94924	.33106	.94361	.34748	.93769	40
21	.29821	.95450	.31482	.94915	.33134	.94351	.34775	.93759	39
22	.29849	.95441	.31510	.94906	.33161	.94342	.34803	.93748	38
23	.29876	.95433	.31537	.94897	.33189	.94332	.34830	.93738	37
24	.29904	.95424	.31565	.94888	.33216	.94322	.34857	.93728	36
25	.29932	.95415	.31593	.94878	.33244	.94313	.34884	.93718	35
26	.29960	.95407	.31620	.94869	.33271	.94303	.34912	.93708	34
27	.29987	.95398	.31648	.94860	.33298	.94293	.34939	.93698	33
28	.30015	.95389	.31675	.94851	.33326	.94284	.34966	.93688	32
29	.30043	.95380	.31703	.94842	.33353	.94274	.34993	.93677	31
30	.30071	.95372	.31730	.94832	.33381	.94264	.35021	.93667	30
31	.30098	.95363	.31758	.94823	.33408	.94254	.35048	.93657	29
32	.30126	.95354	.31786	.94814	.33436	.94245	.35075	.93647	28
33	.30154	.95345	.31813	.94805	.33463	.94235	.35102	.93637	27
34	.30182	.95337	.31841	.94795	.33490	.94225	.35130	.93626	26
35	.30209	.95328	.31868	.94786	.33518	.94215	.35157	.93616	25
36	.30237	.95319	.31896	.94777	.33545	.94206	.35184	.93606	24
37	.30265	.95310	.31923	.94768	.33573	.94196	.35211	.93596	23
38	.30292	.95301	.31951	.94758	.33600	.94186	.35239	.93585	22
39	.30320	.95293	.31979	.94749	.33627	.94176	.35266	.93575	21
40	.30348	.95284	.32006	.94740	.33655	.94167	.35293	.93565	20
41	.30376	.95275	.32034	.94730	.33682	.94157	.35320	.93555	19
42	.30403	.95266	.32061	.94721	.33710	.94147	.35347	.93544	18
43	.30431	.95257	.32089	.94712	.33737	.94137	.35375	.93534	17
44	.30459	.95248	.32116	.94702	.33764	.94127	.35402	.93524	16
45	.30486	.95240	.32144	.94693	.33792	.94118	.35429	.93514	15
46	.30514	.95231	.32171	.94684	.33819	.94108	.35456	.93503	14
47	.30542	.95222	.32199	.94674	.33846	.94098	.35484	.93493	13
48	.30570	.95213	.32227	.94665	.33874	.94088	.35511	.93483	12
49	.30597	.95204	.32254	.94656	.33901	.94078	.35538	.93472	11
50	.30625	.95195	.32282	.94646	.33929	.94068	.35565	.93462	10
51	.30653	.95186	.32309	.94637	.33956	.94058	.35592	.93452	9
52	.30680	.95177	.32337	.94627	.33983	.94049	.35619	.93441	8
53	.30708	.95168	.32364	.94618	.34011	.94039	.35647	.93431	7
54	.30736	.95159	.32392	.94609	.34038	.94029	.35674	.93420	6
55	.30763	.95150	.32419	.94599	.34065	.94019	.35701	.93410	5
56	.30791	.95142	.32447	.94590	.34093	.94009	.35728	.93400	4
57	.30819	.95133	.32474	.94580	.34120	.93999	.35755	.93389	3
58	.30846	.95124	.32502	.94571	.34147	.93989	.35782	.93379	2
59	.30874	.95115	.32529	.94561	.34175	.93979	.35810	.93368	1
60	.30902	.95106	.32557	.94552	.34202	.93969	.35837	.93358	0
′	Cosine 72°	Sine 72°	Cosine 71°	Sine 71°	Cosine 70°	Sine 70°	Cosine 69°	Sine 69°	′

TABLE XIII.—*Continued*

′	21°		22°		23°		24°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60.	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	68°		67°		66°		65°		

TABLE XIII.—*Continued*

′	25° Sine	25° Cosine	26° Sine	26° Cosine	27° Sine	27° Cosine	28° Sine	28° Cosine	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	9
52	.43628	.89981	45192	.89206	.46742	.88404	.48277	.87575	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		

TABLE XIII.—*Continued*

′	29°		30°		31°		32°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.48481	.87462	.50000	.86603	.51504	.85717	.52992	.84805	60
1	.48506	.87448	.50025	.86588	.51529	.85702	.53017	.84789	59
2	.48532	.87434	.50050	.86573	.51554	.85687	.53041	.84774	58
3	.48557	.87420	.50076	.86559	.51579	.85672	.53066	.84759	57
4	.48583	.87406	.50101	.86544	.51604	.85657	.53091	.84743	56
5	.48608	.87391	.50126	.86530	.51628	.85642	.53115	.84728	55
6	.48634	.87377	.50151	.86515	.51653	.85627	.53140	.84712	54
7	.48659	.87363	.50176	.86501	.51678	.85612	.53164	.84697	53
8	.48684	.87349	.50201	.86486	.51703	.85597	.53189	.84681	52
9	.48710	.87335	.50227	.86471	.51728	.85582	.53214	.84666	51
10	.48735	.87321	.50252	.86457	.51753	.85567	.53238	.84650	50
11	.48761	.87306	.50277	.86442	.51778	.85551	.53263	.84635	49
12	.48786	.87292	.50302	.86427	.51803	.85536	.53288	.84619	48
13	.48811	.87278	.50327	.86413	.51828	.85521	.53312	.84604	47
14	.48837	.87264	.50352	.86398	.51852	.85506	.53337	.84588	46
15	.48862	.87250	.50377	.86384	.51877	.85491	.53361	.84573	45
16	.48888	.87235	.50403	.86369	.51902	.85476	.53386	.84557	44
17	.48913	.87221	.50428	.86354	.51927	.85461	.53411	.84542	43
18	.48938	.87207	.50453	.86340	.51952	.85446	.53435	.84526	42
19	.48964	.87193	.50478	.86325	.51977	.85431	.53460	.84511	41
20	.48989	.87178	.50503	.86310	.52002	.85416	.53484	.84495	40
21	.49014	.87164	.50528	.86295	.52026	.85401	.53509	.84480	39
22	.49040	.87150	.50553	.86281	.52051	.85385	.53534	.84464	38
23	.49065	.87136	.50578	.86266	.52076	.85370	.53558	.84448	37
24	.49090	.87121	.50603	.86251	.52101	.85355	.53583	.84433	36
25	.49116	.87107	.50628	.86237	.52126	.85340	.53607	.84417	35
26	.49141	.87093	.50654	.86222	.52151	.85325	.53632	.84402	34
27	.49166	.87079	.50679	.86207	.52175	.85310	.53656	.84386	33
28	.49192	.87064	.50704	.86192	.52200	.85294	.53681	.84370	32
29	.49217	.87050	.50729	.86178	.52225	.85279	.53705	.84355	31
30	.49242	.87036	.50754	.86163	.52250	.85264	.53730	.84339	30
31	.49268	.87021	.50779	.86148	.52275	.85249	.53754	.84324	29
32	.49293	.87007	.50804	.86133	.52299	.85234	.53779	.84308	28
33	.49318	.86993	.50829	.86119	.52324	.85218	.53804	.84292	27
34	.49344	.86978	.50854	.86104	.52349	.85203	.53828	.84277	26
35	.49369	.86964	.50879	.86089	.52374	.85188	.53853	.84261	25
36	.49394	.86949	.50904	.86074	.52399	.85173	.53877	.84245	24
37	.49419	.86935	.50929	.86059	.52423	.85157	.53902	.84230	23
38	.49445	.86921	.50954	.86045	.52448	.85142	.53926	.84214	22
39	.49470	.86906	.50979	.86030	.52473	.85127	.53951	.84198	21
40	.49495	.86892	.51004	.86015	.52498	.85112	.53975	.84182	20
41	.49521	.86878	.51029	.86000	.52522	.85096	.54000	.84167	19
42	.49546	.86863	.51054	.85985	.52547	.85081	.54024	.84151	18
43	.49571	.86849	.51079	.85970	.52572	.85066	.54049	.84135	17
44	.49596	.86834	.51104	.85956	.52597	.85051	.54073	.84120	16
45	.49622	.86820	.51129	.85941	.52621	.85035	.54097	.84104	15
46	.49647	.86805	.51154	.85926	.52646	.85020	.54122	.84088	14
47	.49672	.86791	.51179	.85911	.52671	.85005	.54146	.84072	13
48	.49697	.86777	.51204	.85896	.52696	.84989	.54171	.84057	12
49	.49723	.86762	.51229	.85881	.52720	.84974	.54195	.84041	11
50	.49748	.86748	.51254	.85866	.52745	.84959	.54220	.84025	10
51	.49773	.86733	.51279	.85851	.52770	.84943	.54244	.84009	9
52	.49798	.86719	.51304	.85836	.52794	.84928	.54269	.83994	8
53	.49824	.86704	.51329	.85821	.52819	.84913	.54293	.83978	7
54	.49849	.86690	.51354	.85806	.52844	.84897	.54317	.83962	6
55	.49874	.86675	.51379	.85792	.52869	.84882	.54342	.83946	5
56	.49899	.86661	.51404	.85777	.52893	.84866	.54366	.83930	4
57	.49924	.86646	.51429	.85762	.52918	.84851	.54391	.83915	3
58	.49950	.86632	.51454	.85747	.52943	.84836	.54415	.83899	2
59	.49975	.86617	.51479	.85732	.52967	.84820	.54440	.83883	1
60	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	60°		59°		58°		57°		

TABLE XIII.—*Continued*

′	33° Sine	33° Cosine	34° Sine	34° Cosine	35° Sine	35° Cosine	36° Sine	36° Cosine	′
0	.54464	.83867	.55919	.82904	.57358	.81915	.58779	.80902	60
1	.54488	.83851	.55943	.82887	.57381	.81899	.58802	.80885	59
2	.54513	.83835	.55968	.82871	.57405	.81882	.58826	.80867	58
3	.54537	.83819	.55992	.82855	.57429	.81865	.58849	.80850	57
4	.54561	.83804	.56016	.82839	.57453	.81848	.58873	.80833	56
5	.54586	.83788	.56040	.82822	.57477	.81832	.58896	.80816	55
6	.54610	.83772	.56064	.82806	.57501	.81815	.58920	.80799	54
7	.54635	.83756	.56088	.82790	.57524	.81798	.58943	.80782	53
8	.54659	.83740	.56112	.82773	.57548	.81782	.58967	.80765	52
9	.54683	.83724	.56136	.82757	.57572	.81765	.58990	.80748	51
10	.54708	.83708	.56160	.82741	.57596	.81748	.59014	.80730	50
11	.54732	83692	.56184	.82724	.57619	.81731	.59037	.80713	49
12	.54756	.83676	.56208	.82708	.57643	.81714	.59061	.80696	48
13	.54781	.83660	.56232	.82692	.57667	.81698	.59084	.80679	47
14	.54805	.83645	.56256	.82675	.57691	.81681	.59108	.80662	46
15	.54829	.83629	.56280	.82659	.57715	.81664	.59131	.80644	45
16	.54854	.83613	.56305	.82643	.57738	.81647	.59154	.80627	44
17	.54878	.83597	.56329	.82626	.57762	.81631	.59178	.80610	43
18	.54902	.83581	.56353	.82610	.57786	.81614	.59201	.80593	42
19	.54927	.83565	.56377	.82593	.57810	.81597	.59225	.80576	41
20	.54951	.83549	.56401	.82577	.57833	.81580	.59248	.80558	40
21	.54975	.83533	.56425	.82561	.57857	.81563	.59272	.80541	39
22	.54999	.83517	.56449	.82544	.57881	.81546	.59295	.80524	38
23	.55024	.83501	.56473	.82528	.57904	.81530	.59318	.80507	37
24	.55048	.83485	.56497	.82511	.57928	.81513	.59342	.80489	36
25	.55072	.83469	.56521	.82495	.57952	.81496	.59365	.80472	35
26	.55097	.83453	.56545	.82478	.57976	.81479	.59389	.80455	34
27	.55121	.83437	.56569	.82462	.57999	.81462	.59412	.80438	33
28	.55145	.83421	.56593	.82446	.58023	.81445	.59436	.80420	32
29	.55169	.83405	.56617	.82429	.58047	.81428	.59459	.80403	31
30	.55194	.83389	.56641	.82413	.58070	.81412	.59482	.80386	30
31	.55218	.83373	.56665	.82396	.58094	.81395	.59506	.80368	29
32	.55242	.83356	.56689	.82340	.58118	.81378	.59529	.80351	28
33	.55266	.83340	.56713	.82363	.58141	.81361	.59552	.80334	27
34	.55291	.83324	.56736	.82347	.58165	.81344	.59576	.80316	26
35	.55315	.83308	.56760	.82330	.58189	.81327	.59599	.80299	25
36	.55339	.83292	.56784	.82314	.58212	.81310	.59622	.80282	24
37	.55363	.83276	.56808	.82297	.58236	.81293	.59646	.80264	23
38	.55388	.83260	.56832	.82281	.58260	.81276	.59669	.80247	22
39	.55412	.83244	.56856	.82264	.58283	.81259	.59693	.80230	21
40	.55436	.83228	.56880	.82248	.58307	.81242	.59716	.80212	20
41	.55460	.83212	.56904	.82231	.58330	.81225	.59739	.80195	19
42	.55484	.83195	.56928	.82214	.58354	.81208	.59763	.80178	18
43	.55509	.83179	.56952	.82198	.58378	.81191	.59786	.80160	17
44	.55533	.83163	.56976	.82181	.58401	.81174	.59809	.80143	16
45	.55557	.83147	.57000	.82165	.58425	.81157	.59832	.80125	15
46	.55581	.83131	.57024	.82148	.58449	.81140	.59856	.80108	14
47	.55605	.83115	.57047	.82132	.58472	.81123	.59879	.80091	13
48	.55630	.83098	.57071	.82115	.58496	.81106	.59902	.80073	12
49	.55654	.83082	.57095	.82098	.58519	.81089	.59926	.80056	11
50	.55678	.83066	.57119	.82082	.58543	.81072	.59949	.80038	10
51	.55702	.83050	.57143	.82065	.58567	.81055	.59972	.80021	9
52	.55726	.83034	.57167	.82048	.58590	.81038	.59995	.80003	8
53	.55750	.83017	.57191	.82032	.58614	.81021	.60019	.79986	7
54	.55775	.83001	.57215	.82015	.58637	.81004	.60042	.79968	6
55	.55799	.82985	.57238	.81999	.58661	.80987	.60065	.79951	5
56	.55823	.82969	.57262	.81982	.58684	.80970	.60089	.79934	4
57	.55847	.82953	.57286	.81965	.58708	.80953	.60112	.79916	3
58	.55871	.82936	.57310	.81949	.58731	.80936	.60135	.79899	2
59	.55895	.82920	.57334	.81932	.58755	.80919	.60158	.79881	1
60	.55919	.82904	.57358	.81915	.58779	.80902	.60182	.79864	0
′	Cosine 56°	Sine 56°	Cosine 55°	Sine 55°	Cosine 54°	Sine 54°	Cosine 53°	Sine 53°	′

TABLE XIII.—*Continued*

′	37° Sine	37° Cosine	38° Sine	38° Cosine	39° Sine	39° Cosine	40° Sine	40° Cosine	′
0	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	60
1	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76586	59
2	.60228	.79829	.61612	.78765	.62977	.77678	.64323	.76567	58
3	.60251	.79811	.61635	.78747	.63000	.77660	.64346	.76548	57
4	.60274	.79793	.61658	.78729	.63022	.77641	.64368	.76530	56
5	.60298	.79776	.61681	.78711	.63045	.77623	.64390	.76511	55
6	.60321	.79758	.61704	.78694	.63068	.77605	.64412	.76492	54
7	.60344	.79741	.61726	.78676	.63090	.77586	.64435	.76473	53
8	.60367	.79723	.61749	.78658	.63113	.77568	.64457	.76455	52
9	.60390	.79706	.61772	.78640	.63135	.77550	.64479	.76436	51
10	.60414	.79688	.61795	.78622	.63158	.77531	.64501	.76417	50
11	.60437	.79671	.61818	.78604	.63180	.77513	.64524	.76398	49
12	.60460	.79653	.61841	.78586	.63203	.77494	.64546	.76380	48
13	.60483	.79635	.61864	.78568	.63225	.77476	.64568	.76361	47
14	.60506	.79618	.61887	.78550	.63248	.77458	.64590	.76342	46
15	.60529	.79600	.61909	.78532	.63271	.77439	.64612	.76323	45
16	.60553	.79583	.61932	.78514	.63293	.77421	.64635	.76304	44
17	.60576	.79565	.61955	.78496	.63316	.77402	.64657	.76286	43
18	.60599	.79547	.61978	.78478	.63338	.77384	.64679	.76267	42
19	.60622	.79530	.62001	.78460	.63361	.77366	.64701	.76248	41
20	.60645	.79512	.62024	.78442	.63383	.77347	.64723	.76229	40
21	.60668	.79494	.62046	.78424	.63406	.77329	.64746	.76210	39
22	.60691	.79477	.62069	.78405	.63428	.77310	.64768	.76192	38
23	.60714	.79459	.62092	.78387	.63451	.77292	.64790	.76173	37
24	.60738	.79441	.62115	.78369	.63473	.77273	.64812	.76154	36
25	.60761	.79424	.62138	.78351	.63496	.77255	.64834	.76135	35
26	.60784	.79406	.62160	.78333	.63518	.77236	.64856	.76116	34
27	.60807	.79388	.62183	.78315	.63540	.77218	.64878	.76097	33
28	.60830	.79371	.62206	.78297	.63563	.77199	.64901	.76078	32
29	.60853	.79353	.62229	.78279	.63585	.77181	.64923	.76059	31
30	.60876	.79335	.62251	.78261	.63608	.77162	.64945	.76041	30
31	.60899	.79318	.62274	.78243	.63630	.77144	.64967	.76022	29
32	.60922	.79300	.62297	.78225	.63653	.77125	.64989	.76003	28
33	.60945	.79282	.62320	.78206	.63675	.77107	.65011	.75984	27
34	.60968	.79264	.62342	.78188	.63698	.77088	.65033	.75965	26
35	.60991	.79247	.62365	.78170	.63720	.77070	.65055	.75946	25
36	.61015	.79229	.62388	.78152	.63742	.77051	.65077	.75927	24
37	.61038	.79211	.62411	.78134	.63765	.77033	.65100	.75908	23
38	.61061	.79193	.62433	.78116	.63787	.77014	.65122	.75889	22
39	.61084	.79176	.62456	.78098	.63810	.76996	.65144	.75870	21
40	.61107	.79158	.62479	.78079	.63832	.76977	.65166	.75851	20
41	.61130	.79140	.62502	.78061	.63854	.76959	.65188	.75832	19
42	.61153	.79122	.62524	.78043	.63877	.76940	.65210	.75813	18
43	.61176	.79105	.62547	.78025	.63899	.76921	.65232	.75794	17
44	.61199	.79087	.62570	.78007	.63922	.76903	.65254	.75775	16
45	.61222	.79069	.62592	.77988	.63944	.76884	.65276	.75756	15
46	.61245	.79051	.62615	.77970	.63966	.76866	.65298	.75738	14
47	.61268	.79033	.62638	.77952	.63989	.76847	.65320	.75719	13
48	.61291	.79016	.62660	.77934	.64011	.76828	.65342	.75700	12
49	.61314	.78998	.62683	.77916	.64033	.76810	.65364	.75680	11
50	.61337	.78980	.62706	.77897	.64056	.76791	.65386	.75661	10
51	.61360	.78962	.62728	.77879	.64078	.76772	.65408	.75642	9
52	.61383	.78944	.62751	.77861	.64100	.76754	.65430	.75623	8
53	.61406	.78926	.62774	.77843	.64123	.76735	.65452	.75604	7
54	.61429	.78908	.62796	.77824	.64145	.76717	.65474	.75585	6
55	.61451	.78891	.62819	.77806	.64167	.76698	.65496	.75566	5
56	.61474	.78873	.62842	.77788	.64190	.76679	.65518	.75547	4
57	.61497	.78855	.62864	.77769	.64212	.76661	.65540	.75528	3
58	.61520	.78837	.62887	.77751	.64234	.76642	.65562	.75509	2
59	.61543	.78819	.62909	.77733	.64256	.76623	.65584	.75490	1
60	.61566	.78801	.62932	.77715	.64279	.76604	.65606	.75471	0
′	Cosine 52°	Sine 52°	Cosine 51°	Sine 51°	Cosine 50°	Sine 50°	Cosine 49°	Sine 49°	′

TABLE XIII.—*Concluded*

′	41°		42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	48°		47°		46°		45°		

TABLE XIV

′	0° Tan.	0° Co-tan.	1° Tan.	1° Co-tan.	2° Tan.	2° Co-tan.	3° Tan.	3° Co-tan.	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.750	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.870	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.920	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00814	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04832	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	89° Co-tan.	89° Tan.	88° Co-tan.	88° Tan.	87° Co-tan.	87° Tan.	86° Co-tan.	86° Tan.	′

TABLE XIV.—*Continued*

′	4°		5°		6°		7°		′
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33154	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	85°		84°		83°		82°		

TABLE XIV.—*Continued*

′	8° Tan.	8° Co-tan.	9° Tan.	9° Co-tan.	10° Tan.	10° Co-tan.	11° Tan.	11° Co-tan.	′
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18383	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91235	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Co-tan. 81°	Tan. 81°	Co-tan. 80°	Tan. 80°	Co-tan. 79°	Tan. 79°	Co-tan. 78°	Tan. 78°	′

TABLE XIV.—*Continued*

′	12° Tan.	12° Co-tan.	13° Tan.	13° Co-tan.	14° Tan.	14° Co-tan.	15° Tan.	15° Co-tan.	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27920	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03075	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Co-tan. 77°	Tan. 77°	Co-tan. 76°	Tan. 76°	Co-tan. 75°	Tan. 75°	Co-tan. 74°	Tan. 74°	′

TABLE XIV.—*Continued*

′	16° Tan.	16° Co-tan.	17° Tan.	17° Co-tan.	18° Tan.	18° Co-tan.	19° Tan.	19° Co-tan.	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35019	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33160	3.01489	.35117	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94590	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Co-tan. 73°	Tan. 73°	Co-tan. 72°	Tan. 72°	Co-tan. 71°	Tan. 71°	Co-tan. 70°	Tan. 70°	′

TABLE XIV.—*Continued*

′	20°		21°		22°		23°		′
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37124	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38862	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	69°		68°		67°		66°		

TABLE XIV.—*Continued*

′	24°		25°		26°		27°		
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45537	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97680	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52984	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	65°		64°		63°		62°		

TABLE XIV.—*Continued*

	28°		29°		30°		31°		
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1. 6209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65534	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56500	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58904	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76630	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1·76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	
	61°		60°		59°		58°		

Table XIV.—*Continued*

′	32° Tan.	32° Co-tan.	33° Tan.	33° Co-tan.	34° Tan.	34° Co-tan.	35° Tan.	35° Co-tan.	′
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65023	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72166	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Co-tan. 57°	Tan. 57°	Co-tan 56°	Tan. 56°	Co-tan. 55°	Tan. 55°	Co-tan. 54°	Tan. 54°	′

TABLE XIV.—*Continued*

	36°		37°		38°		39°		
′	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	′
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36133	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN	′
	53°		52°		51°		50°		

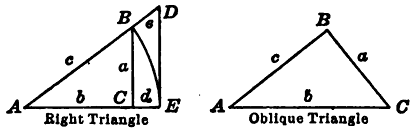
TABLE XIV.—*Continued*

′	40° Tan.	40° Co-tan.	41° Tan.	41° Co-tan.	42° Tan.	42° Co-tan.	43° Tan.	43° Co-tan.	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85307	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	49°		48°		47°		46°		

TABLE XIV.—*Concluded*

′	44° Tan.	44° Co-tan.	′	′	44° Tan.	44° Co-tan.	′	′	44° Tan.	44° Co-tan.	′
0	.96569	1.03553	60	21	.97756	1.02295	39	41	.98901	1.01112	19
1	.96625	1.03493	59	22	.97813	1.02236	38	42	.98958	1.01053	18
2	.96681	1.03433	58	23	.97870	1.02176	37	43	.99016	1.00994	17
3	.96738	1.03372	57	24	.97927	1.02117	36	44	.99073	1.00935	16
4	.96794	1.03312	56	25	.97984	1.02057	35	45	.99131	1.00876	15
5	.96850	1.03252	55	26	.98041	1.01998	34	46	.99189	1.00818	14
6	.96907	1.03192	54	27	.98098	1.01939	33	47	.99247	1.00759	13
7	.96963	1.03132	53	28	.98155	1.01879	32	48	.99304	1.00701	12
8	.97020	1.03072	52	29	.98213	1.01820	31	49	.99362	1.00642	11
9	.97076	1.03012	51	30	.98270	1.01761	30	50	.99420	1.00583	10
10	.97133	1.02952	50	31	.98327	1.01702	29	51	.99478	1.00525	9
11	.97189	1.02892	49	32	.98384	1.01642	28	52	.99536	1.00467	8
12	.97246	1.02832	48	33	.98441	1.01583	27	53	.99594	1.00408	7
13	.97302	1.02772	47	34	.98499	1.01524	26	54	.99652	1.00350	6
14	.97359	1.02713	46	35	.98556	1.01465	25	55	.99710	1.00291	5
15	.97416	1.02653	45	36	.98613	1.01406	24	56	.99768	1.00233	4
16	.97472	1.02593	44	37	.98671	1.01347	23	57	.99826	1.00175	3
17	.97529	1.02533	43	38	.98728	1.01288	22	58	.99884	1.00116	2
18	.97586	1.02474	42	39	.98786	1.01229	21	59	.99942	1.00058	1
19	.97643	1.02414	41	40	.98843	1.01170	20	60	1	1	0
20	.97700	1.02355	40								
′	Co-tan. 45°	Tan. 45°	′	′	Co-tan. 45°	Tan. 45°	′	′	Co-tan. 45°	Tan. 45°	′

Table XV.—Trigonometric Formulas



Right Triangles

$$\sin A = \frac{a}{c} = \cos B \qquad \sec A = \frac{c}{b} = \operatorname{cosec} B$$

$$\cos A = \frac{b}{c} = \sin B \qquad \operatorname{cosec} A = \frac{c}{a} = \sec B$$

$$\tan A = \frac{a}{b} = \cot B \qquad \operatorname{vers} A = \frac{c-b}{c} = \frac{d}{c}$$

$$\cot A = \frac{b}{a} = \tan B \qquad \operatorname{exsec} A = \frac{e}{c}$$

$$a = c \sin A = c \cos B = b \tan A = b \cot B = \sqrt{c^2 - b^2}$$

$$b = c \cos A = c \sin B = a \cot A = a \tan B = \sqrt{c^2 - a^2}$$

$$c = \frac{a}{\sin A} = \frac{a}{\cos B} = \frac{b}{\sin B} = \frac{b}{\cos A} = \frac{d}{\operatorname{vers} A} = \frac{e}{\operatorname{exsec} A} = \sqrt{a^2 + b^2}$$

$$d = c \operatorname{vers} A \qquad e = c \operatorname{exsec} A$$

Oblique Triangles

Given	Sought	Formulas
A, B, a	b, c	$b = \frac{a}{\sin A} \cdot \sin B \qquad c = \frac{a}{\sin A} \cdot \sin (A + B)$
A, a, b	B, c	$\sin B = \frac{\sin A}{a} \cdot b \qquad c = \frac{a}{\sin A} \cdot \sin C$
C, a, b	$\frac{1}{2}(A + B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2}C$
	$\frac{1}{2}(A - B)$	$\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \cdot \tan \frac{1}{2}(A + B)$
a, b, c	A	If $s = \frac{1}{2}(a + b + c)$, $\sin \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}$ $\cos \frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$, $\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ $\sin A = 2\sqrt{\frac{s(s-a)(s-b)(s-c)}{bc}}$ $\operatorname{vers} A = \frac{2(s-b)(s-c)}{bc}$
	area	*area* $= \sqrt{s(s-a)(s-b)(s-c)}$
C, a, b	*area*	*area* $= \frac{1}{2}ab \sin C$
A, B, C, a	*area*	*area* $= \frac{a^2 \sin B \cdot \sin C}{2 \sin A}$

INDEX

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HEI-INV

TRA-TRI